Foundation of Diagnosis and Predictability in Probabilistic Systems

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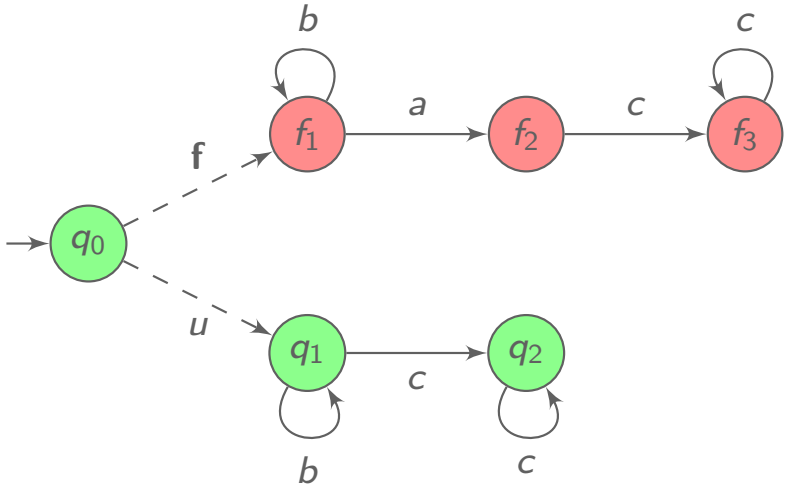
Diagnosis of discrete event systems

Objective: tell whether a fault **f** occurred, based on observations.



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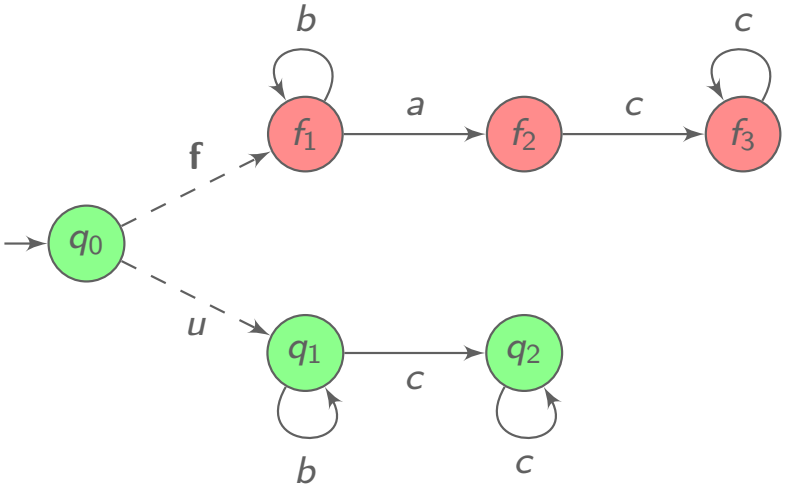

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ac^+	✗	faulty
b^+	?	ambiguous

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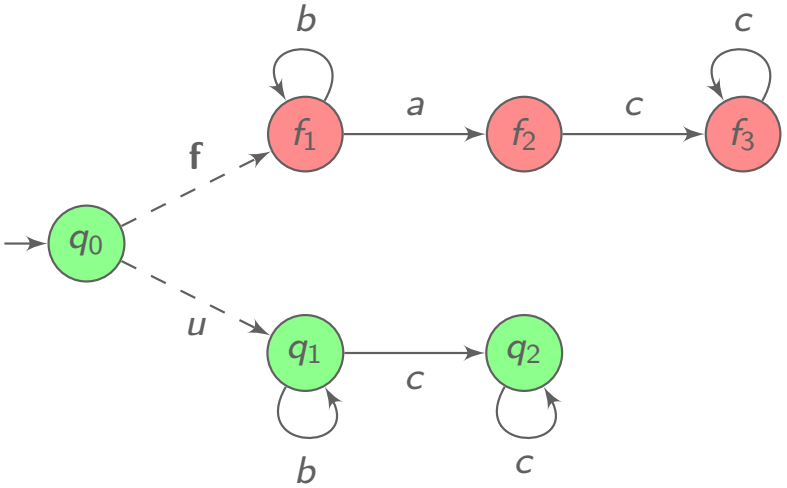

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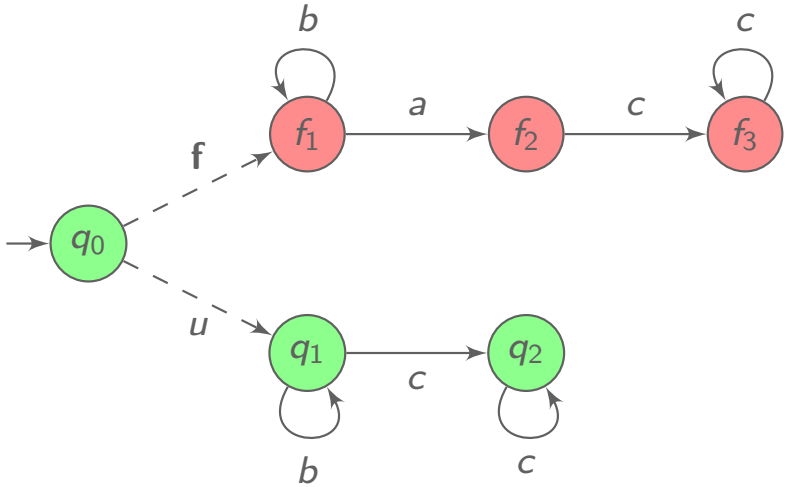

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Diagnoser: assigns verdicts to observed sequences $D : \Sigma_o^* \to \{✓, ✗, ?\}$

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- **Reactivity**: every fault will be detected.

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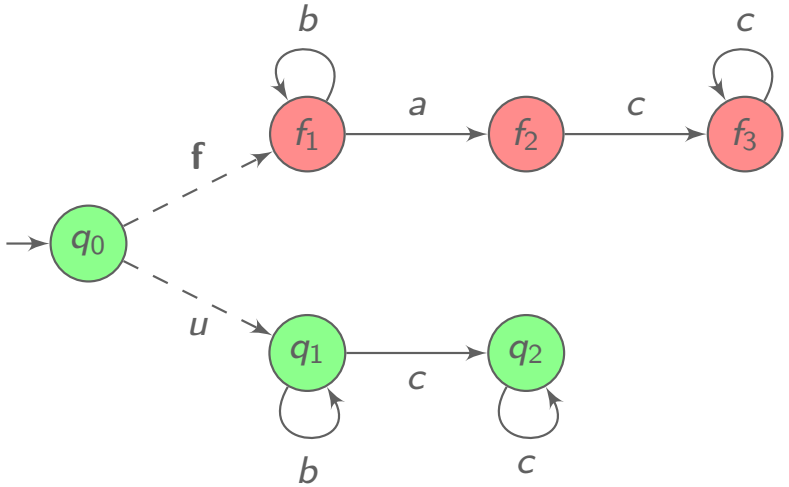

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Diagnosability and diagnoser synthesis in PTIME [Jiang et al. TAC 2001]

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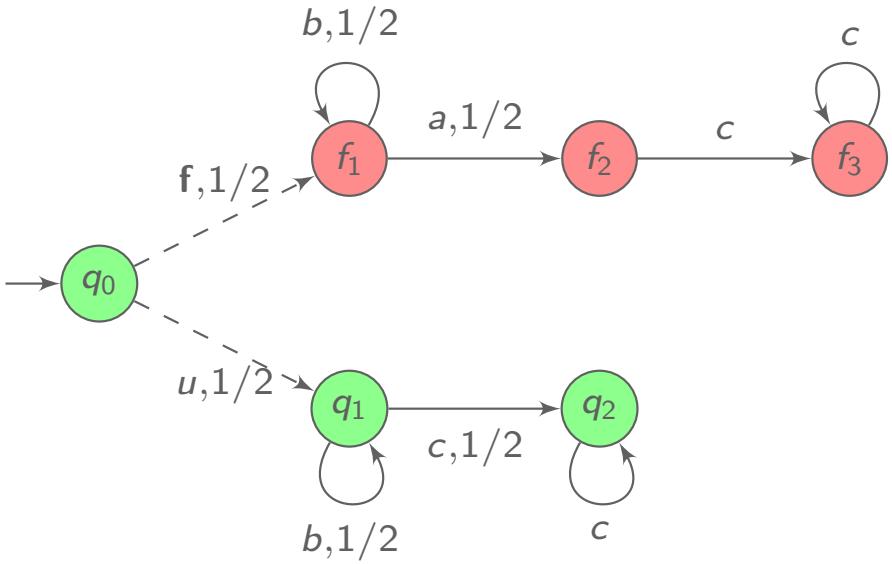

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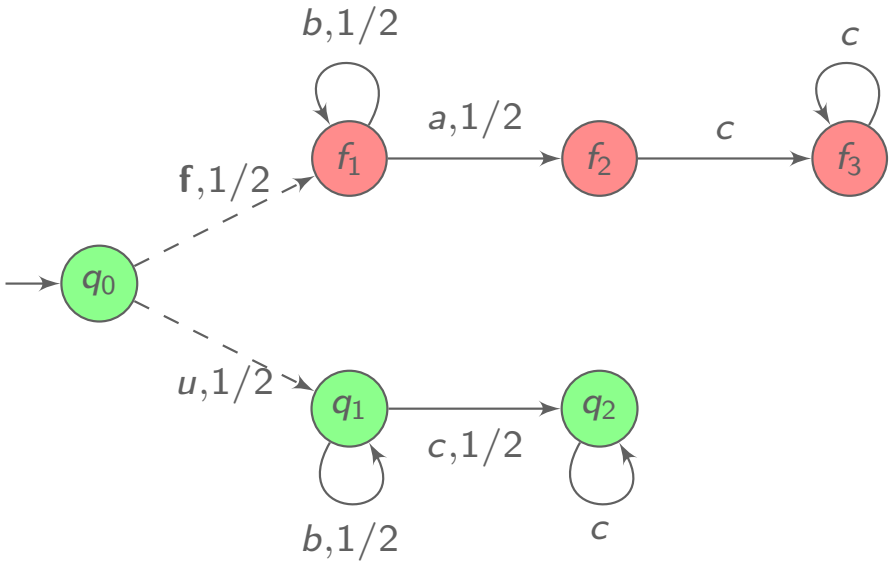

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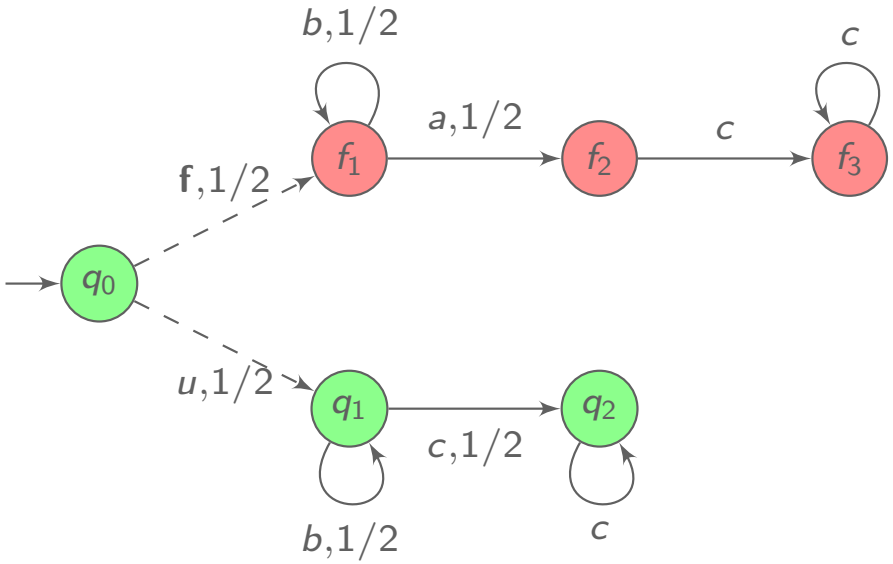

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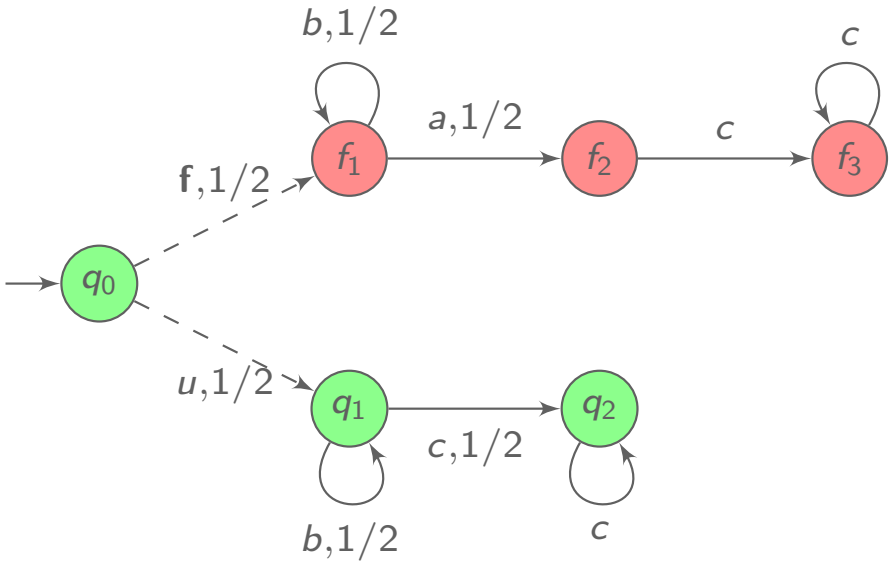

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Our contribution

- Relevant soundness and reactivity criteria in probabilistic setting
- Decidability and complexity of diagnosability
- Optimal diagnoser construction
- Beyond diagnosis: predictability and prediagnosis

Outline

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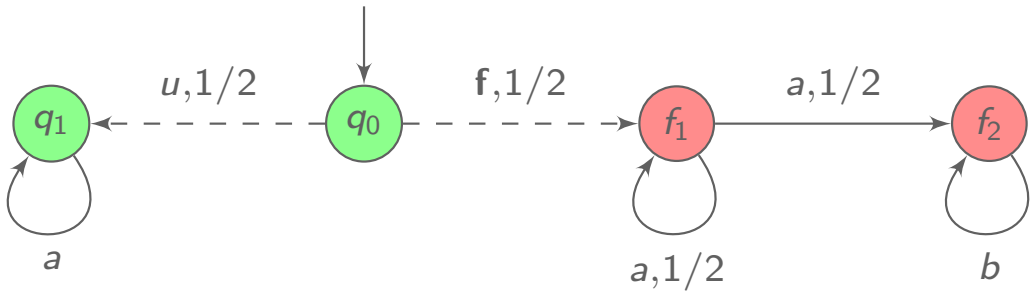
All runs or faulty runs?



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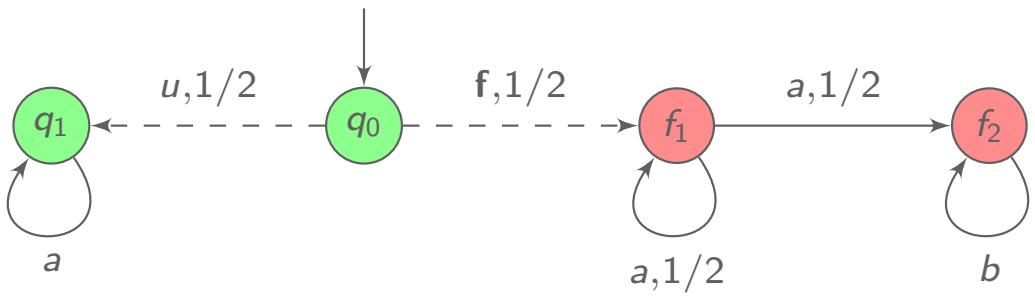

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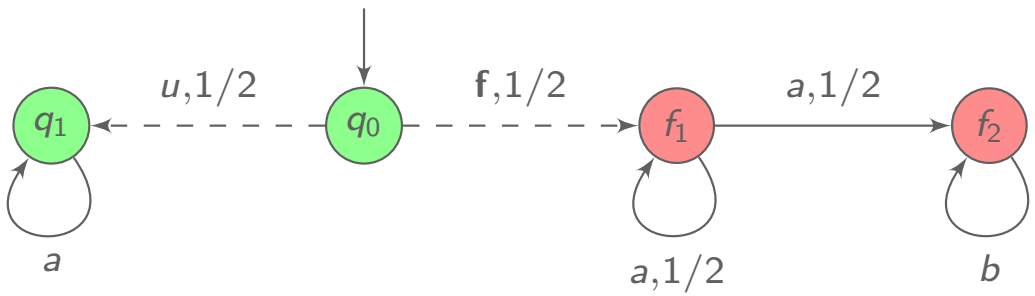

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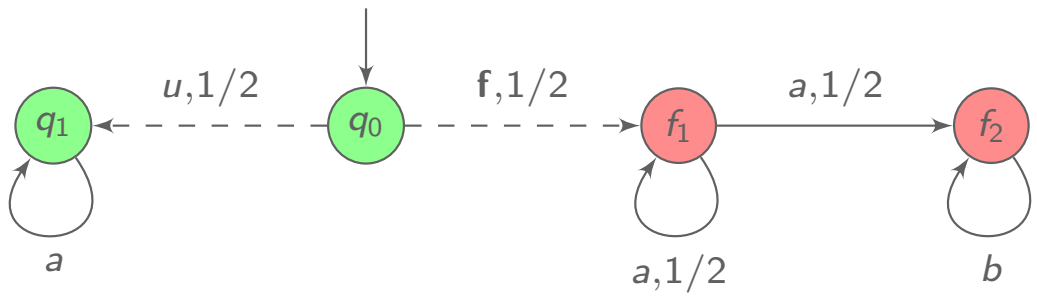

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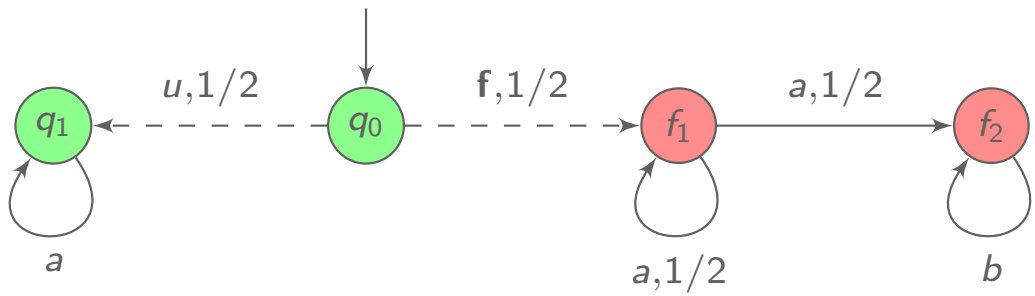

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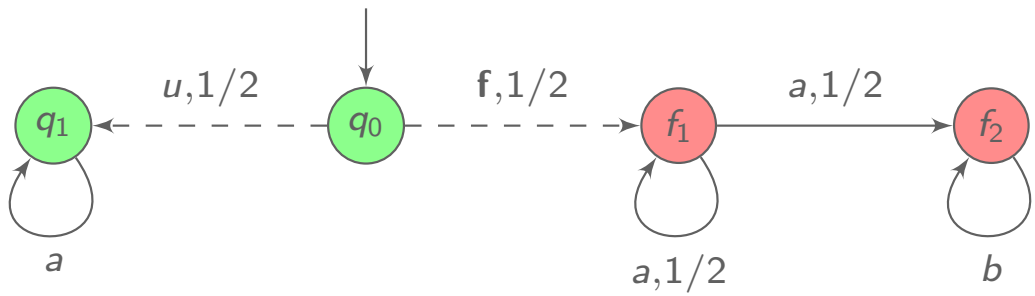

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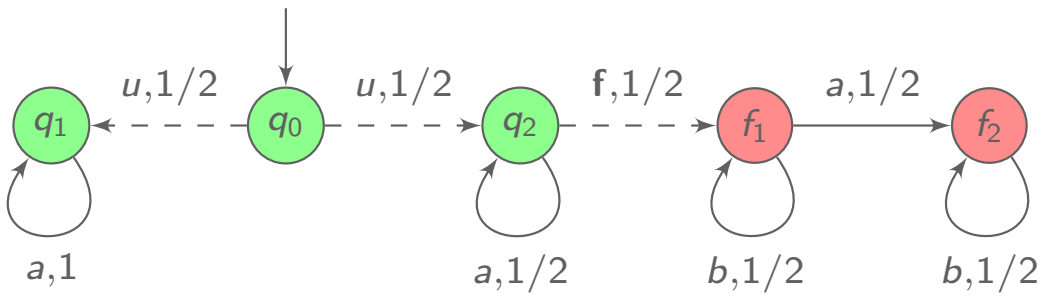
Reactivity specifications:

- Detect a fault, almost surely.
- Detect if a run is faulty or correct, almost surely.

Infinite sequences or their finite prefixes?



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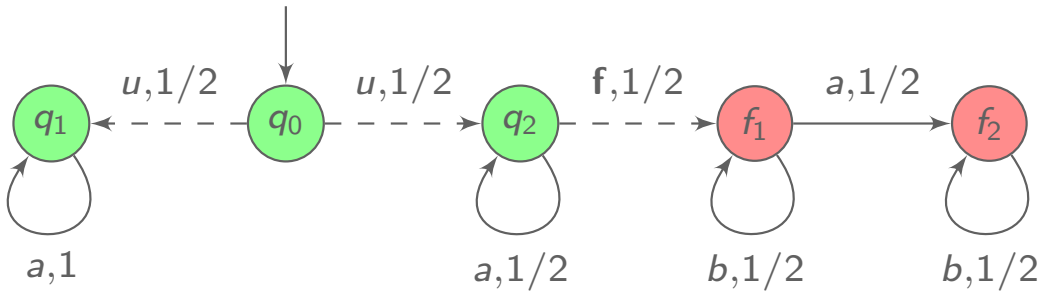

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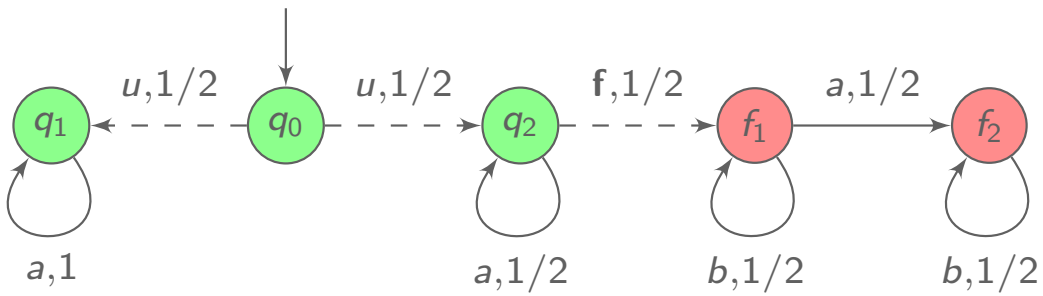

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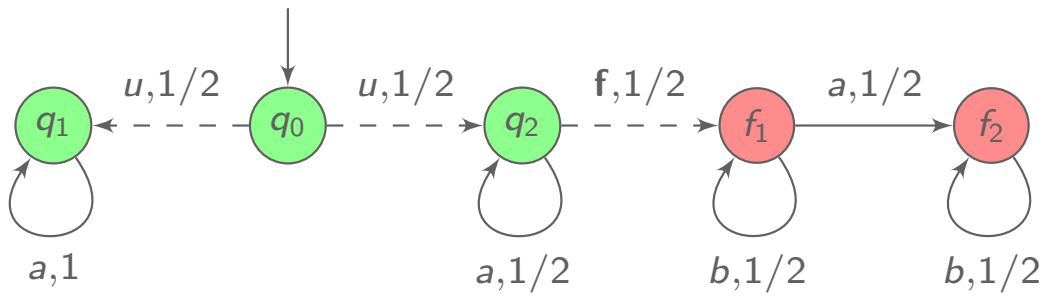

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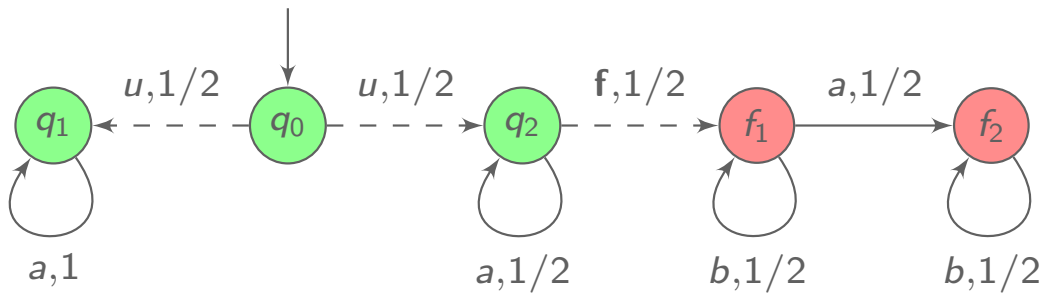

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- Infinite sequences are almost surely non ambiguous.
- The probability of ambiguous prefixes tends to 0.

Four diagnosability notions

Diagnosability	All runs		Faulty runs
Finite prefixes	FA	$\Rightarrow$ $\not\Leftarrow$	FF
	$\Downarrow\not\Uparrow$		$\Downarrow\Uparrow$
Infinite sequences	IA	$\Rightarrow$ $\not\Leftarrow$	IF

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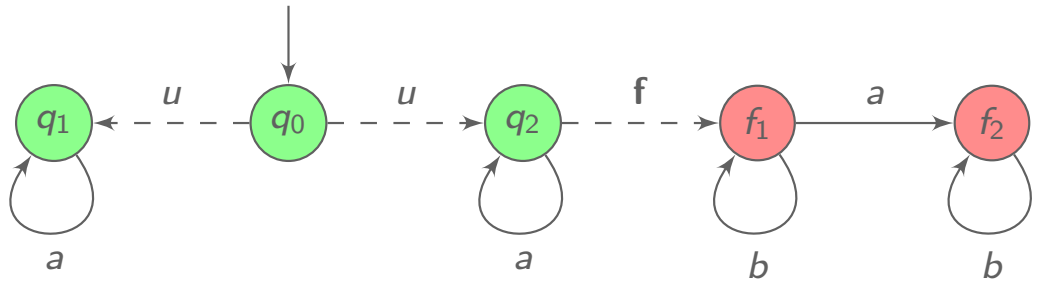
Focus on IF in this talk.

Characterisation of diagnosability

Specification of IF-diagnosability: Infinite sequences, Fault diagnosis

$\mathcal{A}$



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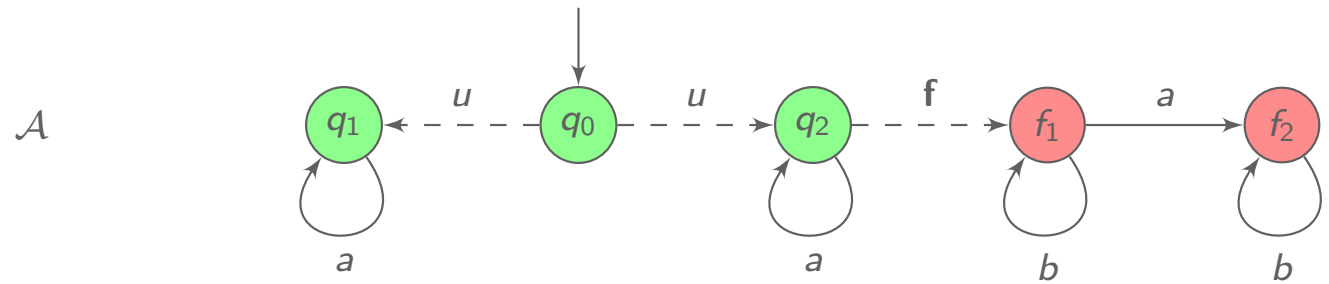
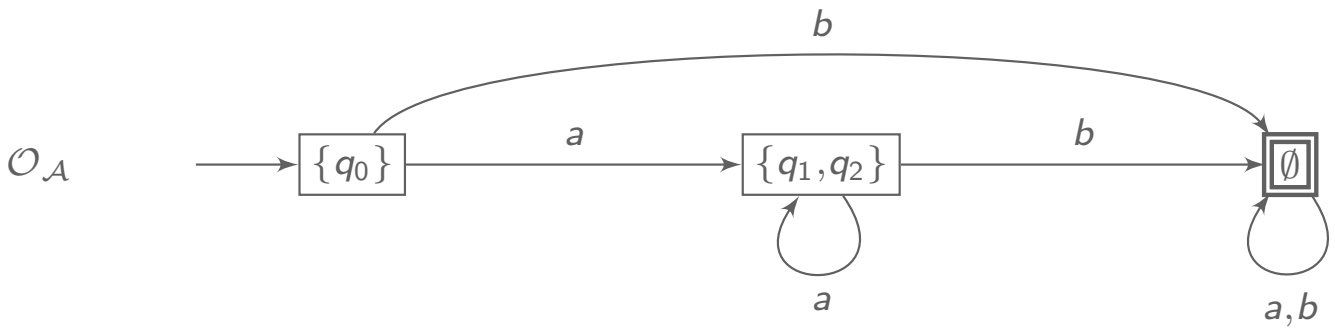


Observer: tracks possible correct states after given observed sequence.



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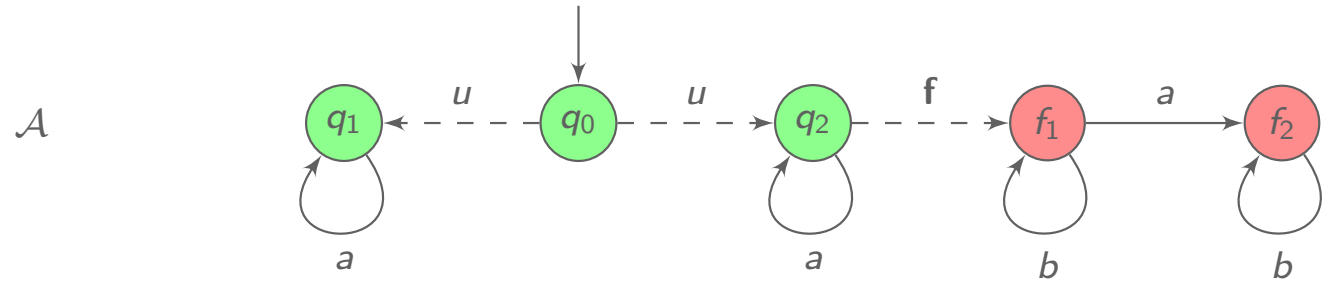

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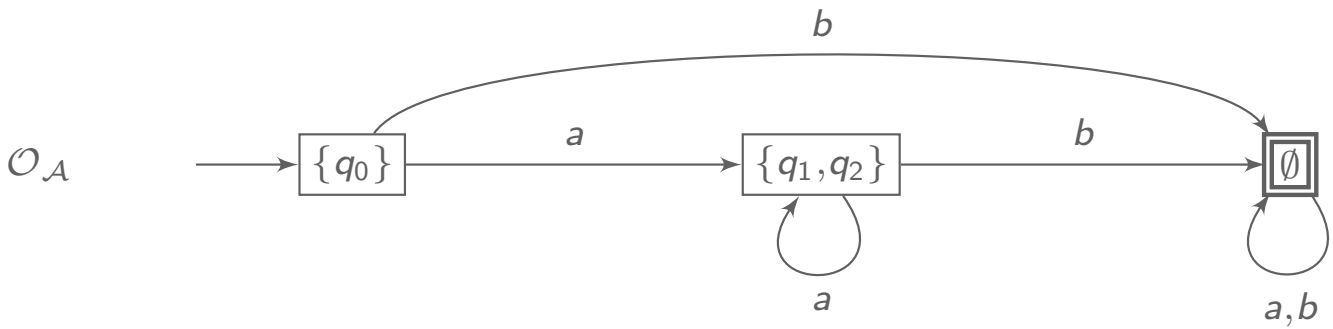

$\mathcal{A}$ is not IF-diagnosable
iff
there exists a state (q, U) in a BSCC of $\mathcal{A} \times \mathcal{O}_{\mathcal{A}}$ with q faulty and $U \neq \emptyset$.

Diagnoser synthesis

For every IF-diagnosable system with n correct states
one can build an IF-diagnoser with at most 2^n states.

Diagnoser derived from observer $\mathcal{O}_{\mathcal{A}}$: emits ✗ in state $\emptyset$.

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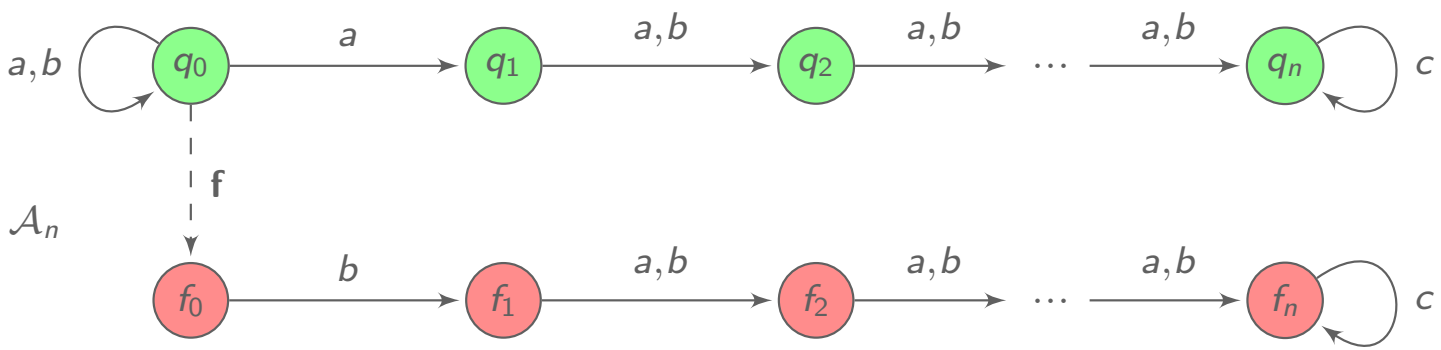
There is a family $(\mathcal{A}_n)$ of IF-diagnosable systems such that
$\mathcal{A}_n$ has $n+1$ correct states and any IF-diagnoser needs 2^n states.



Diagnosability is in PSPACE

Diagnosability is decidable in PSPACE for probabilistic systems.

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Sketch of proof

- relies on the characterisation on $\mathcal{A} \times \mathcal{O}_{\mathcal{A}}$
- avoids building the product
- uses Savitch's theorem for appropriate guesses

Diagnosability is PSPACE-hard

$\mathcal{L} \subseteq \Sigma^*$ is *eventually universal* if $\exists v \in \Sigma^*, v^{-1}\mathcal{L} = \Sigma^*$.

The eventual universality problem for NFA is PSPACE-hard.

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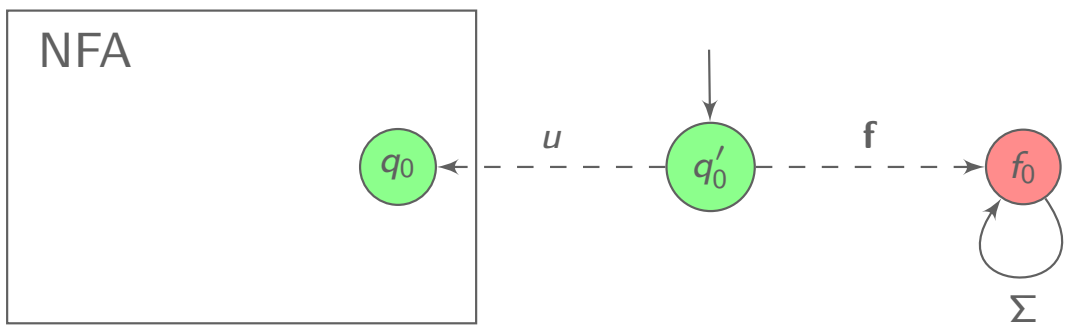
Reduction from eventual universality to diagnosability.



$\mathcal{A}$ not diagnosable iff
$\mathcal{A} \times \mathcal{O}_{\mathcal{A}}$ contains a BSCC where each state has the form (f_0, U) with $U \neq \emptyset$

Comparison with non-probabilistic discrete event systems

Diagnosability is PSPACE-complete for probabilistic systems.

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Erroneous adaptation to probabilistic case in [Chen, Kumar TASE 2013].

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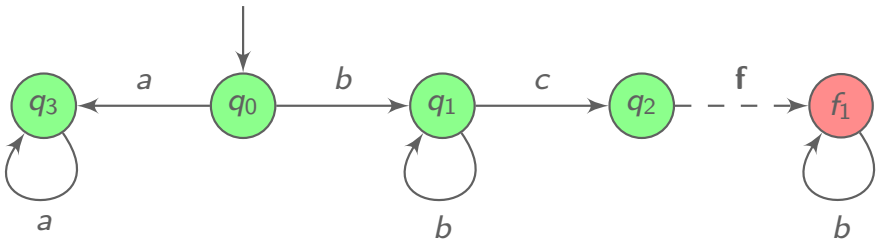

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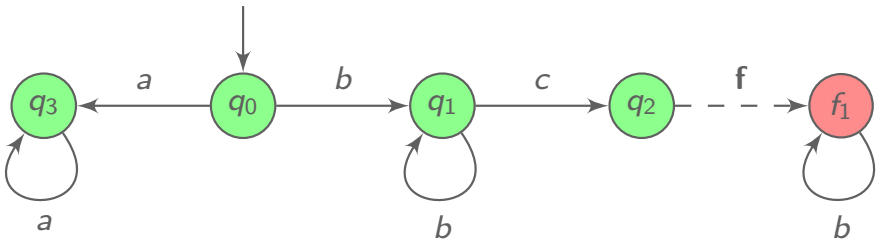

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Two notions of **soundness**:

- sure: if a fault is claimed, a fault will occur
- almost-sure: if a fault is claimed, a fault will almost-surely occur

Reactivity: a fault is detected at least k steps before occurrence.

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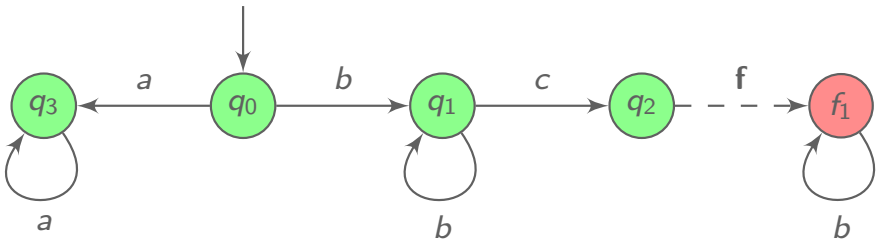

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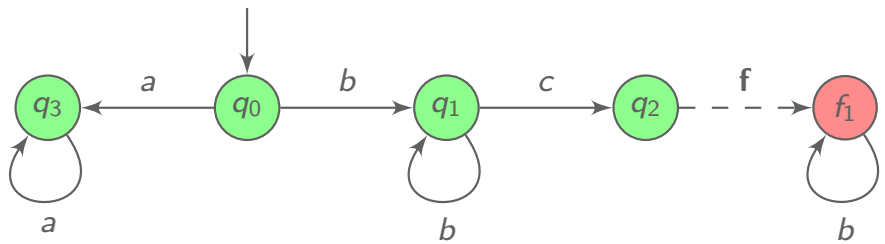

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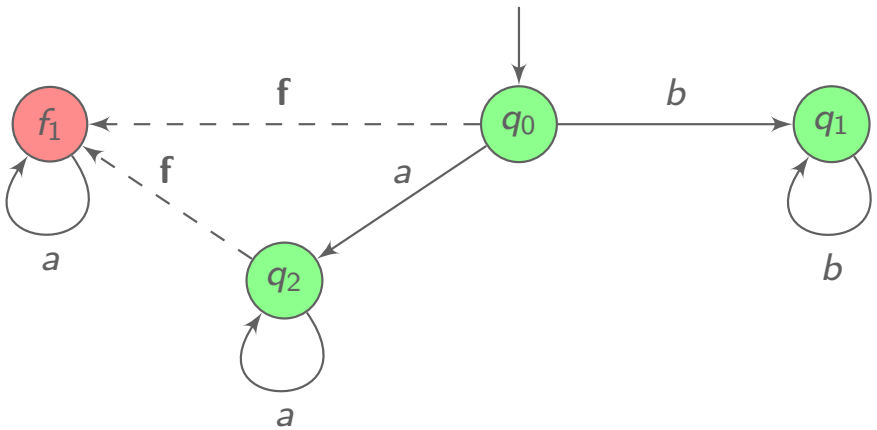

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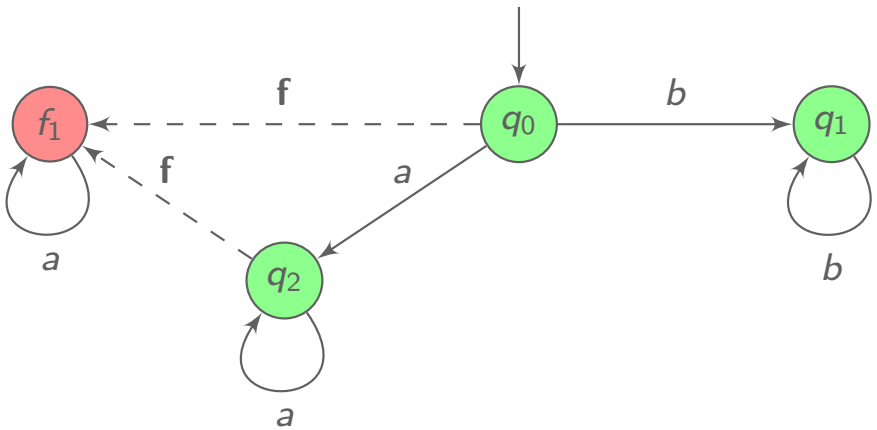

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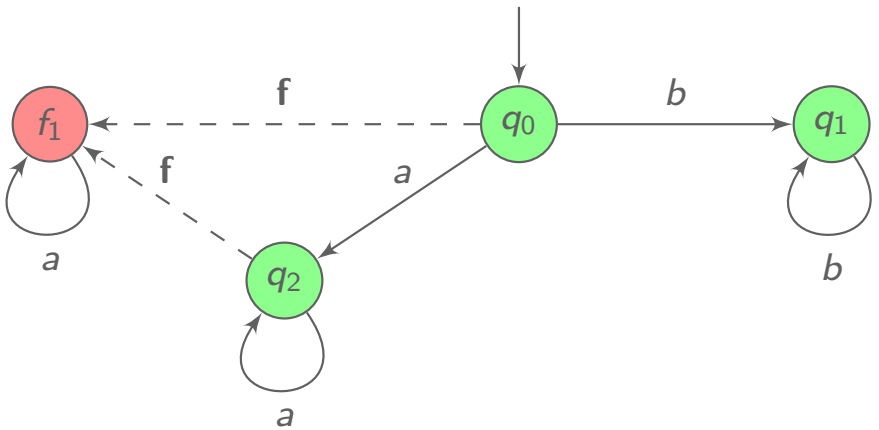

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- Investigation of semantical issues
- Tight complexity bounds for diagnosability and diagnoser synthesis problems
- Introduction of prediagnosability

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Future work

- Approximate diagnosis (relaxing soundness)
- Other paradigms related to partial observation (detectability, opacity, etc.)
- Space and time optimisation of observations