Fault diagnosis for Probabilistic Systems a semantical and algorithmic journey

Nathalie Bertrand

Inria Rennes, France

based on joint work with Éric Fabre, Stefan Haar, Serge Haddad, Loïc Hélouët and Engel Lefaucheux

Two tales of smoke and observation





Original idea by Stefan Schwoon

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Assuming the behaviour of a system is known, an observer may deduce the occurrence of internal events from the outputs.

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Assuming the behaviour of a system is known, an observer may deduce the occurrence of internal events from the outputs.

Diagnosis, non-interference, information flow, opacity, etc.

Outline

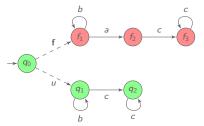
Introduction to fault diagnosis

Diagnosability in probabilistic systems Exact Diagnosis Approximate diagnosis

Control for probabilistic diagnosability

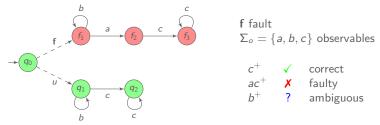
Conclusion

Objective: tell whether a fault occurred, based on observations.

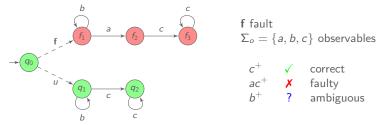


f fault $\Sigma_o = \{a, b, c\}$ observables

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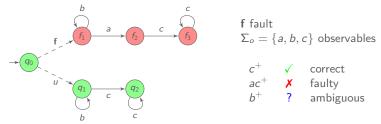


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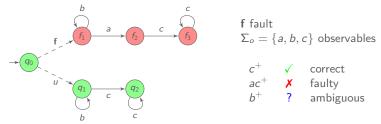
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Diagnosability: all observed sequences are unambiguous.

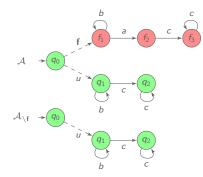
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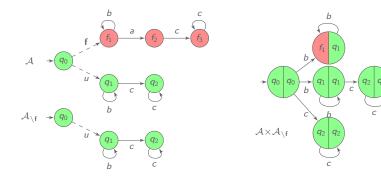


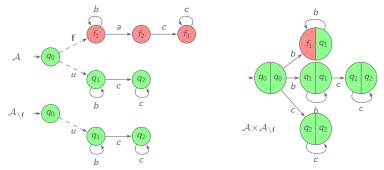
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Remark: w.l.o.g. state space partitionned into correct and faulty states

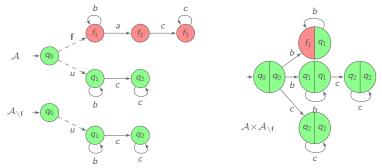






indeterminate cycle: $(f_0, q_0) \cdots \rightarrow (f_n, q_n) \rightarrow (f_0, q_0)$ s.t. f_i faulty and q_i correct

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Decidability and complexity of diagnosability [JHCK01] Diagnosability is decidable in PTIME.

[JHCK01] Jiang, Huang, Chandra and Kumar, A polynomial algorithm for testing diagnosability of discrete-event systems, TAC, 2001.

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Diagnoser: assigns verdicts to observed sequences $D: \Sigma_o^* \to {\checkmark, \bigstar, ?}$

Diagnoser requirements

- **Soundness**: if a fault is claimed **X**, a fault occurred.
- **Reactivity**: every fault is eventually claimed.

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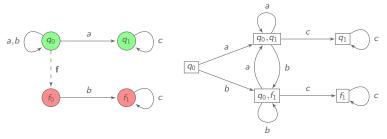
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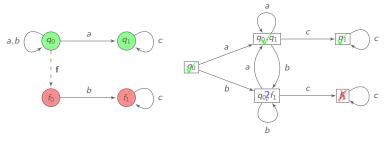
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Diagnoser synthesis

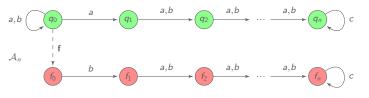
Complexity of diagnoser synthesis Diagnoser synthesis is in EXPTIME.

intuition: subset construction to track possible correct and faulty states

[JHCK01] Jiang, Huang, Chandra and Kumar, A polynomial algorithm for testing diagnosability of discrete-event systems, TAC, 2001. [HHMS13] Haar, Haddad, Melliti and Schwoon, Optimal constructions for active diagnosis, FSTTCS'13. **Complexity of diagnoser synthesis** Diagnoser synthesis is in EXPTIME.

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There is a family (A_n) of diagnosable systems such that A_n has 2n + 2 states and any diagnoser needs 2^n states.



diagnoser must remember the last n events: 2^n possibilities

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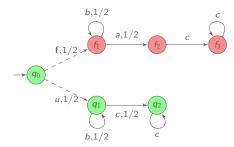
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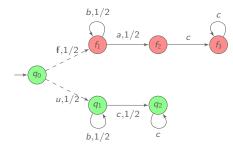
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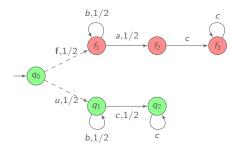
Control for probabilistic diagnosability

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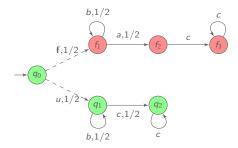


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How to adapt the framework to probabilistic systems?

- diagnosability notion(s)
- soundness and correctness for diagnosers
- algorithms for diagnosability checking and diagnoser synthesis

[TT05] Thorsley and Teneketzis, Diagnosability of stochastic discrete-event systems, TAC, 2005.
[CK13] Chen and Kumar, Polynomial test for stochastic diagnosability of dicrete-event systems, TASE, 2013.
[BHL14] B., Hadada and Lefaucheux, Foundation of diagnosis and predictability in probabilistic systems, FSTTCS'14.

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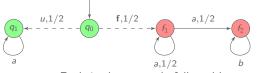
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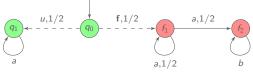


- Fault is almost surely followed by occurrence of b.
 - Ambiguous sequences have probability $\frac{1}{2}$.

Specifying diagnosability for probabilistic systems

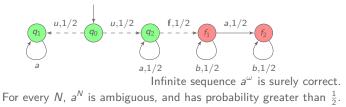
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- 2. Consider infinite observed sequences or their finite prefixes?

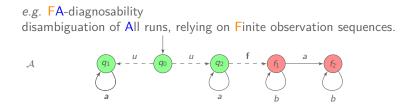


Four diagnosability specifications

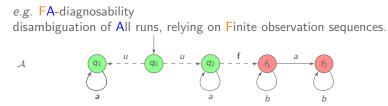
Diagnosability	All runs		Faulty runs
Finite prefixes	FA	$\Rightarrow \not=$	FF
	↓ ¥	/	$\Downarrow \Uparrow^*$
Infinite sequences	IA	⇒ ∉	IF

* assuming finitely-branching models

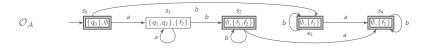
Characterizing diagnosability



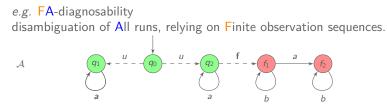
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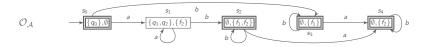
Observer $\mathcal{O}_\mathcal{A} \colon$ tracks possible correct and faulty states in two subsets



Characterizing diagnosability



Observer $\mathcal{O}_\mathcal{A}$: tracks possible correct and faulty states in two subsets



 $\begin{array}{l} \mathcal{A} \text{ is not FA-diagnosable iff} \\ \text{there exists a BSCC of } \mathcal{A} \times \mathcal{O}_{\mathcal{A}} \text{ where every state } (q, C, F) \text{ satisfies} \\ q \text{ faulty and } C \neq \emptyset \quad \text{ or } \quad q \text{ correct and } F \neq \emptyset. \end{array}$

[BHL14] B., Haddad and Lefaucheux, Foundation of diagnosis and predictability in probabilistic systems, FSTTCS'14.

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Solving diagnosability

Methodology to decide all diagnosability notions for probabilistic systems:

- \blacktriangleright build a deterministic observer $\mathcal{O}_{\mathcal{A}}$ by an ad hoc subset construction
- \blacktriangleright form the product $\mathcal{A}\times\mathcal{O}_{\mathcal{A}}$ to recover probabilistic behaviour
- ▶ check graph-based characterization on $\mathcal{A} \times \mathcal{O}_{\mathcal{A}}$

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Diagnosability is PSPACE-complete for probabilistic systems. [BHL14]

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For every diagnosable system with n states one can build a diagnoser with at most 3^n states.

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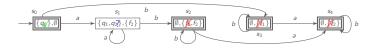
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Diagnoser derived from observer $\mathcal{O}_{\mathcal{A}}$:



[BHL14] B., Haddad and Lefaucheux, Foundation of diagnosis and predictability in probabilistic systems, FSTTCS'14. Fault Diagnosis for Probabilistic Systems – Nathalie Bertrand May 25th 2016 – MFPS XXXII – CMU Pittsburgh – 15/26

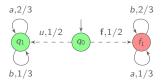
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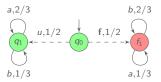
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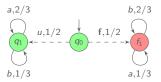
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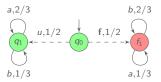


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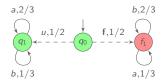
Relaxed soundness: if a fault is claimed, the probability of error is small.

[TT05] Thorsley and Teneketzis, Diagnosability of stochastic discrete-event systems, TAC, 2005.

Formalisation of accurate approximate diagnosability

Correcness proportion of an observation sequence σ

$$\mathsf{CorP}(\sigma) = \frac{\mathbb{P}(\{\pi^{-1}(\sigma) \cap \mathsf{correct}\})}{\mathbb{P}(\{\pi^{-1}(\sigma)\})}$$

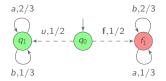


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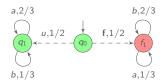


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CorP(a) = 2/3, CorP(ab) = 1/2, CorP(abb) = 1/3, CorP(abbb) = 1/5, ...

Accurate approximate diagnosers

ε -Diagnoser requirements

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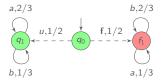
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admits ε -diagnoser, for every $\varepsilon > 0$ has no uniform ε -diagnoser, for any $\varepsilon > 0$

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Simple case: initial-fault models

$$\mathcal{A}^{c}$$
 q_{c} q_{c} q_{f} \mathcal{A}^{f}

 \mathcal{A} is accurate approximate diagnosable iff dist $(\mathcal{A}^c, \mathcal{A}^f) = 1$ *i.e.* there exists an event $E \subseteq \Sigma_o^{\omega}$ s.t. $|\mathbb{P}_{\mathcal{A}^c}(E) - \mathbb{P}_{\mathcal{A}^c}(E)| = 1$

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[CK14]

Uniform accurate approximate diagnosability is undecidable. [BHL16]

[CK14] Chen and Kiefer, On the Total Variation Distance of Labelled Markov Chains, CSL-LICS'14.
[BHL16] B., Haddad and Lefaucheux, Accurate approximate diagnosability of stochastic systems, LATA'16.

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From passive to active diagnosis

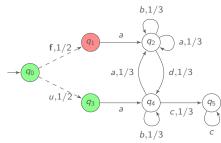


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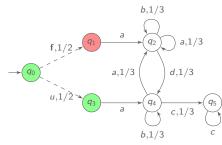


Original idea by Stefan Schwoon

Objective: control the probabilistic system so that it is diagnosable

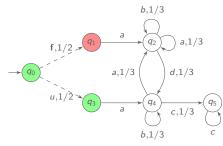


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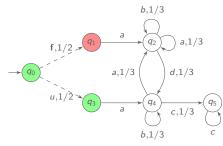
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 $\{a, b, c, d\}$ controllable

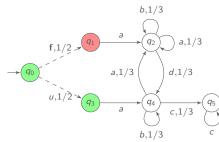
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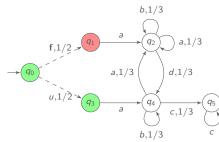
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 Controller: based on observation, decides which actions are allowed

 Active probabilistic diagnosis problem
 [BFHHH14]

 does there exist a controller such that the system is almost-surely
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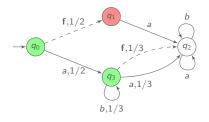
The active probabilistic diagnosis problem is **EXPTIME-complete**.

[BFHHH14] B., Fabre, Haar, Haddad and Hélouët, Active diagnosis for probabilistic systems, FoSSaCS'14.

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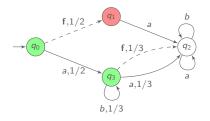
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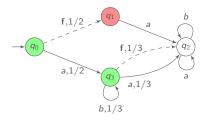
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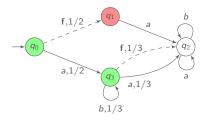
forbid a after first a \implies diagnosable... but almost all sequences faulty!

Safe active probabilistic diagnosis

[BFHHH14]

does there exist a controller such that the system is almost-surely diagnosable **and** correct runs have positive probability?

Objective: avoid fault-provocative controllers



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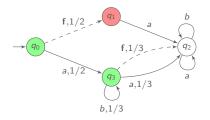
Safe active probabilistic diagnosis

[BFHHH14]

does there exist a controller such that the system is almost-surely diagnosable **and** correct runs have positive probability?

The safe active probabilistic diagnosis problem is undecidable.

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The safe active probabilistic diagnosis problem restricted to **finite memory controllers** is **EXPTIME-complete**.

 [BFHHH14] B., Fabre, Haar, Haddad and Hélouët, Active diagnosis for probabilistic systems, FoSSaCS'14.

 Fault Diagnosis for Probabilistic Systems – Nathalie Bertrand
 May 25th 2016 – MFPS XXXII – CMU Pittsburgh – 24/ 26

Outline

Introduction to fault diagnosis

Diagnosability in probabilistic systems Exact Diagnosis Approximate diagnosis

Control for probabilistic diagnosability

Conclusion

Concluding remarks

Contributions: Foundations of stochastic diagnosis

- Investigation of semantical issues
- Exact diagnosis: tight complexity bounds for diagnosability and diagnoser synthesis problems
- Accurate approximate diagnosis: PTIME algorithm
- Active diagnosability

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Perspectives: Towards more quantitative questions

- Bounded-delay diagnosis tradeoff: delay vs diagnosability precision
- Space and time optimisation of observations tradeoff: observation cost vs diagnosability probability
- ► Challenge: control, partial observation, quantitative properties