Diagnosis in Infinite-State Probabilistic Systems

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Infinity 2016, Singapore

Two tales of smoke and observation

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Assuming the behaviour of a system is known, an observer may deduce the occurrence of internal events from the outputs.

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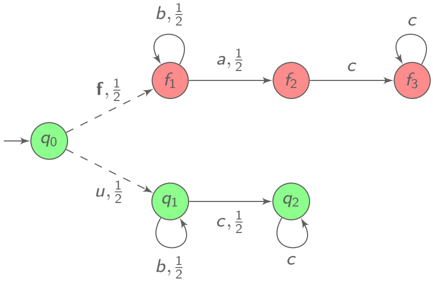
Assuming the behaviour of a system is known, an observer may deduce the occurrence of internal events from the outputs.

Diagnosis, non-interference, information flow, opacity, etc.

Fault Diagnosis in Probabilistic systems

Diagnoser: must tell whether a fault **f** occurred, based on observations.



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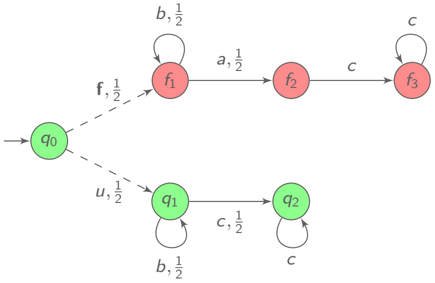

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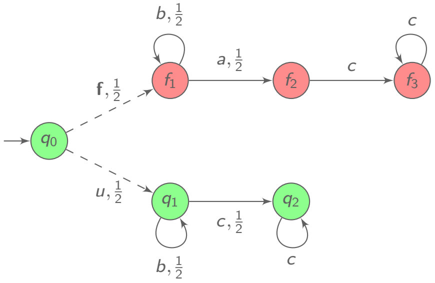

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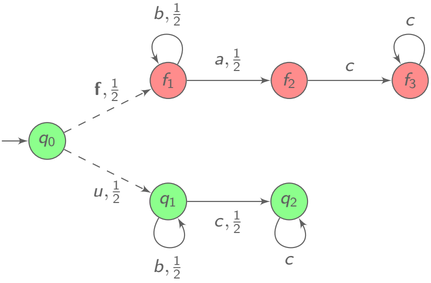

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? b is *ambiguous* since $\mathcal{P}^{-1}(b) = \{q_0 \mathbf{f} f_1 b f_1, q_0 u q_1 b q_1\}$.

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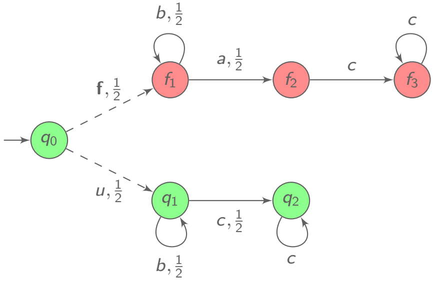

sound and reactive diagnoser: claim a fault when *a* occurs.

Outline

Diagnosability specifications

Characterising diagnosability for infinite-state systems

Deciding diagnosability of visibly pushdown models

Specifying diagnosability for probabilistic systems

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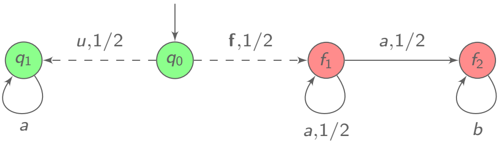

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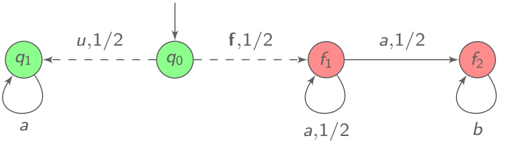

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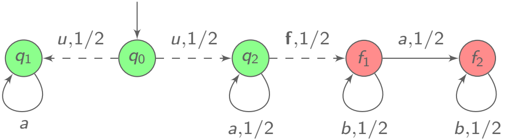
2. Consider infinite observed sequences or their finite prefixes?



Infinite sequence a^ω is surely correct.

For every n, a^n is ambiguous, and has probability greater than $\frac{1}{2}$.

Four diagnosability specifications [BHL 14]

[BHL 14] Bertrand, Haddad and Lefaucheux, *Foundation of Diagnosis and Predictability in Probabilistic Systems*, FSTTCS'14

Diagnosability	All runs		Faulty runs
Finite prefixes	FA	$\Rightarrow$ $\not\Leftarrow$	FF
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All diagnosability problems are PSPACE-complete.

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What about infinite-state probabilistic systems?

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Quest for a characterisation

Objective: simple qualitative charac., independent of probability values

$\mathcal{N}$ is diagnosable iff $\mathbb{P}_{\mathcal{N}}(B) \bowtie p$, where:

- $p \in \{0, 1\}$, $\bowtie \in \{<, =, >\}$;
- B ($\star$) belongs to a low level of Borel hierarchy and
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Characterisation in pathL: expressive linear temporal logic

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also valid for IF-*diagnosability if* $\mathcal{N}$ *is finitely-branching*

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There is no F_σ set B s.t. $\mathbb{P}(B) = 0$ characterises FA-diagnosability
There is no Borel set B s.t. $\mathbb{P}(B) > 0$ characterises FA-diagnosability

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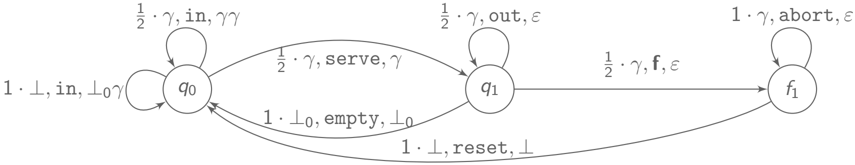
Probabilistic Visibly Pushdown Automata (pVPA)

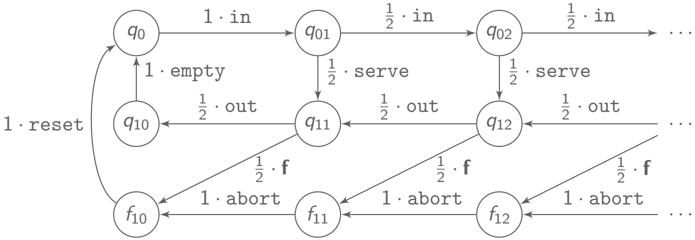


The action determines the operation on the stack.
i.e. the size of the stack is always known.

Iterative behaviour of a server.

1. A server takes an arbitrary list of requests.
2. It starts serving them until
    2.1 all of them are satisfied.
    2.2 or an error occurred then it drops all the following requests.

Semantics of pVPA



Observation of pop events: $\mathcal{P}(\texttt{out}) = \mathcal{P}(\mathbf{f}) = \mathcal{P}(\texttt{abort}) = \texttt{pop}$.

$$(q_0, |\bot_0|) \xrightarrow{\texttt{in}} \left(q_0, \left|\begin{matrix}\gamma\\ \bot_0\end{matrix}\right|\right) \xrightarrow{\texttt{in}} \left(q_0, \left|\begin{matrix}\gamma\\ \gamma\\ \bot_0\end{matrix}\right|\right) \xrightarrow{\texttt{serve}} \left(q_1, \left|\begin{matrix}\gamma\\ \gamma\\ \bot_0\end{matrix}\right|\right) \xrightarrow{\texttt{out}} \left(q_1, \left|\begin{matrix}\gamma\\ \bot_0\end{matrix}\right|\right) \xrightarrow{\texttt{out}} (q_1, |\bot_0|) \xrightarrow{\texttt{empty}} (q_0, |\bot_0|)$$

$$\left(q_1, \left|\begin{matrix}\gamma\\ \gamma\\ \bot_0\end{matrix}\right|\right) \xrightarrow{\mathbf{f}} \left(f_1, \left|\begin{matrix}\gamma\\ \bot_0\end{matrix}\right|\right) \xrightarrow{\texttt{abort}} (f_1, |\bot_0|) \xrightarrow{\texttt{reset}} (q_0, |\bot_0|)$$

Reducing to pLTL model checking on pPDA

Reduction in 4 steps:

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- model checking of qualitative pLTL formulae [EY 12]

[EY 12] Etessami and Yannakakis, *Model checking recursive probabilistic systems*, ACMToCL 2012

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FF-diagnosability, IF-diagnosability and IA-diagnosability are decidable in EXPSPACE for pVPA.

Details on the determinisation

- Inspired by original determinisation of [AM 04]
- With tags customized for diagnosis borrowed from [HHMS 13]

[AM 04] Alur and Madhusudan. *Visibly pushdown languages*, STOC'04

[HHMS 13] Haar, Haddad, Melliti and Schwoon. *Optimal constructions for active diagnosis*, FSTTCS'13.

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stack symbol = set of tuples $\frac{\gamma,\mathsf{X},q}{\gamma^-,\mathsf{X}^-,q^-}$ corresponding to possible runs:

• states q, q^-: q reached after the last action;
q^- reached after the last push;

• tags X, X^-: X status after last action
U = correct, V = recent fault, W = old fault;
X^- status after the last push

• original stack symbols γ, γ^-: γ the top stack symbol;
γ^- last but top stack symbol

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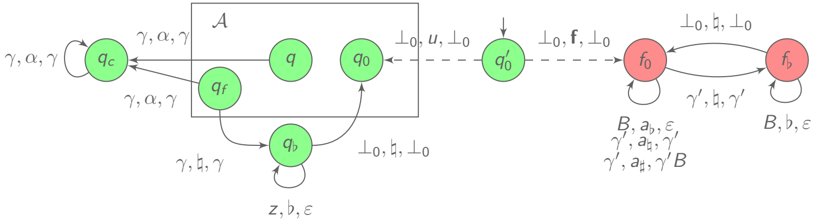
Hardness of diagnosis

Diagnosability is EXPTIME-hard for pVPA.

Reduction from the universality problem for VPA.



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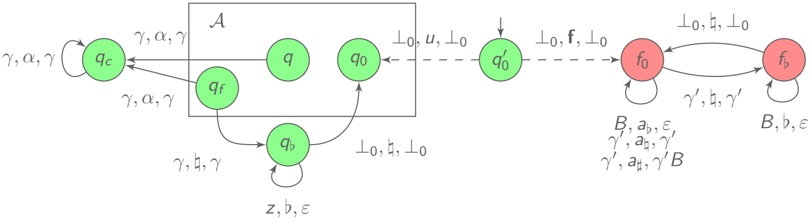

Diagnosability is undecidable for probabilistic pushdown automata.

Reduction from the Post Correspondence Problem.
Already holds for restricted classes of pPDA (constant nb of phases).

Conclusion

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Future work

- Reduction of the complexity gap between lower and upper bounds;
- Diagnosis of other infinite state stochastic systems;
- Diagnosis for continuous-time stochastic systems.