Determinizing timed automata

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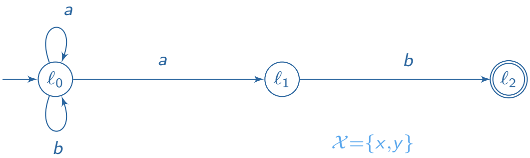
Timed automata on example

Timed automaton: Finite automaton enriched with clocks.



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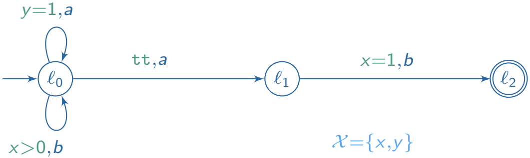

Transitions are equipped with guards

Timed automata on example

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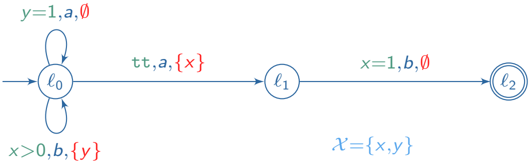

Transitions are equipped with guards and sets of reset clocks.

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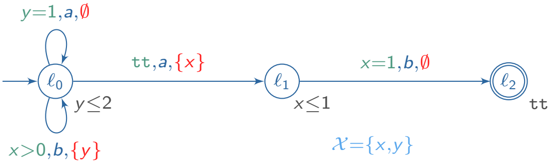
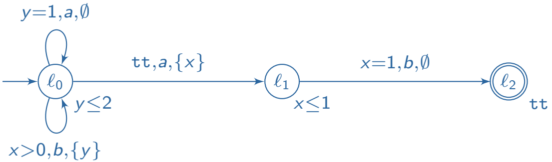


Transitions are equipped with guards and sets of reset clocks.
Locations are equipped with invariants.

Syntax



Timed automata

A timed automaton is a tuple $\mathcal{A} = (L, L_0, L_{acc}, \Sigma, X, E, Inv)$ with

- L finite set of locations
- $L_0 \subseteq L$ initial locations and $L_{acc} \subseteq L$ set of accepting locations
- Σ finite alphabet and X finite set of clocks
- $E \subseteq L \times \mathcal{G} \times \Sigma \times 2^X \times L$ set of edges is the set of guards. (with $\bowtie \in \{<, \leq, =, \geq, >\}$)
- $Inv : L \to \mathcal{G}$ invariant function

Semantics

Valuation: $v \in \mathbb{R}_+^X$ assigns to each clock a clock-value

State: $(\ell, v) \in L \times \mathbb{R}_+^X$ composed of a location and a valuation.

Transitions between states of $\mathcal{A}$:

- Delay transitions: $(\ell, v) \xrightarrow{\tau} (\ell, v + \tau)$
- Discrete transitions: $(\ell, v) \xrightarrow{a} (\ell', v')$

  if $\exists(\ell, g, a, Y, \ell') \in E$ with $v \models g$ and $\begin{cases} v'(x) = 0 & \text{if } x \in Y, \\ v'(x) = v(x) & \text{otherwise.} \end{cases}$

Run of $\mathcal{A}$:
$(\ell_0, v_0) \xrightarrow{\tau_1} (\ell_0, v_0 + \tau_1) \xrightarrow{a_1} (\ell_1, v_1) \xrightarrow{\tau_2} (\ell_1, v_1 + \tau_2) \xrightarrow{a_2} \cdots \xrightarrow{a_k} (\ell_k, v_k)$
or simply: $(\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \xrightarrow{\tau_2, a_2} \cdots \xrightarrow{\tau_k, a_k} (\ell_k, v_k)$

Timed language

Time sequence: $\mathbf{t} = (t_i)_{1 \le i \le k}$ finite non-decreasing sequence over $\mathbb{R}_+$.

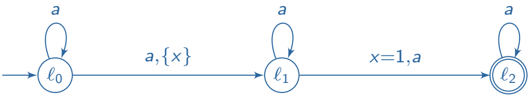
Timed word: $w = (\sigma, \mathbf{t}) = (a_i, t_i)_{1 \le i \le k}$ where $a_i \in \Sigma$ and $\mathbf{t}$ time sequence.

Accepted timed word

A timed word $w = (a_0, t_0)(a_1, t_1) \dots (a_k, t_k)$ is accepted in $\mathcal{A}$, if there is a run $\rho = (\ell_0, v_0) \xrightarrow{\tau_0, a_0} (\ell_1, v_1) \xrightarrow{\tau_1, a_1} \dots (\ell_{k+1}, v_{k+1})$ with $\ell_0 \in L_0$, $\ell_{k+1} \in L_{acc}$, and $t_i = \sum_{j<i} \tau_j$.

Accepted timed language: $\mathcal{L}(\mathcal{A}) = \{w \mid w \text{ accepted by } \mathcal{A}\}$.

An example



$w = (a, 0.1)(a, 0.3)(a, 1.1)(a, 1.2)(a, 1.3)$ is an accepted timed word

Example of an accepting run for w:

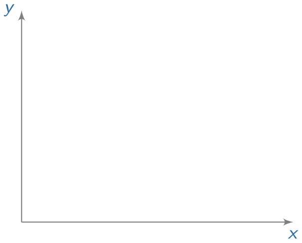
$(\ell_0, 0) \xrightarrow{0.1,a} (\ell_1, 0) \xrightarrow{0.2,a} (\ell_1, 0.2) \xrightarrow{0.8,a} (\ell_2, 0) \xrightarrow{0.1,a} (\ell_2, 0.1) \xrightarrow{0.1,a} (\ell_2, 0.3)$

$$\mathcal{L}(\mathcal{A}) = \{(a, t_1) \cdots (a, t_k) \mid \exists i < j,\ t_j - t_i = 1\}$$

Regions

Problem: Timed automata have infinite state space.

Regions form a finite partition of the set of valuations



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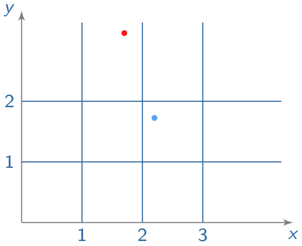

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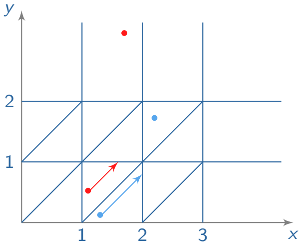

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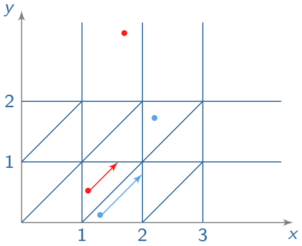
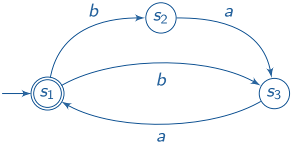


Emptiness problem

Emptiness is decidable for timed automata and is PSPACE-complete.

Determinizing finite automata: Subset construction



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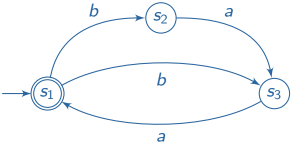
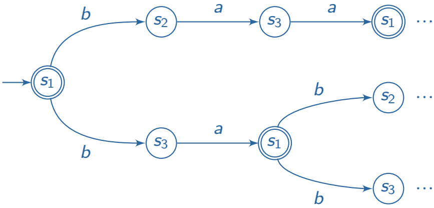


Unfolding the automaton



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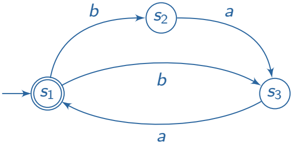

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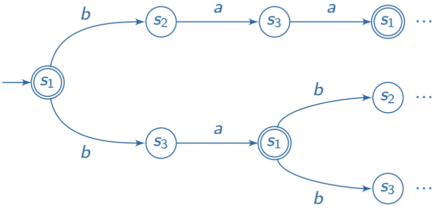
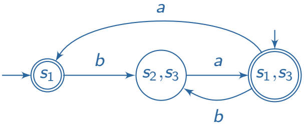
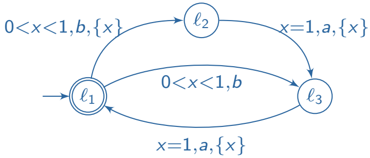


Deterministic equivalent



Naive adaptation to timed automata



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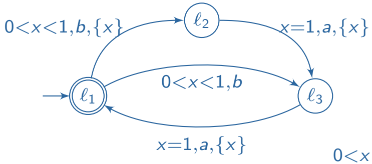

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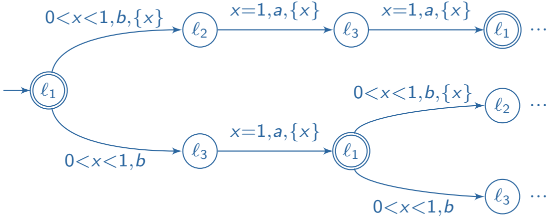

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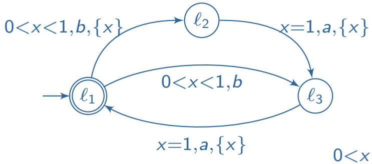

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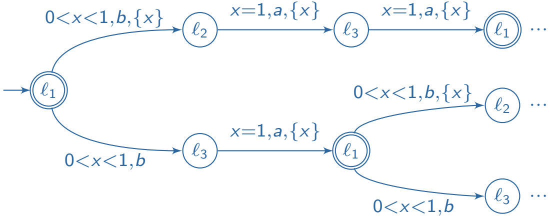
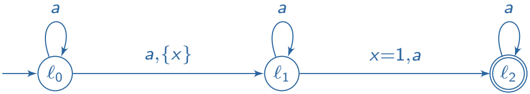


Subset construction fails because of non-uniform clock resets.

Determinizability of timed automata

Some timed automata are not determinizable [AD90].



$$\mathcal{L}(\mathcal{A}) = \{(a, t_1)\ldots(a, t_n) \mid n \geq 2 \text{ and } \exists i < j \text{ s.t. } t_j - t_i = 1\}$$

Theorem [Finkel 06]

Checking whether a given timed automata is determinizable is undecidable, even under fixed resources.

Workarounds

Two approaches to overcome unfeasible determinization:

- exhibit determinizable subclasses
- perform an approximate determinization

1. Determinizable subclasses
2. Determinization procedure
3. Overapproximate determinization
4. A game approach

1. Determinizable subclasses
   - Event-recording automata
   - Integer-reset timed automata
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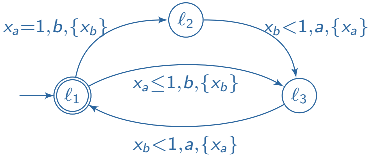
Event-recording automata [AFH94]

Finite automata with one clock associated with each action $a \in \Sigma$ which is reset exactly when action a occurs.



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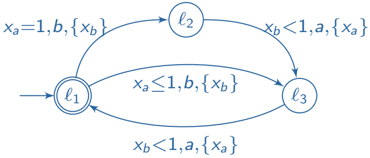

Input-determinacy property

Valuation only depends on input word, not on the precise execution.

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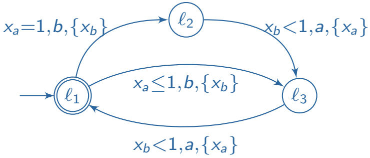

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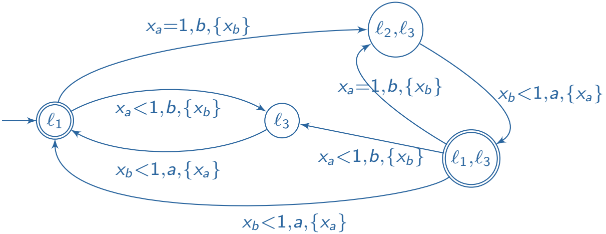
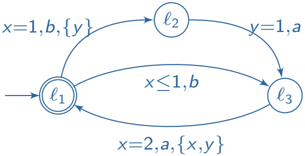
Determinization via subset construction



Integer-reset timed automata [SPKM08]

Resets only allowed when some clock value equals a constant.



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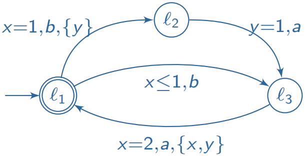

Tick property All clocks share the same fractional part.

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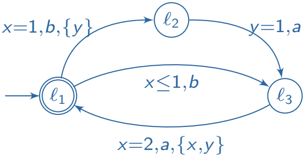

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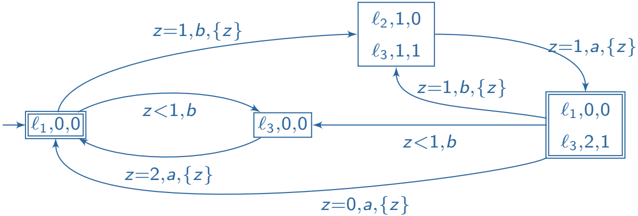
Determinization with a single clock!



1. Determinizable subclasses
2. Determinization procedure
   - Overview
   - Details
3. Overapproximate determinization
4. A game approach

Determinization procedure

joint work w. Baier, Bouyer, Brihaye

Approach overview

- unfolding of the automaton, introducing a fresh clock at each step, into a timed tree with infinitely many clocks and nodes
- symbolic determinization
- reduction of the number of clocks (under some assumption) and folding back into an automaton
- effective algorithm with fixed upper bound on resources.

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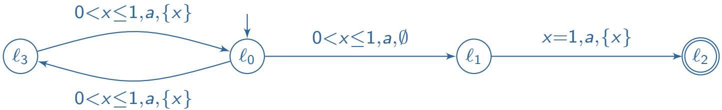
Essential features

- in each location of the new automaton, original clocks are mapped to new clocks
- termination of the procedure is not guaranteed
- exact determinization

Unfolding

- Unfolding into an infinite timed tree with a fresh clock at each step.
- Original clocks are mapped to their reference in the new set of clocks.



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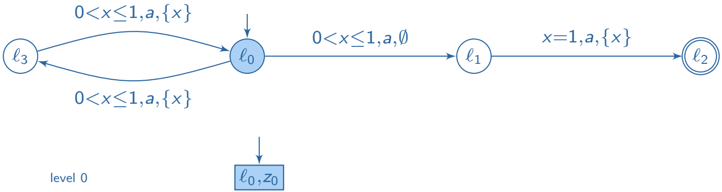

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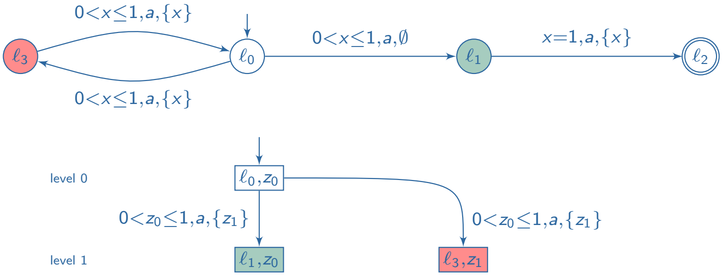

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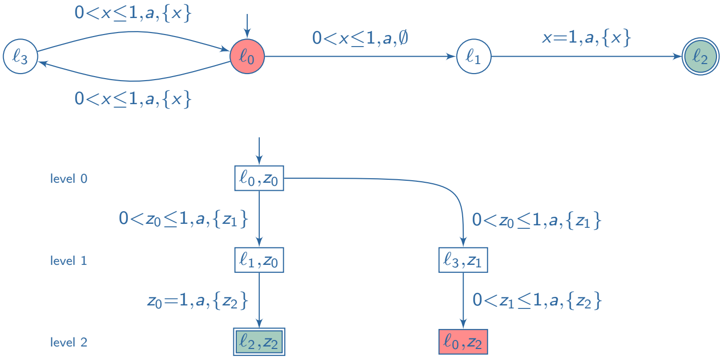

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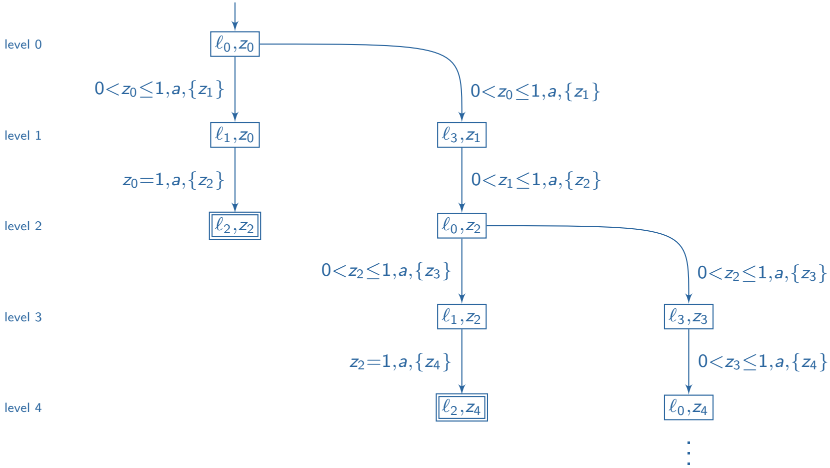
level 0
level 1
level 2
level 3
level 4
ℓ_0, z_0
$0 < z_0 \leq 1, a, \{z_1\}$
$0 < z_0 \leq 1, a, \{z_1\}$
ℓ_1, z_0
ℓ_3, z_1
$z_0 = 1, a, \{z_2\}$
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ℓ_2, z_2
ℓ_0, z_2
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ℓ_2, z_4
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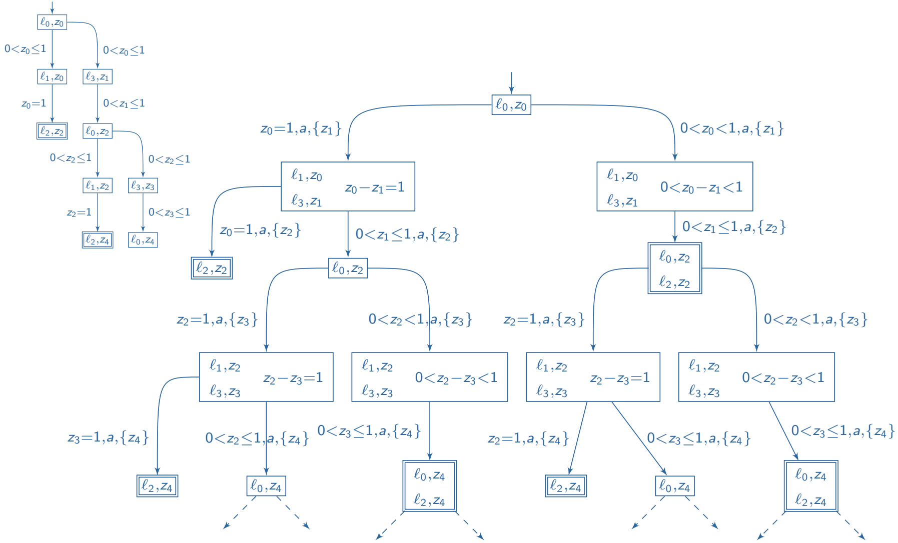
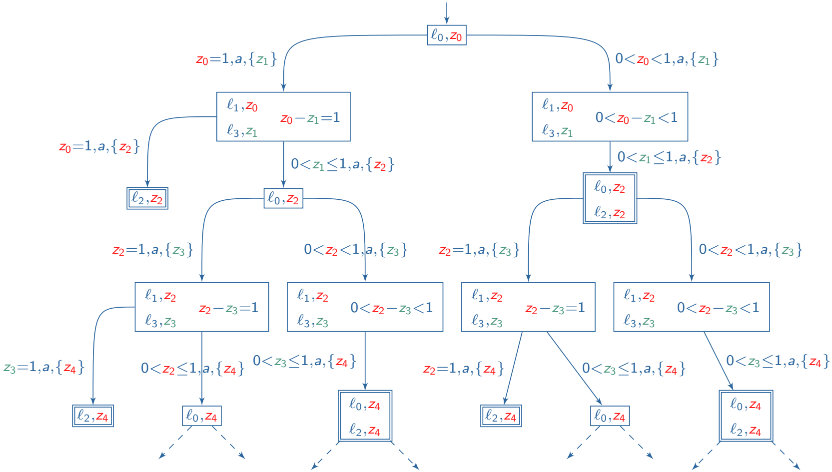
Symbolic determinization



Clock reduction

Two clocks are sufficient to encode all timing information!



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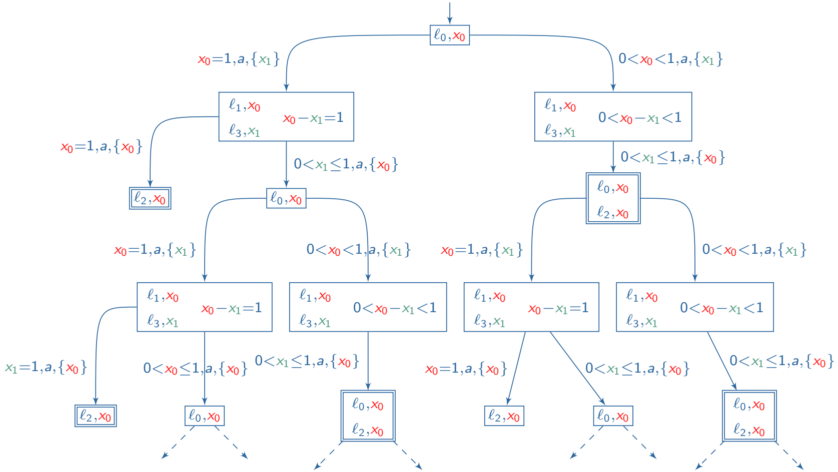
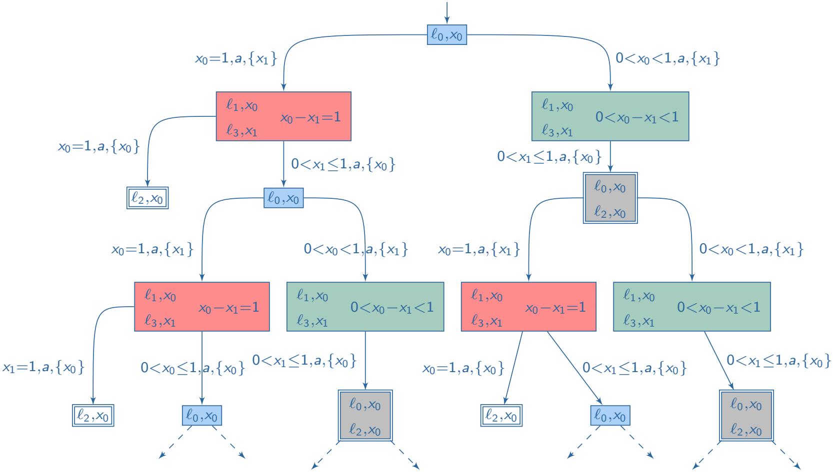


Renaming: $z_{2n} \leftarrow x_0$ and $z_{2n+1} \leftarrow x_1$

Location reduction

Finitely many node labels.



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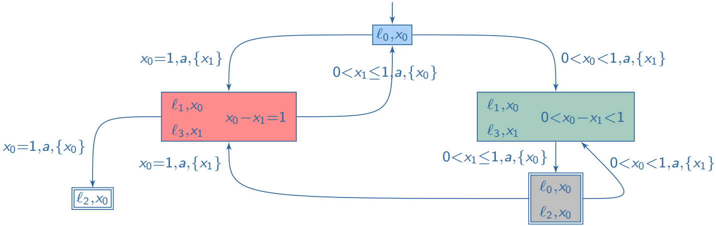

Merging of isomorphic nodes

Overapproximate determinization [KT09]

Approach overview

- observation of the behaviour using a new clock, reset at each step
- over-approximation of the guards according to the new clock
- estimation of the possible current states

- can be extended to several observation clocks (resets fixed by DFA)

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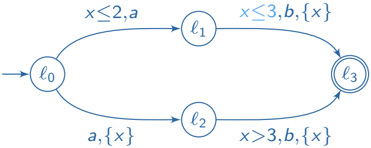
Essential features

- fixed resources (number of clocks and maximal constant)
- flexible relations between old and new clocks
- no assumptions for termination
- deterministic over-approximation

Overapproximation on an example



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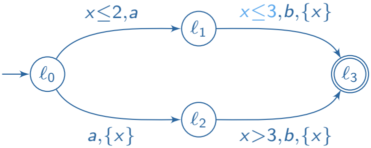

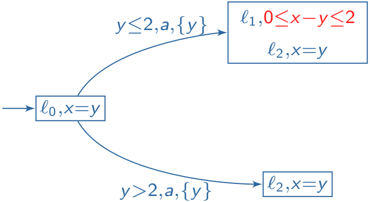

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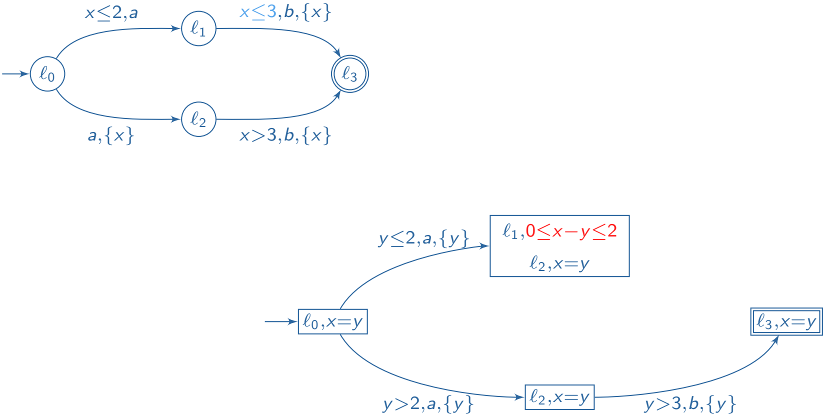

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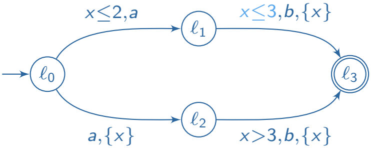

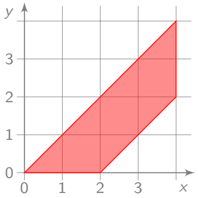

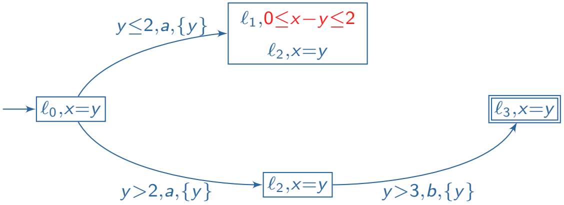

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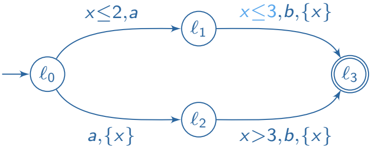
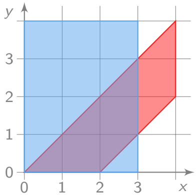
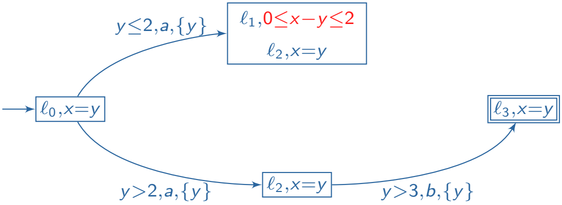
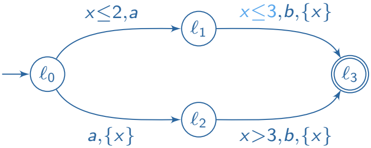
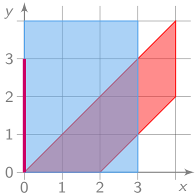


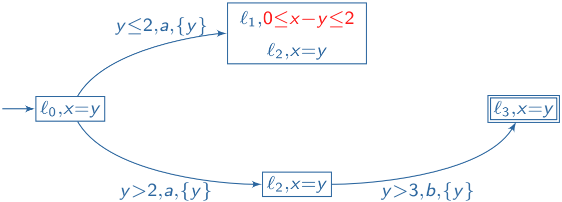




Overapproximation on an example







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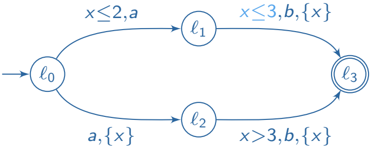

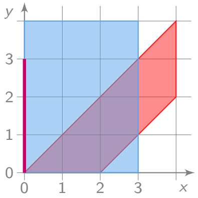

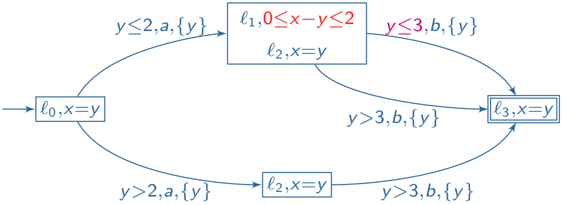

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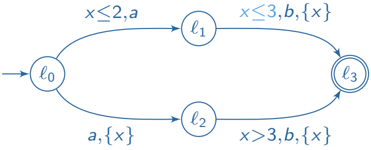
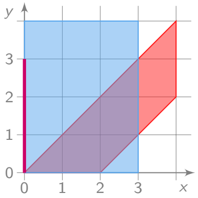
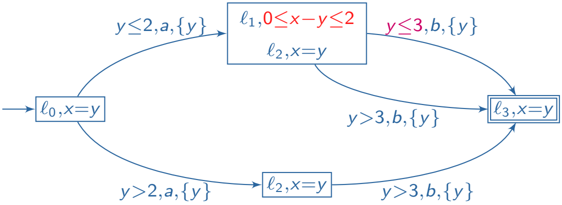






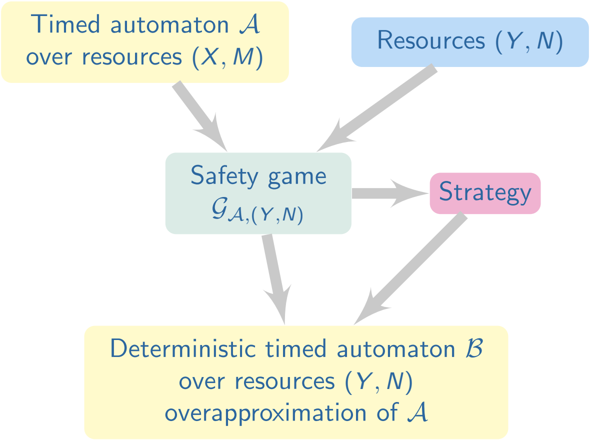
Timed word $(a, 0.6)(b, 3.2)$ is accepted in the deterministic over-approximation but not in the original timed automaton.

A game approach

joint work w. Stainer, Jéron, Krichen

- Goal: extend existing approaches
  - fixed resources (number of clocks and maximal constant)
  - determinization or deterministic over-approximation
- Method
  - inspired by [BCD05] for diagnosis of timed automata
  - turn-based game to choose when to reset the new clocks
  - coding of the relations between old and new clocks similar to [KT09]

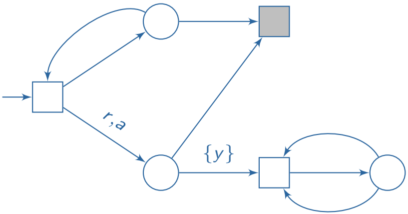
Overview of the approach



Closer look to the game

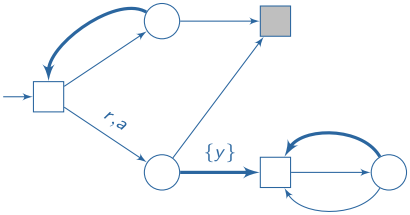
Finite turn-based safety game between Spoiler and Determinizator.

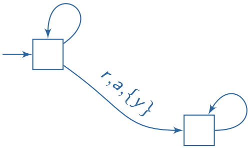
- First, Spoiler chooses an action and when to fire it (region over the new clocks)
- Then, Determinizator chooses which (new) clocks to reset
- States are unsafe when an over-approximation possibly happened
- Determinizator wants to avoid unsafe states



Properties of the game

Any strategy for Determinizator yields a timed automaton by merging each move of Spoiler with the next move of Determinizator.





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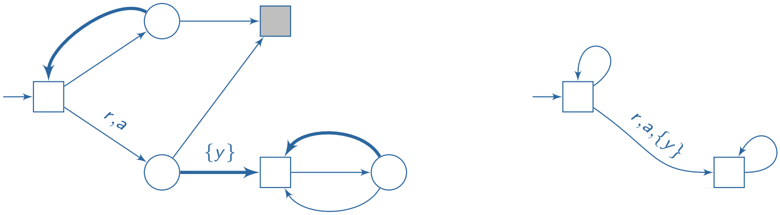


Properties

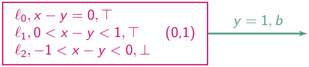
- Every strategy for Determinizator yields a deterministic over-approximation.
- Every **winning** strategy for Determinizator yields a deterministic equivalent.

States and moves: Spoiler

States of Spoiler (□-states):

- a set of configurations each with a marker
  - configuration: location + relation between old and new clocks
    relation: conjunction of $x - y \equiv c$
  - marker: $\top$ or $\perp$ to indicate possible over-approximations
- a region (over the new set of clocks)

Moves: Spoiler chooses a successor region and an action.



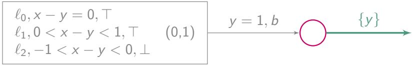
Unsafe states: □-states of the form $(\{\ell_i, C_i, \perp\}_{i \in I}, r)$

States and moves: Determinizator

States of Determinizator (◯-states):

- a state of Spoiler + a region over new clocks + an action

Moves: Determinizator chooses a reset set.



States' update

Given a ◯-state and a reset set, how to compute the next □-state?

States' update

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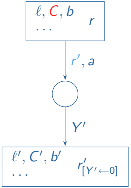
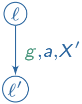
- For each configuration ℓ, C, b, given moves (r', a) of ☐ and Y' of ◯





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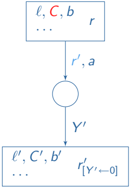
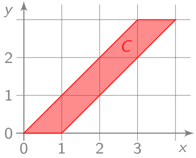
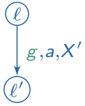




C: $0 < x - y < 1$



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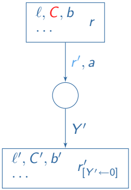
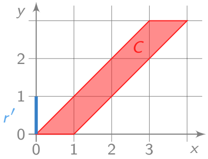
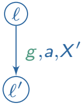




C: $0<x-y<1$ r': $0<y<1$



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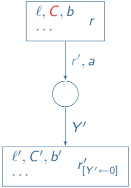

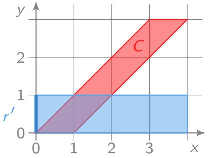

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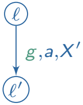

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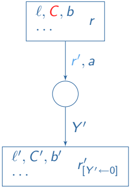

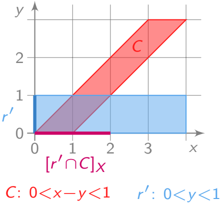

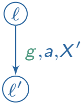

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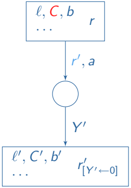
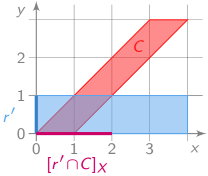
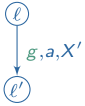




C: $0 < x - y < 1$ $\quad$ r': $0 < y < 1$



$[r' \cap C]_X \cap g \neq \emptyset$

States' update

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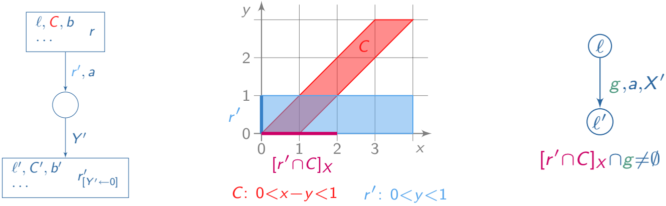
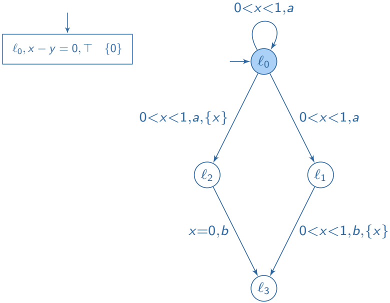


- for each transition $\ell \xrightarrow{g,a,X'} \ell'$ with $[r' \cap C]_{|X} \cap g \neq \emptyset$ build a successor configuration ℓ', C', b'
  - C' updates C according to r', g, X', Y': $C' = \overleftrightarrow{(r' \cap C \cap g)}_{[X' \leftarrow 0][Y' \leftarrow 0]}$
  - b' indicates possible over-approximations: $b' = b \wedge ([r' \cap C]_{|X} \cap \neg g = \emptyset)$

Game on the example

Construction of the game with resources $(y, 1)$



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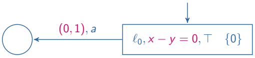

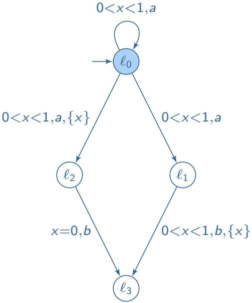

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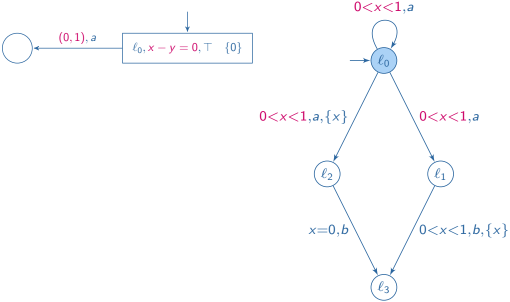

$y \in (0,1) \wedge x - y = 0 \implies x \in (0,1)$

no overapproximation

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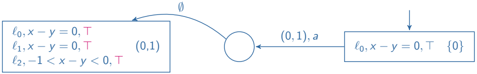

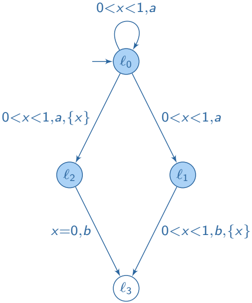

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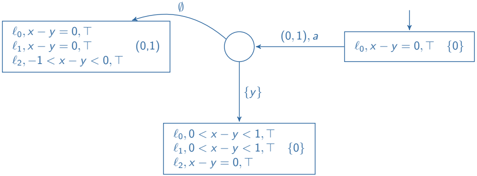

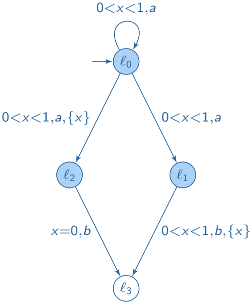

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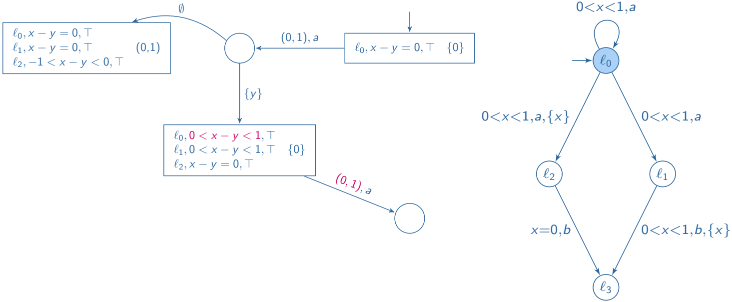

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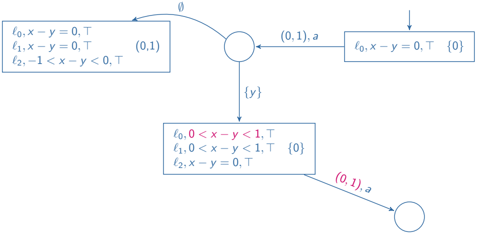

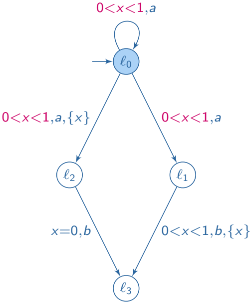

$y \in (0,1) \wedge 0 < x-y < 1 \implies 0 < x < 2$

overapproximation

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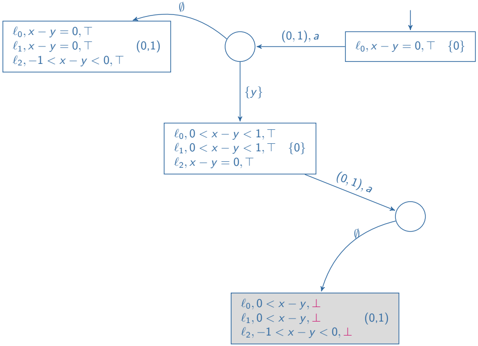

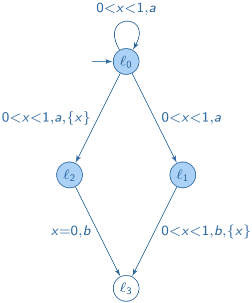

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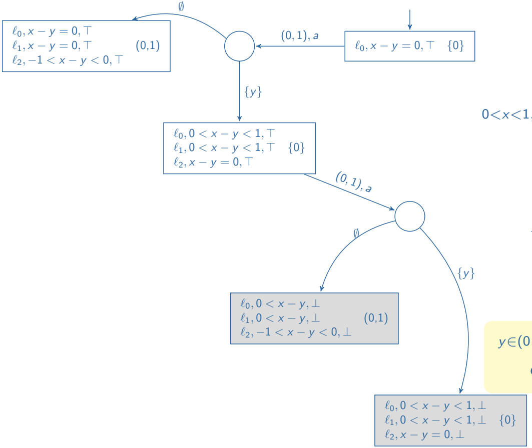

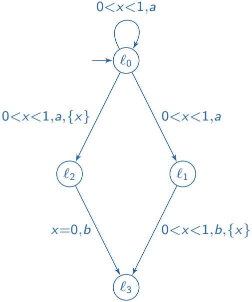

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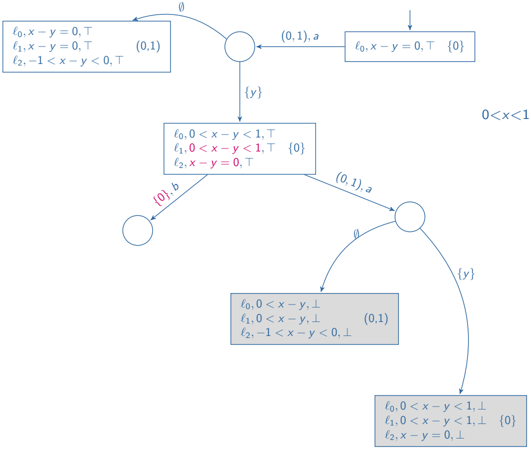

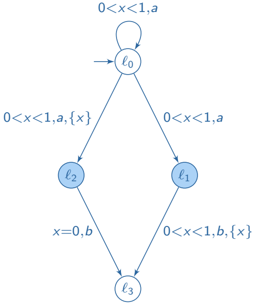

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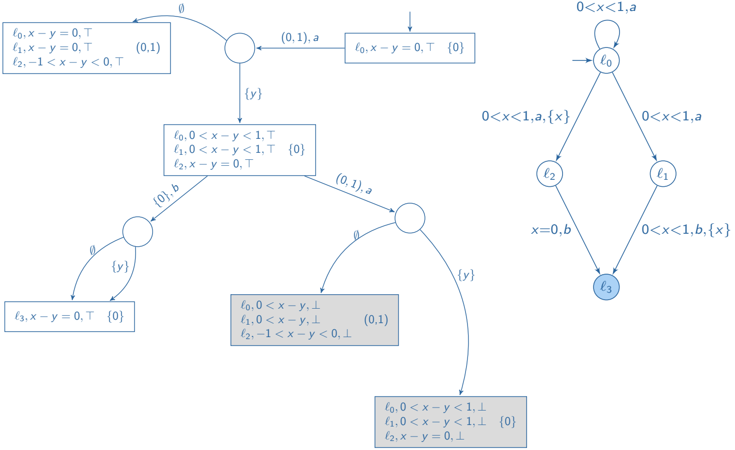

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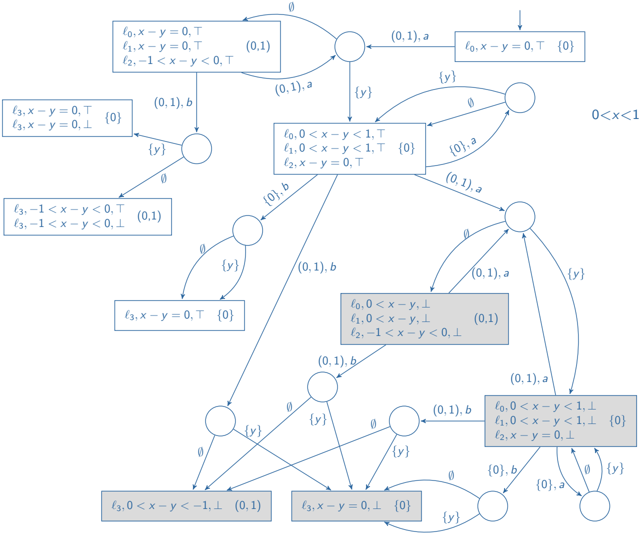
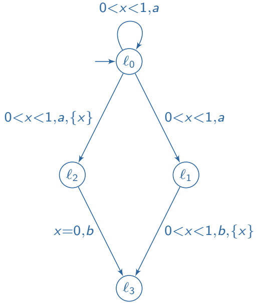
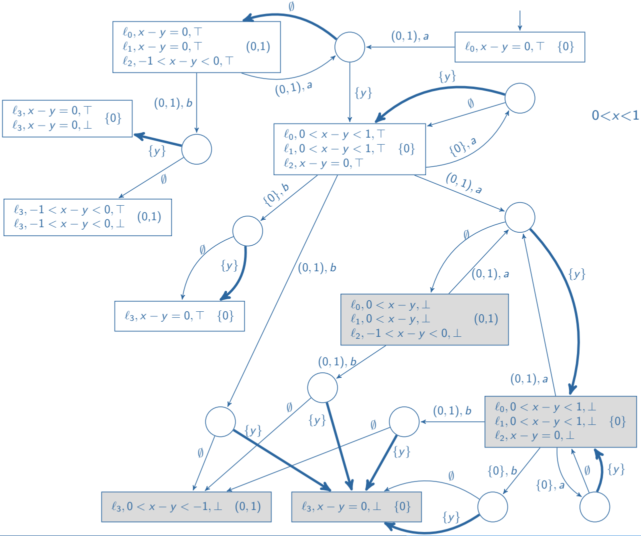
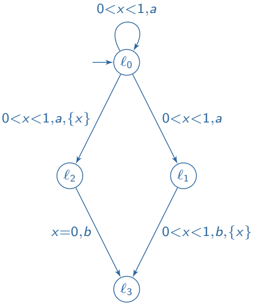




Resolution of the game

Winning strategy for Determinizator





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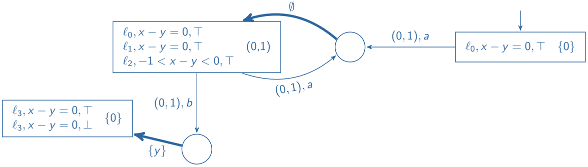
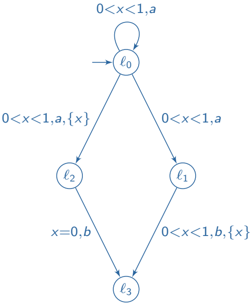
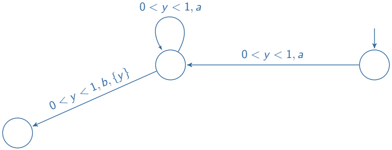




Deterministic equivalent



Comparison with other approaches

- More precise than the over-approximation of [KT09]
  - general strategies compared to *a priori* fixed blind ones
  - determinism is preserved (under sufficient resources)

→ Exact determinization in more cases.

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- More precise than the over-approximation of [KT09]
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$\longrightarrow$ Exact determinization in more cases.

- More general than the determinization procedure of [BBBB09]
  - relations are more expressive than mapping
  - dealing with some traces inclusion thanks to marking

$\longrightarrow$ Strictly more timed automata can be determinized.
$\longrightarrow$ Some timed automata are determinized with less resources.

Details

Limits

No winning strategy $\not\Rightarrow$ no deterministic equivalent

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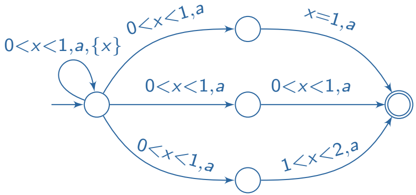
- Example

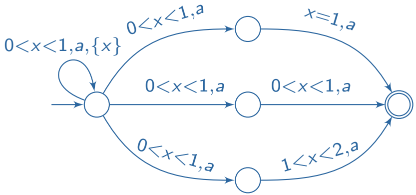


  - no winning strategy (with resources (1,1))
  - but some losing strategy yields a deterministic equivalent

Limits

No winning strategy $\not\Rightarrow$ no deterministic equivalent

- Example



  - no winning strategy (with resources (1,1))
  - but some losing strategy yields a deterministic equivalent

- How to choose a good losing strategy?
  - when possible, use language inclusion
  - heuristic: maximize distance to unsafe states
  - other possibilities: use quantities on timed languages such as volume

Summary

Determinization of timed automata is impossible in general, and determinizability is undecidable.

Approaches:

- determinizable subclasses (event-clock, integer reset)
- determinization procedure
  - terminates under some condition
  - produces a deterministic equivalent on known determinizable subclasses
- overapproximate determinization
  - produces a deterministic overapproximation
  - always terminates
- game-based approach

Recent contribution

Game-based approach to (approximately) determinize timed automata

- improves existing approaches
  - more timed automata determinized
  - exact determinization in more cases
  - less resources needed
- deals with timed automata with ε-transitions and invariants
- extension to timed automata with inputs and outputs
  $\longrightarrow$ application to testing

References

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When are timed automata determinizable?
ICALP 2009

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Conformance testing for real-time systems
Formal Methods in System Design, 2009

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Fault diagnosis using timed automata
FoSSaCS 2005

B., Stainer, Jéron, Krichen
A game approach to determinize timed automata
FoSSaCS 2011

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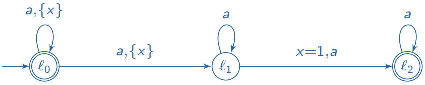
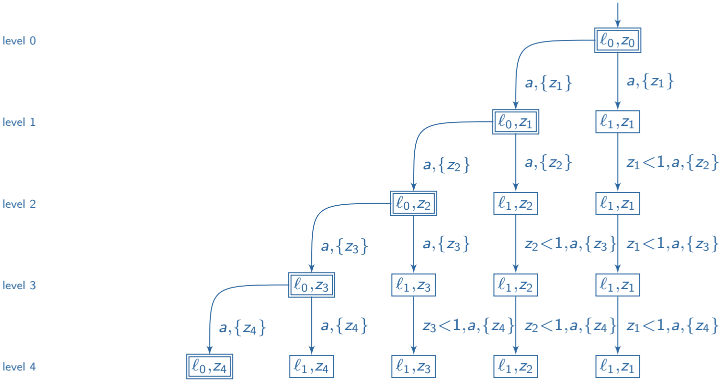
Off-line test selection with test purposes for non-deterministic timed automata
TACAS 2011

Detailed comparison





Determinization procedure fails.

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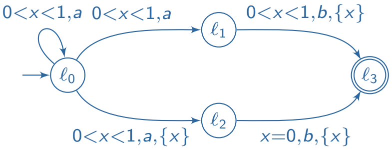
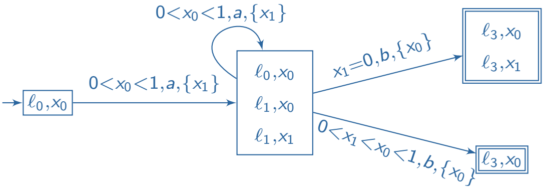
Detailed comparison





Determinization procedure produces a 2-clock automaton.

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