When are timed automata determinizable?

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Outline

1. Timed automata
2. A determinization procedure
   - Unfolding into an infinite tree
   - Region equivalence
   - Symbolic determinization
   - Clock reduction
   - Location reduction
3. The abstract procedure applied
   - Determinizable classes
   - Algorithmic issues and complexity

Syntax and semantics

Timed automata

A timed automaton is a tuple $\mathcal{A} = (L, \Sigma, X, E)$ with

- L finite set of locations
- X finite set of clocks
- Σ finite alphabet
- $E \subseteq L \times \Sigma \times \mathcal{G} \times 2^X \times L$ set of edges

where $\mathcal{G} = \{\bigwedge x \sim c \mid x \in X, c \in \mathbb{N}\}$ is the set of guards.

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States of $\mathcal{A}$: $L \times (\mathbb{R}_+)^X$

Transitions between states of $\mathcal{A}$:

- Delay transitions: $(\ell, v) \xrightarrow{t} (\ell, v + t)$
- Discrete transitions: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists(\ell, a, g, Y, \ell') \in E$ with $v \models g$, $v'(x) = 0$ if $x \in Y$, and $v'(x) = v(x)$ otherwise.

Run of $\mathcal{A}$:

$(\ell_0, v_0) \xrightarrow{\tau_0} (\ell_0, v_0 + \tau_0) \xrightarrow{a_0} (\ell_1, v_1) \xrightarrow{\tau_1} (\ell_1, v_1 + \tau_1) \xrightarrow{a_1} (\ell_2, v_2) \dots$

or simply: $(\ell_0, v_0) \xrightarrow{\tau_0, a_0} (\ell_1, v_1) \xrightarrow{\tau_1, a_1} (\ell_2, v_2) \dots$

Timed language

Timed word: $w = (a_0, t_0)(a_1, t_1) \dots (a_k, t_k)$
with $a_i \in \Sigma$ and $(t_i)_{0 \leq i \leq k}$ nondecreasing sequence in $\mathbb{R}_+$.

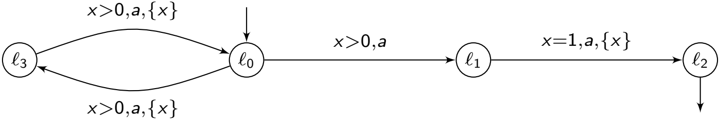
$\mathcal{A} = (L, \ell_0, L_{acc}, \Sigma, X, E)$ timed automaton equipped with ℓ_0 initial location, and L_{acc} set of accepting locations.

Accepted timed word

A timed word $w = (a_0, t_0)(a_1, t_1) \dots (a_k, t_k)$ is accepted in $\mathcal{A}$, if there is a run $\rho = (\ell_0, v_0) \xrightarrow{\tau_0, a_0} (\ell_1, v_1) \xrightarrow{\tau_1, a_1} \dots (\ell_{k+1}, v_{k+1})$ in $\mathcal{A}$ with $\ell_{k+1} \in L_{acc}$, and $t_i = \sum_{j<i} \tau_j$.

Accepted timed language: $\mathcal{L}(\mathcal{A}) = \{w \mid w \text{ accepted by } \mathcal{A}\}$.

A running example



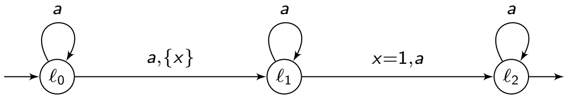
$$\mathcal{L}(A) = \{(a, t_1)(a, t_2) \cdots (a, t_{2n}) \mid 0 < t_1 < t_2 < \cdots < t_{2n-1} \text{ and } t_{2n} - t_{2n-2} = 1\}$$

Deterministic timed automata

Deterministic timed automata

$\mathcal{A}$ is deterministic whenever for every timed word w, there is at most one initial run on w in $\mathcal{A}$.

Some timed automata are not determinizable [AD90].



$\mathcal{L}(\mathcal{A}) = \{(a, t_1) \dots (a, t_n) \mid n \geq 2 \text{ and } \exists i < j \text{ s.t. } t_j - t_i = 1\}$
$\longrightarrow$ infinitely many clocks needed

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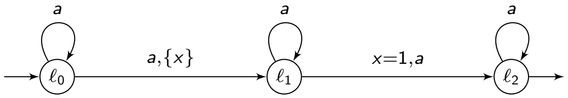

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Theorem [Finkel 06]

Checking whether a given timed automata is determinizable is undecidable.

About universality

$\mathcal{A}$ is universal if $\mathcal{L}(\mathcal{A}) = (\Sigma \times \mathbb{R}_+)^*$

Theorem [AD90]

Universality is undecidable for timed automata.

However, universality is decidable for some subclasses

- event-clock timed automata [AFH94]
- one-clock timed automata [OW04]

Strong timed bisimulation

Strong timed (bi)simulation

$\mathfrak{R}$ is a strong timed simulation between transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$ if for every $s_1 \mathfrak{R} s_2$ and $s_1 \xrightarrow{t_1,a} s_1'$ for some $t_1 \in \mathbb{R}_+$ and $a \in \Sigma$, then there exists $s_2' \in S_2$ such that $s_2 \xrightarrow{t_1,a} s_2'$ and $s_1' \mathfrak{R} s_2'$.
$\mathfrak{R}$ is a strong timed bisimulation if $\mathfrak{R}$ and $\mathfrak{R}^{-1}$ are strong timed simulations.

Strong timed bisimulation (preserving initial and accepting states) implies language equivalence.

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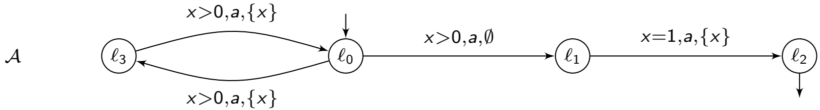
Unfolding

- $\mathcal{A}$ unfolded into a tree $\mathcal{A}^\infty$ with a fresh clock at each step.
- clocks of $\mathcal{A}$ are mapped to their reference in the new set of clocks.



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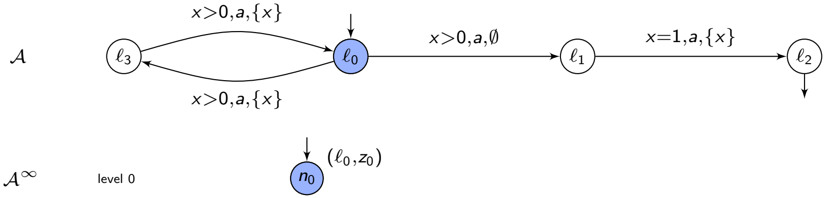

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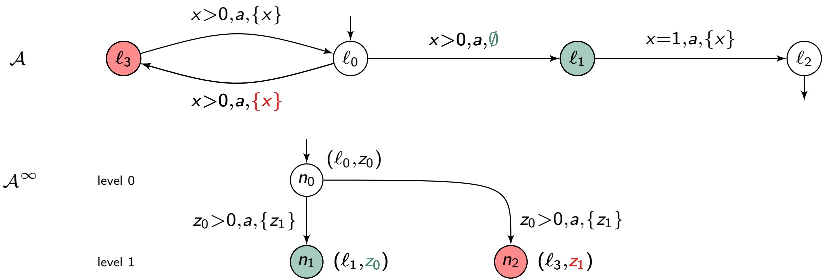

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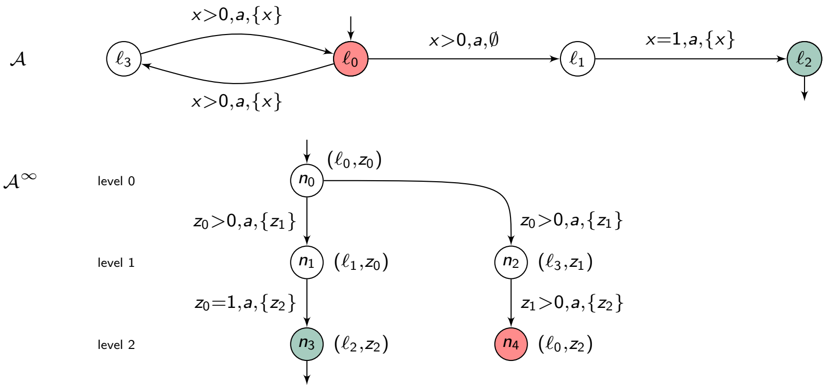
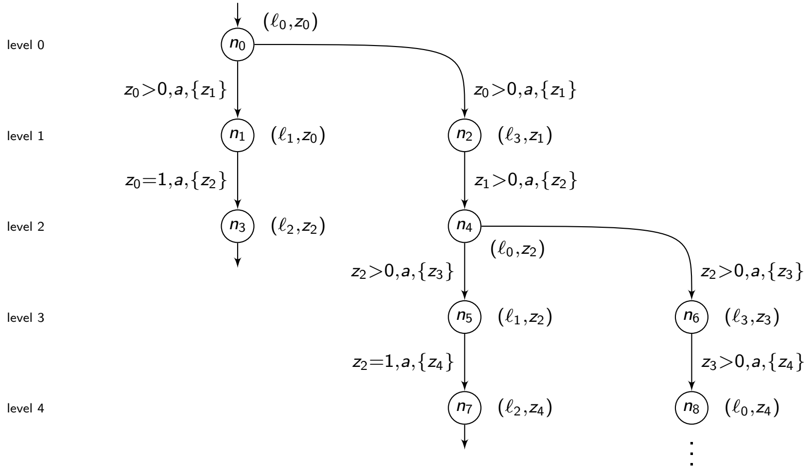


Unfolding


level 0
level 1
level 2
level 3
level 4
(ℓ_0,z_0)
n_0
$z_0>0,a,\{z_1\}$
$z_0>0,a,\{z_1\}$
n_1
(ℓ_1,z_0)
n_2
(ℓ_3,z_1)
$z_0=1,a,\{z_2\}$
$z_1>0,a,\{z_2\}$
n_3
(ℓ_2,z_2)
n_4
(ℓ_0,z_2)
$z_2>0,a,\{z_3\}$
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n_5
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n_6
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$z_2=1,a,\{z_4\}$
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n_7
(ℓ_2,z_4)
n_8
(ℓ_0,z_4)

Properties of the unfolding

Input-determinacy property:
for every timed word w, there is a unique valuation v_w s.t. every initial run on w ends in some (n, v_w) with $level(n) = |w|$.

Lemma

$\mathcal{A}$ and $\mathcal{A}^\infty$ are strongly timed bisimilar; in particular $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}^\infty)$.

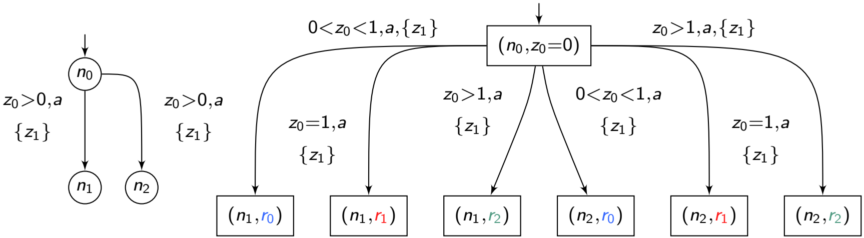
Drawbacks:

- $\mathcal{A}^\infty$ has infinitely many locations.
- $\mathcal{A}^\infty$ has infinitely many clocks.

Outline

Region equivalence

Region construction on $\mathcal{A}^\infty$: at level i regions over $\{z_0, \cdots, z_i\}$.



where $r_0=0=z_1<z_0<1$, $r_1=0=z_1<z_0=1$ and $r_2=0=z_1<1<z_0$

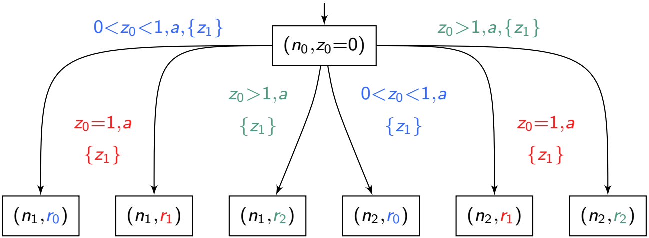
Lemma

$\mathcal{A}^\infty$ and $R(\mathcal{A}^\infty)$ are strongly timed bisimilar; thus $\mathcal{L}(\mathcal{A}) = \mathcal{L}(R(\mathcal{A}^\infty))$.

Outline

Symbolic determinization

Determinization at level i on the alphabet $\mathrm{Reg}_i \times \Sigma \times Z$.



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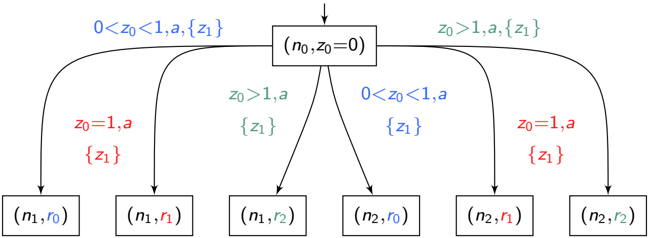
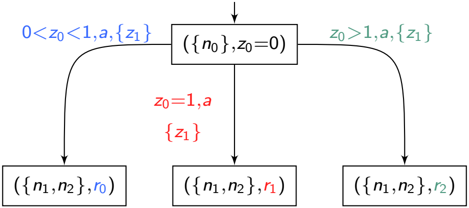




Properties of the symbolic determinization

The symbolic determinization corresponds to determinization of the timed system.

SymbDet($\mathcal{A}$) is deterministic!

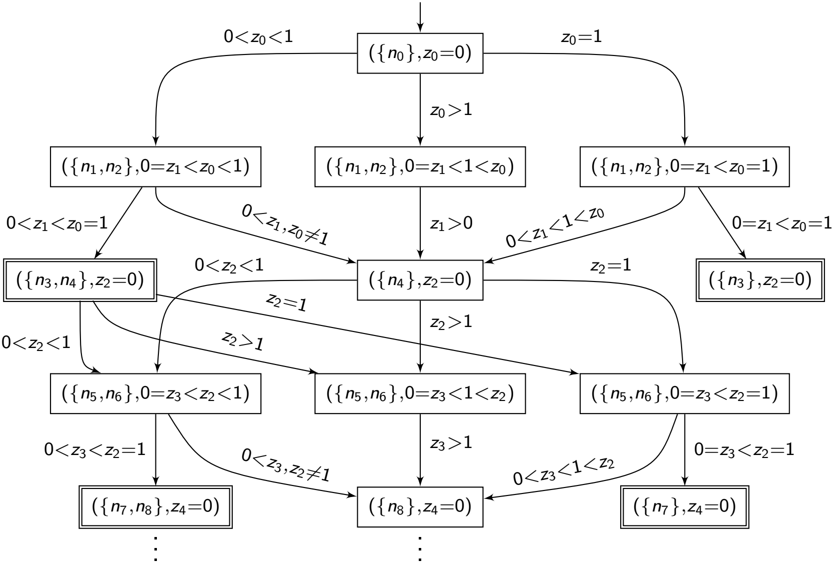
Lemma

$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathrm{SymbDet}(R(\mathcal{A}^\infty)))$.

Drawbacks:

- SymbDet($R(\mathcal{A}^\infty)$) has infinitely many locations.
- SymbDet($R(\mathcal{A}^\infty)$) has infinitely many clocks.

Symbolic determinization on the example



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Clock reduction

Active clocks: given a node of SymbDet($R(\mathcal{A})$), its active clocks is the set of clocks appearing in the region of the node.

Clock boundedness

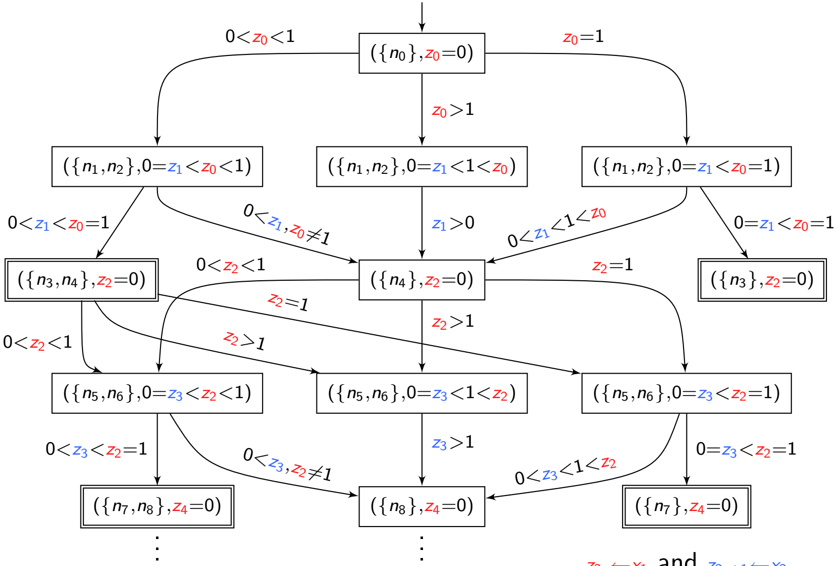
SymbDet($R(\mathcal{A}^\infty)$) is γ-clock bounded if in every node the number of active clocks is bounded by γ.

Under the clock-boundedness assumption: $\Gamma_\gamma(\text{SymbDet}(R(\mathcal{A}^\infty))) =$ reduction of SymbDet($R(\mathcal{A}^\infty)$) to set of clocks $\{x_1, \cdots, x_\gamma\}$.

Lemma

$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\Gamma_\gamma(\text{SymbDet}(R(\mathcal{A}^\infty))))$

Clock reduction on the example



$z_{2n} \leftarrow x_1$ and $z_{2n+1} \leftarrow x_2$

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Location reduction

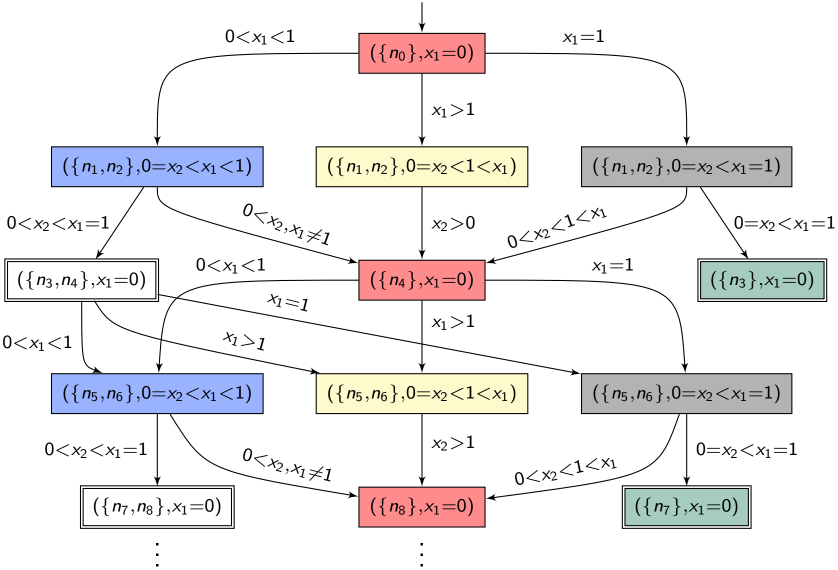
Property of $\Gamma_\gamma(\mathsf{SymbDet}(R(\mathcal{A}^\infty)))$:
Nodes sharing the same label (= set of locations + region + assignment of the clocks) are isomorphic.

$\mathcal{B}_{\mathcal{A},\gamma}$: $\Gamma_\gamma(\mathsf{SymbDet}(R(\mathcal{A}^\infty)))$ after merging isomorphic nodes.

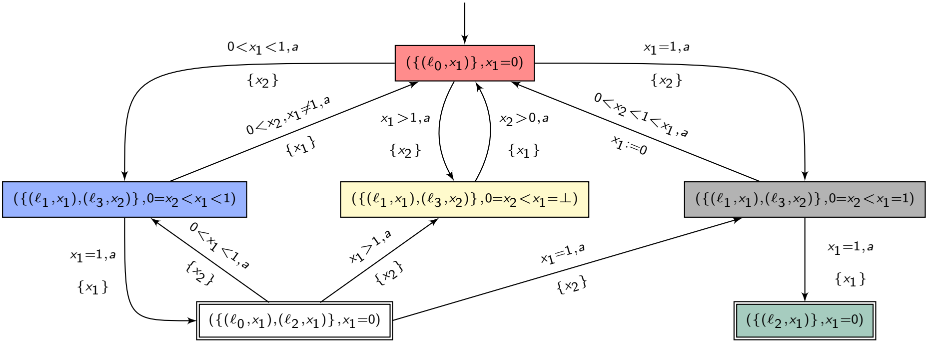
Theorem

$\mathcal{B}_{\mathcal{A},\gamma}$ is a deterministic timed automaton such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B}_{\mathcal{A},\gamma})$.

Back to the example



A deterministic version of the example



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Recap of the procedure

1. Unfolding into a timed tree with infinitely many clocks and nodes
2. Region construction on the timed tree
   (still infinitely many clocks and nodes)
3. Symbolic determinization of the region tree
   (corresponding to a determinization of the timed system)
4. Reduction of the number of clocks
   (under the γ-clock bounded hypothesis)
5. Reduction of the number of locations

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Key hypothesis: γ-clock boundedness

Outline

When are TA γ-clock bounded?

Event-clock timed automata

For every $a \in \Sigma$ there is a clock x_a reset at each occurrence of a.

Given $\mathcal{A}$ an event-clock TA, the number of active clocks is bounded by Σ.

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Integer-reset timed automata

For every edge (ℓ, g, a, Y, ℓ')
$Y \neq \emptyset$ if and only if g contains some constraint $x = c$.

The deterministic timed tree associated with an integer reset TA is $(M+1)$-clock bounded, where M is the maximal constant in $\mathcal{A}$.

Sufficient condition

p-assumption

Let $p \in \mathbb{N}$. $\mathcal{A}$ satisfies the p-assumption if for every $n \geq p$, for every run

$$\rho = (\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \ldots \xrightarrow{\tau_n, a_n} (\ell_n, v_n)$$

for every clock x, either x is reset along ρ, of $v_n(x) > M$.

$\mathcal{A}$ satisfies the p-assumption $\Longrightarrow$ SymbDet($R(\mathcal{A}^\infty)$) is p-clock bounded.

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$\mathcal{A}$ satisfies the p-assumption $\Longrightarrow$ $\mathsf{SymbDet}(R(\mathcal{A}^\infty))$ is p-clock bounded.

Strongly non-Zeno

A timed automaton $\mathcal{A}$ is *strongly non-Zeno* if there exists $K \in \mathbb{N}$ s.t. for every run $s_0 \xrightarrow{\tau_1, a_1} s_1 \cdots \xrightarrow{t_k, a_k} s_k$ in $\mathcal{A}$, $k \geq K$ implies $\sum_{i=1}^{k} \tau_i \geq 1$.

If $\mathcal{A}$ is strongly non-Zeno, then it satisfies the p-assumption for some p exponential in the size of $\mathcal{A}$.

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Algorithmic issues

Given $\mathcal{A} = (L, \ell_0, L_{acc}, X, M, E)$ s.t. SymbDet($R(\mathcal{A}^\infty)$) is γ-clock bounded, locations in $\mathcal{B}_{\mathcal{A},\gamma}$ are characterized by:

- a finite set of pairs in $L_\mathcal{A} \times X_\gamma^X$, and
- a region over X_γ.

Hence $\mathcal{B}_{\mathcal{A},\gamma}$ has $2^{|L|} \cdot \gamma^{|X|} \cdot \left((2M+2)^{(\gamma+1)^2} \cdot \gamma!\right)$ locations.

Algorithmic issues

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Size of the deterministic TA

- TA under the *p*-assumption: doubly exponential
- event-clock TA: exponential
- integer reset TA: doubly exponential

Complexity of universality

Lower bound

Checking universality in timed automata either satisfying the *p*-assumption or with integer resets is EXPSPACE-hard.

Proof idea: given an EXPSPACE Turing machine and an input word, build a timed automaton which is universal if and on ly if the machine does not halt. Executions are coded by timed-words, actions (representing letters) are separated by 1 time unit.

Remark: same lower bound for the inclusion problem (also for SnZTA).

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Upper bound

Checking universality is in EXPSPACE for timed automata satisfying the *p*-assumption, and for integer resets timed automata.

Proof idea: the complement of $\mathcal{B}_{\mathcal{A},\gamma}$ can be computed on the fly. Checking for emptiness can be done in logarithmic space in the number of locations.

Summary complexity

	size of the det. TA	universality problem	inclusion problem
TA_p	*doubly exp.*	*EXPSPACE-compl.*	*EXPSPACE-compl.*
SnZTA	*doubly exp.*	trivial	*EXPSPACE-compl.*
ECTA	exp.	PSPACE-compl.	PSPACE-compl.
IRTA	doubly exp.	EXPSPACE-*compl.*	EXPSPACE-*compl.*

Conclusion

Contribution

- general procedure for the determinization of TA
- new determinizable class(es)
- tight complexity bounds

Future work

- adapt the procedure to infinite timed words
- recover decidability of universality for 1-clock TA
- find other determinizable classes