Fault diagnosis for Probabilistic Systems

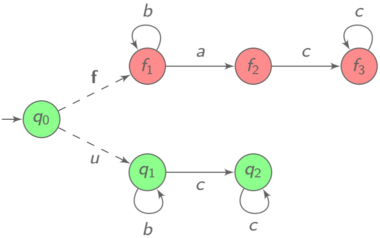
Nathalie Bertrand

Inria Rennes, France

based on joint work with Éric Fabre, Stefan Haar, Serge Haddad, Loïc Hélouët and Engel Lefaucheux

Fault diagnosis in discrete event systems [SSLST95]

Objective: tell whether a fault occurred, based on observations.



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$\Sigma_o = \{a, b, c\}$ observables

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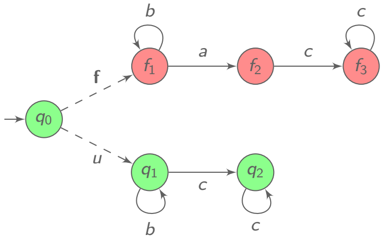

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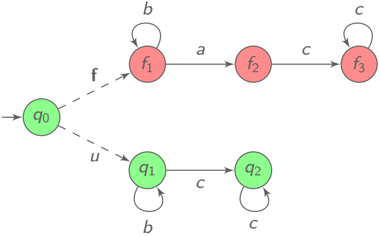

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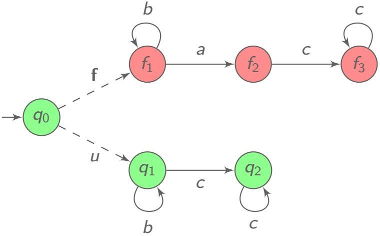

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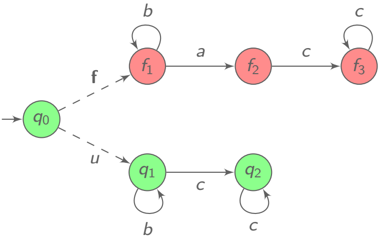

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Remark: w.l.o.g. state space partitionned into correct and faulty states

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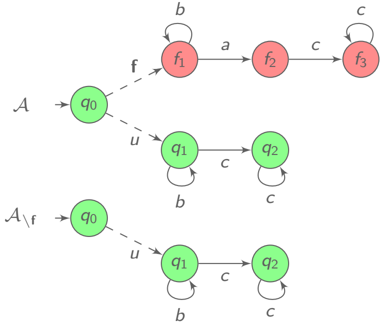

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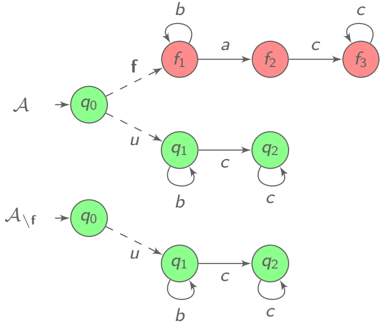

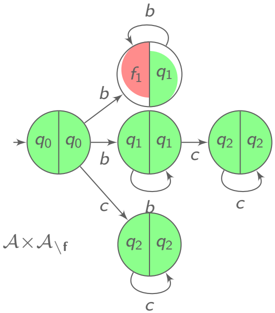

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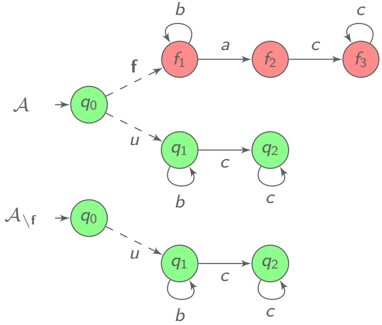

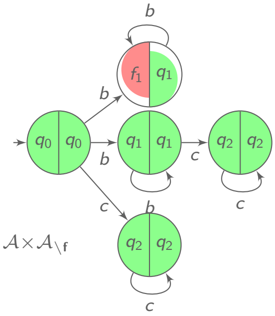

indeterminate cycle: $(f_0, q_0) \cdots \to (f_n, q_n) \to (f_0, q_0)$ s.t. f_i faulty and q_i correct

$\mathcal{A}$ is not diagnosable iff
there exists a reachable indeterminate cycle in $\mathcal{A} \times \mathcal{A}_{\setminus \mathbf{f}}$.

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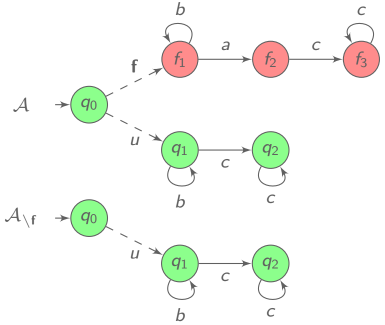

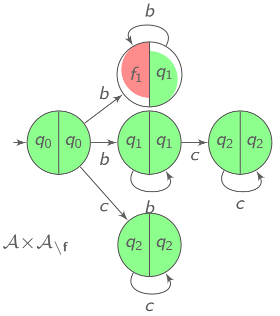

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Decidability and complexity of diagnosability [JHCK01]
Diagnosability is decidable in PTIME.

[JHCK01] Jiang, Huang, Chandra and Kumar, *A polynomial algorithm for testing diagnosability of discrete-event systems*, TAC, 2001.

Diagnosers

Diagnoser: assigns verdicts to observed sequences $D : \Sigma_o^* \to \{\checkmark, \text{✗}, ?\}$

Diagnoser requirements

- **Soundness**: if a fault is claimed, a fault occurred.
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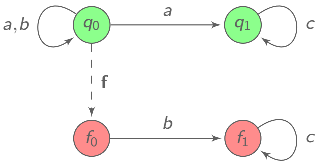

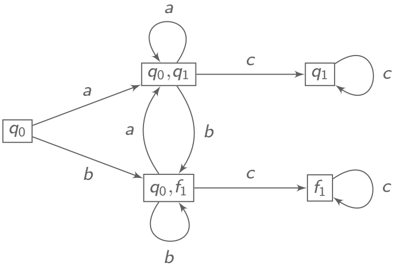

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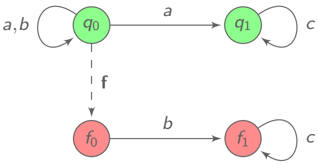

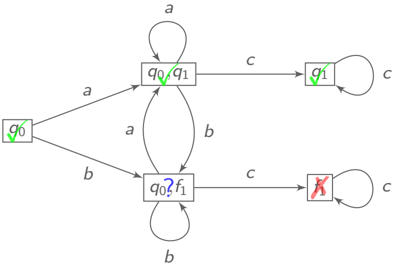

Diagnoser synthesis

Complexity of diagnoser synthesis [JHCK01]
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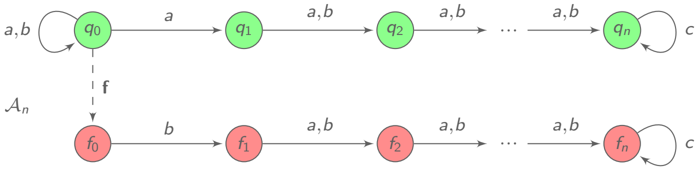
There is a family $(\mathcal{A}_n)$ of diagnosable systems such that $\mathcal{A}_n$ has $2n+2$ states and any diagnoser needs 2^n states.



diagnoser must remember the last n events: 2^n possibilities

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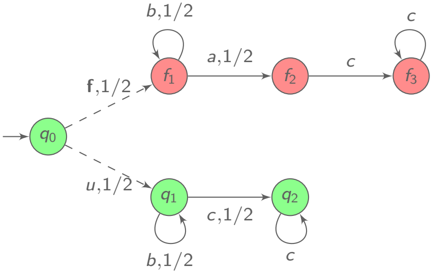
Outline

Diagnosis of probabilistic systems



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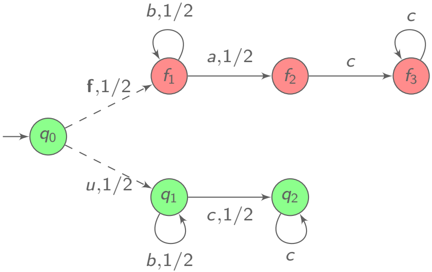

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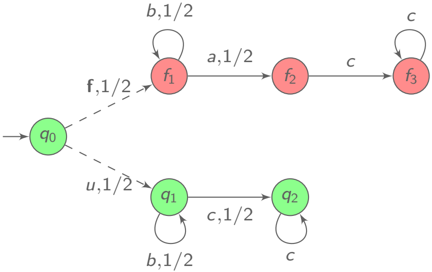

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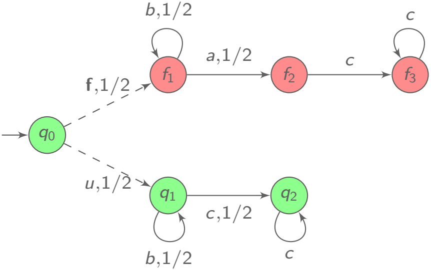

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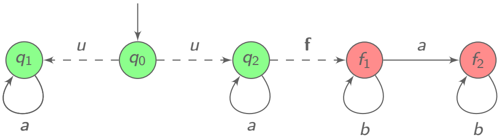
Diagnosability: Probability of infinite ambiguous sequences is zero.

[TT05] Thorsley and Teneketzis, *Diagnosability of stochastic discrete-event systems*, TAC, 2005.
[CK13] Chen and Kumar, *Polynomial test for stochastic diagnosability of dicrete-event systems*, TASE, 2013.
[BHL14] **B.**, Haddad and Lefaucheux, *Foundation of diagnosis and predictability in probabilistic systems*, FSTTCS'14.

Characterisation of diagnosability

$\mathcal{A}$



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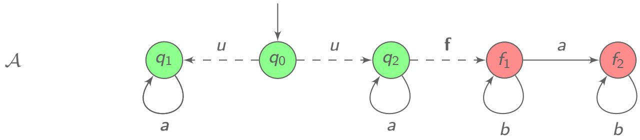
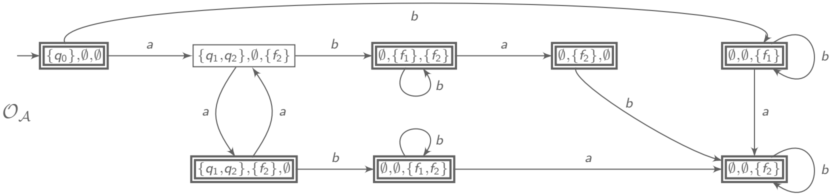


deterministic Büchi automaton $\mathcal{O}_\mathcal{A}$ characterizes non-ambiguous sequences



states (U, V, W) with U possible correct states; V waiting room for possible faulty states; W possible faulty states for latest faults [HHMS13]

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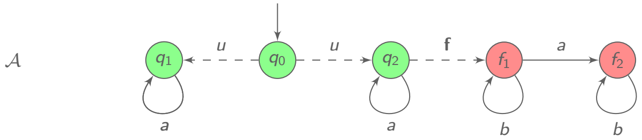

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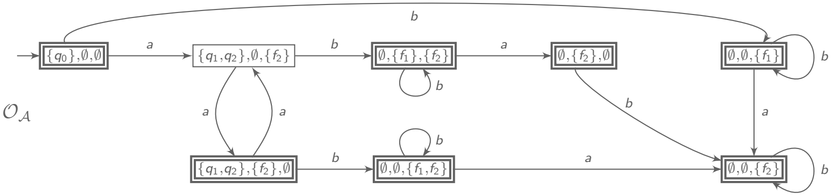

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$\mathcal{A}$ is not diagnosable iff
there exists a BSCC of $\mathcal{A} \times \mathcal{O}_\mathcal{A}$ where every state (q, U, V, W) satisfies
q faulty and $U \neq \emptyset$ or q correct and $W \neq \emptyset$.

[HHMS13] Haar, Haddad, Melliti and Schwoon, *Optimal constructions for active diagnosis*, FSTTCS'13.

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Diagnosability is in PSPACE for probabilistic systems.

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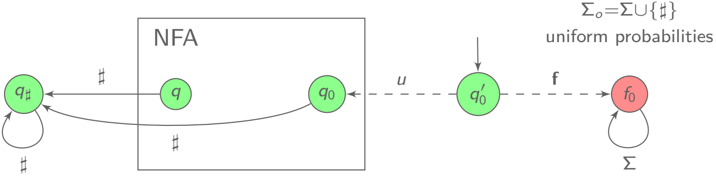

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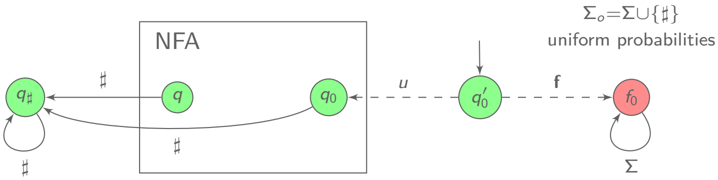

Diagnosability is PSPACE-complete for probabilistic systems. [BHL14]

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For every diagnosable system with n correct states
one can build a diagnoser with at most 3^n states.

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Diagnoser derived from observer $\mathcal{O}_{\mathcal{A}}$: ✗ in states $(\emptyset, V, W)$
✓ in states $(U, V, \emptyset)$ with $U \neq \emptyset$
? otherwise.

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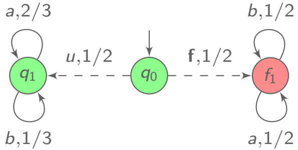
Accurate approximate diagnosis



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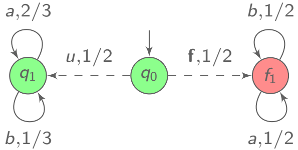

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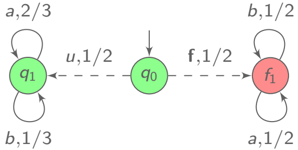

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[TT05] Thorsley and Teneketzis, *Diagnosability of stochastic discrete-event systems*, TAC, 2005.
[BHL16?] **B.**, Haddad and Lefaucheux, *Accurate approximate diagnosability of stochastic systems* , submitted.

Outline

Introduction to fault diagnosis

Diagnosability in probabilistic systems

Control for probabilistic diagnosability

Conclusion

Active diagnosis

Objective: control the system so that it is diagnosable

[SLT98] Sampath, Lafortune and Teneketzis, *Active diagnosis of discrete-event systems*, TAC, 1998.
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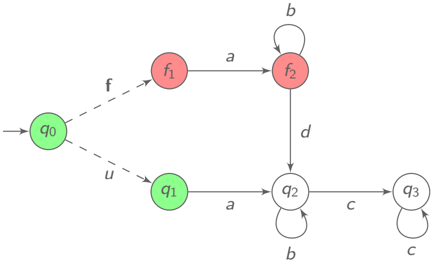

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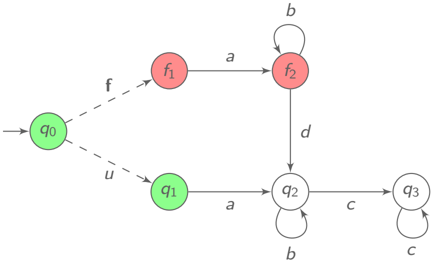

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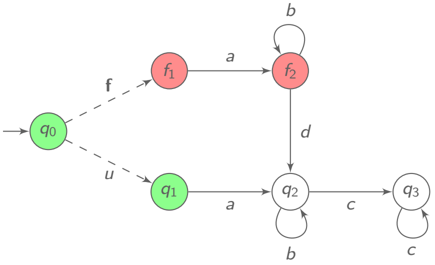

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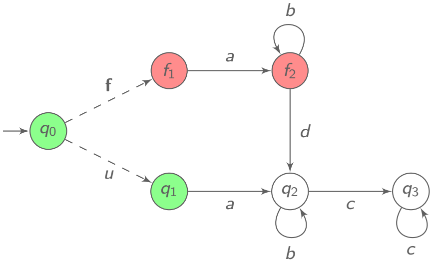

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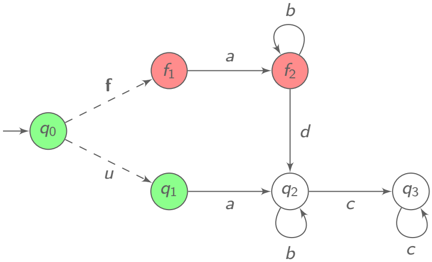

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Active diagnosis problem
does there exist a controller such that the system is diagnosable?

caution: the system must remain *live*.

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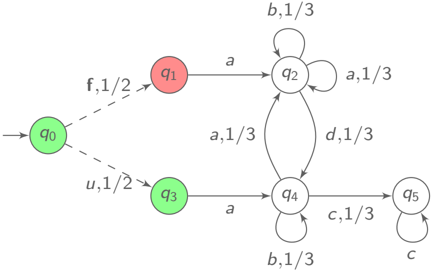
Active probabilistic diagnosis

Objective: control the system so that it is almost-surely diagnosable



Active probabilistic diagnosis

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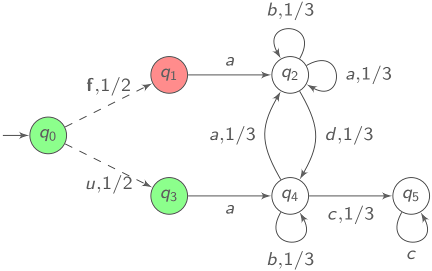

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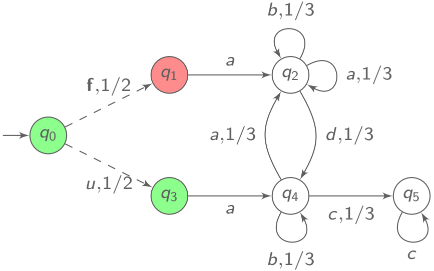

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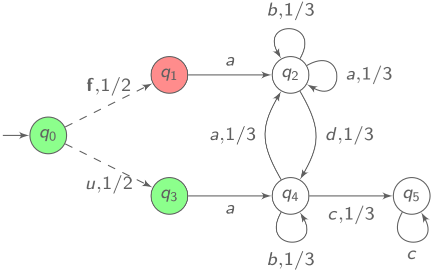

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Controller $\sigma : \Sigma_o^* \to \mathsf{Dist}(2^{\Sigma_c})$

Active probabilistic diagnosis problem [BFHHH14]
does there exist a controller such that the system is almost-surely diagnosable?

[BFHHH14] **B.**, Fabre, Haar, Haddad and Hélouët, *Active diagnosis for probabilistic systems*, FoSSaCS'14.

Solving active probabilistic diagnosis

The active probabilistic diagnosis problem is EXPTIME-complete.

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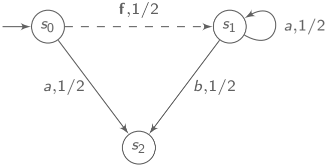
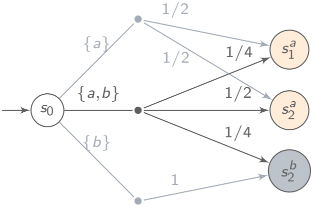
Proof idea (upper bound)

- characterize unambiguous sequences by deterministic Büchi automaton $\mathcal{B}$ [HHMS13]
- build the product of probabilistic LTS with $\mathcal{B}$
- view it as POMDP $\mathcal{P}$





- decide whether there is an almost-surely winning strategy for the Büchi condition on $\mathcal{P}$ [BBG08,CDGH10]

[HHMS13] Haar, Haddad, Melliti and Schwoon, *Optimal constructions for active diagnosis*, FSTTCS'13.
[BBG08] Baier, **B.** and Größer, *On decision problems for probabilistic Büchi automata*, FoSSaCS'08.
[CDGH10] Chatterjee, Doyen, Gimbert and Henzinger, *Randomness for free*, MFCS'10.

Safe active probabilistic diagnosis

Objective: avoid fault-provocative controllers

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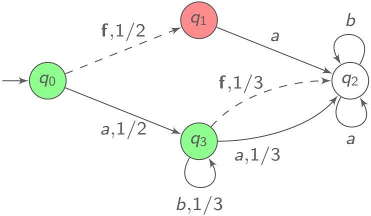

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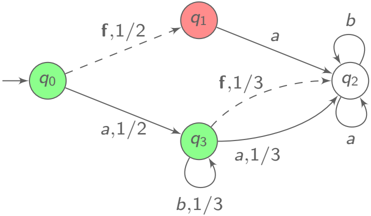

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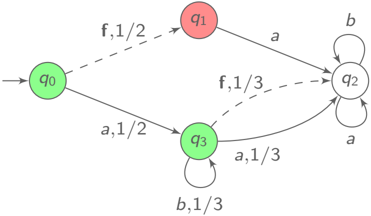

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Safe active probabilistic diagnosis [BFHHH14]
does there exist a controller such that the system is almost-surely diagnosable **and** correct runs have positive probability?

[BFHHH14] **B.**, Fabre, Haar, Haddad and Hélouët, *Active diagnosis for probabilistic systems*, FoSSaCS'14.

Beliefs are not enough!

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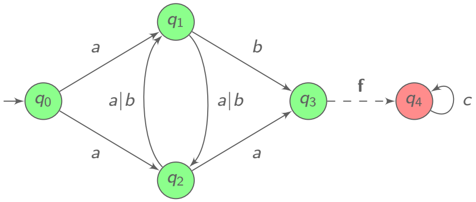

Finite-memory is not enough!

Infinite memory is needed for safe probabilistic diagnosis.

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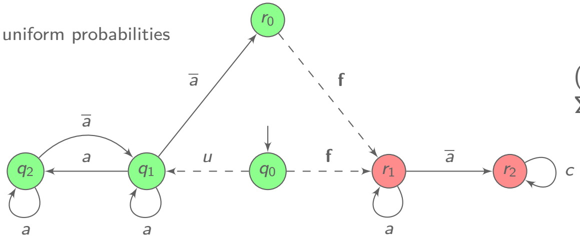

$(a^+\overline{a})^\omega$	✓	correct
$\Sigma_{\mathrm{obs}}^* c^\omega$	✗	faulty

- Safe controller: infinitely many $\overline{a}$'s to diagnose all faults...
  but not too often, to have non-negligible correct runs

- Finite-memory controllers almost-surely force a fault.

Solving safe active probabilistic diagnosis?

The safe active probabilistic diagnosis problem is undecidable.

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Proof idea

- reduction from the existence, in a blind POMDP, of a strategy ensuring a Büchi objective with positive probability
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The safe active probabilistic diagnosis problem restricted to **finite memory strategies** is EXPTIME-complete.

Outline

Introduction to fault diagnosis

Diagnosability in probabilistic systems

Control for probabilistic diagnosability

Conclusion

Concluding remarks

Contributions: Foundations of stochastic diagnosis

- Investigation of semantical issues
- Tight complexity bounds for diagnosability
  and diagnoser synthesis problems
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Perspectives: Towards more quantitative questions

- Bounded-delay diagnosis
  tradeoff: delay *vs* diagnosability probability
- Space and time optimisation of observations
  tradeoff: observation cost *vs* diagnosability probability
- Active approximate diagnosis