Active diagnosis for probabilistic systems

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Serge Haddad, Loïc Hélouët

Inria

Diagnosis

Objective: tell whether a fault occurred, based on observations.

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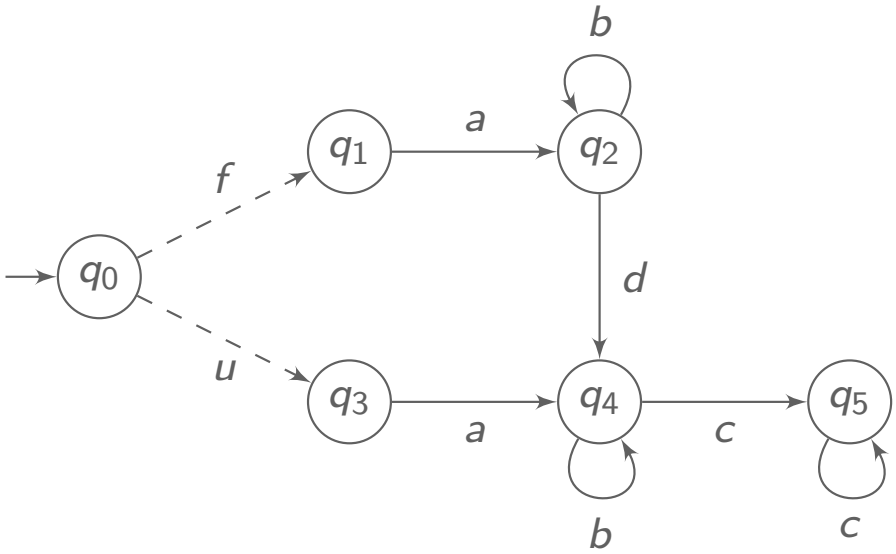

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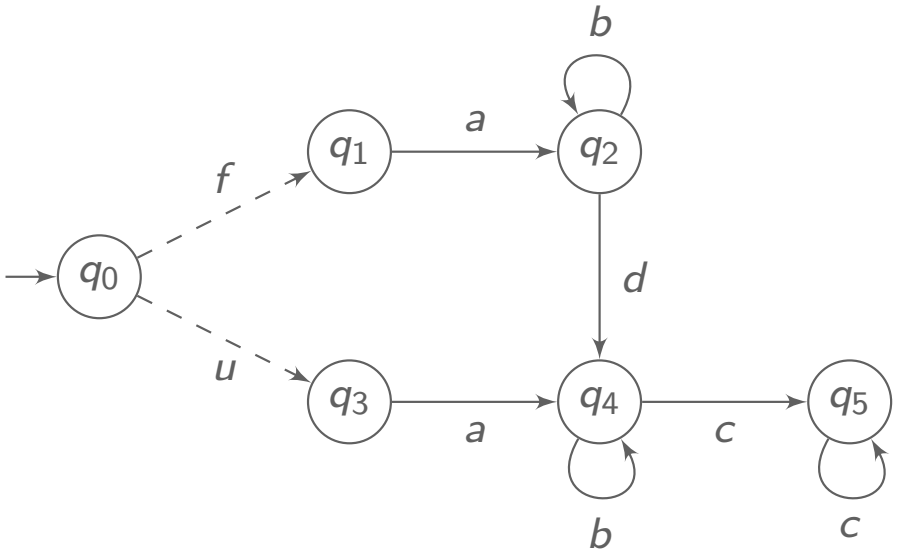

ac^ω	✓	correct
adc^ω	✗	faulty
ab^ω	?	ambiguous

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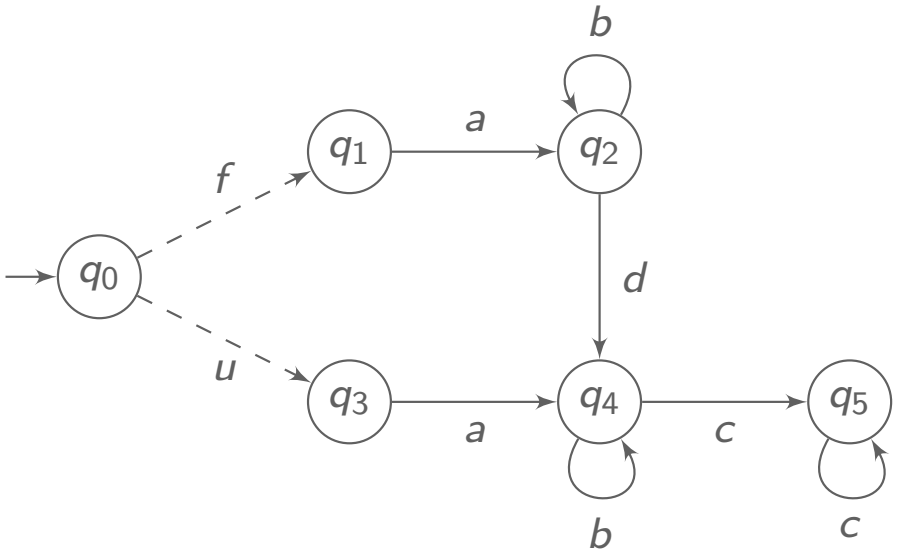

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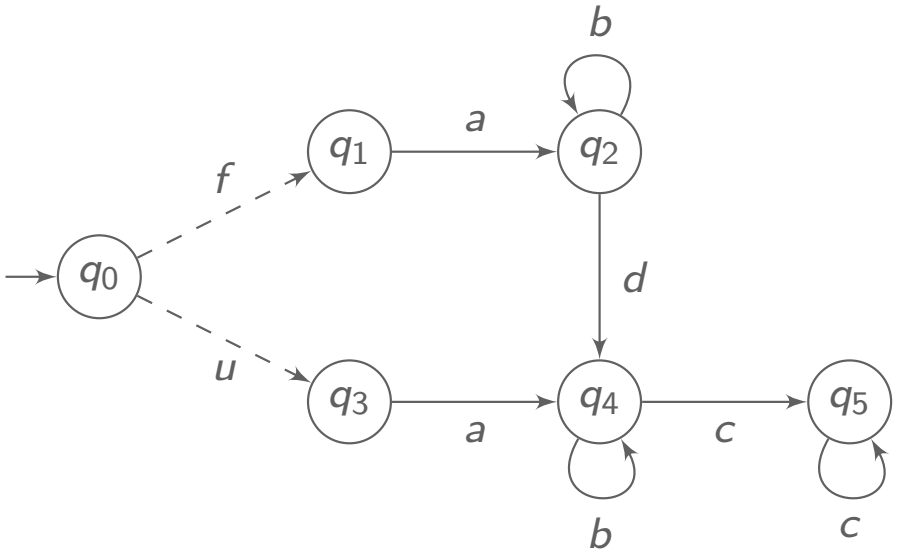

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Diagnosability: all infinite observed sequences are unambiguous.

Active diagnosis [HHMS-fsttcs13]

Objective: control the system so that it is diagnosable

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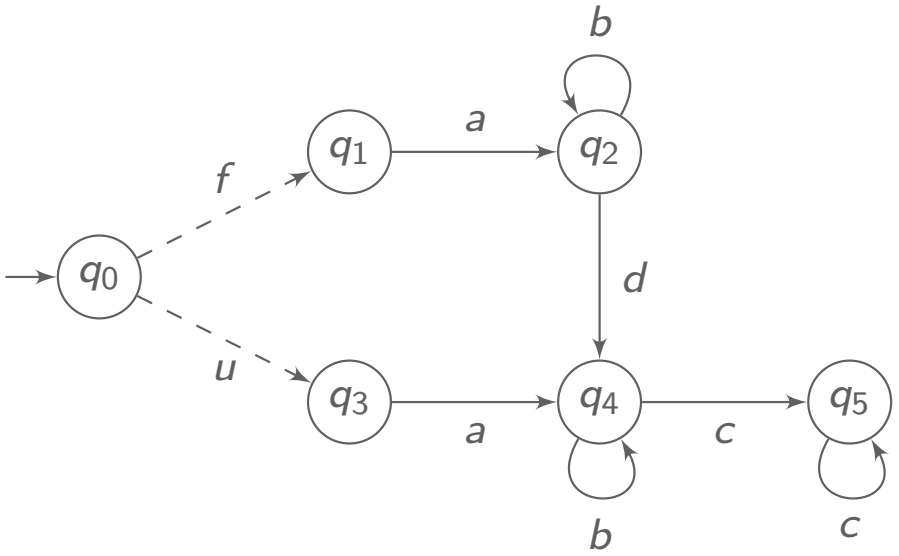

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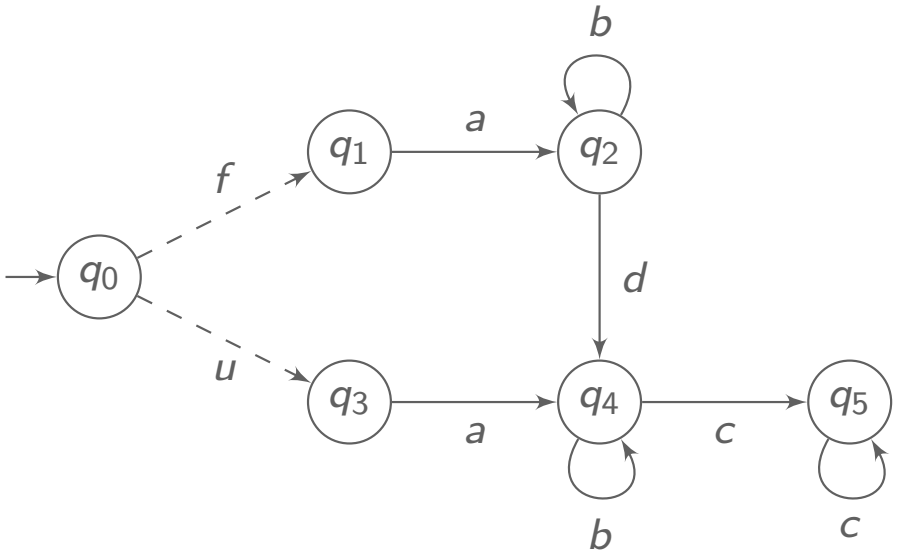

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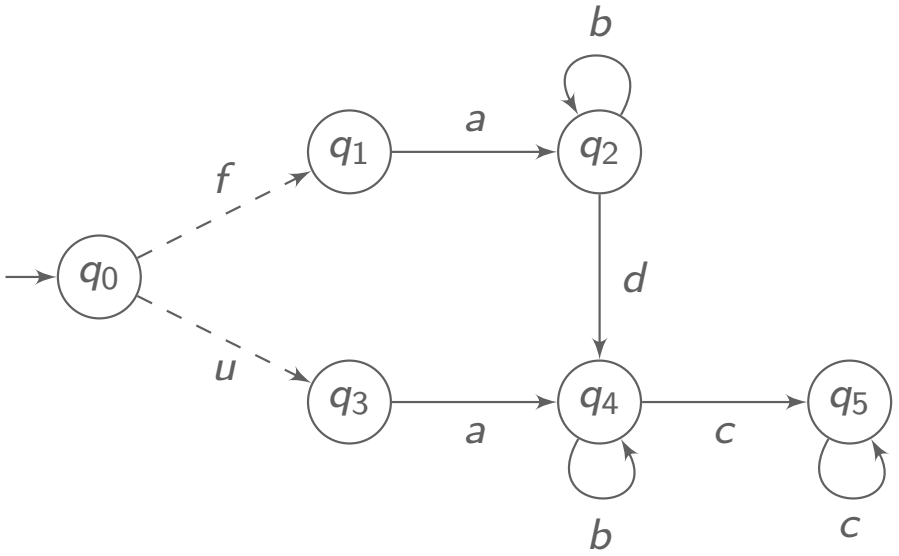

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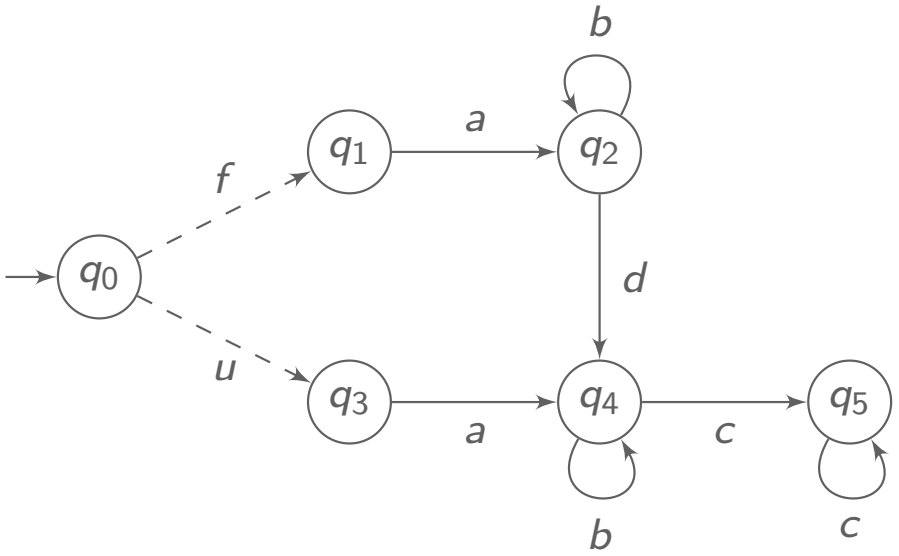

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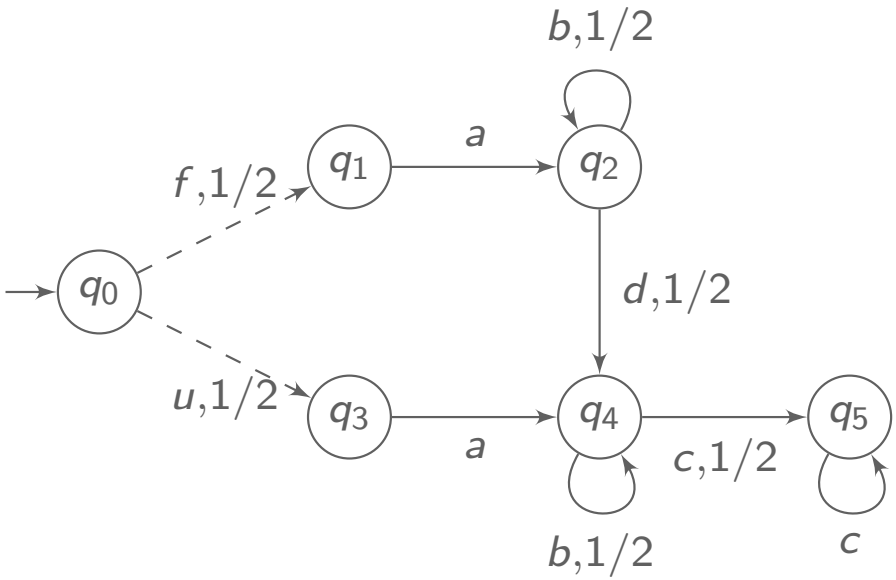
Active diagnosis problem
does there exist a controller such that the system is diagnosable?

caution: the system must remain *live*.

Diagnosis of probabilistic systems



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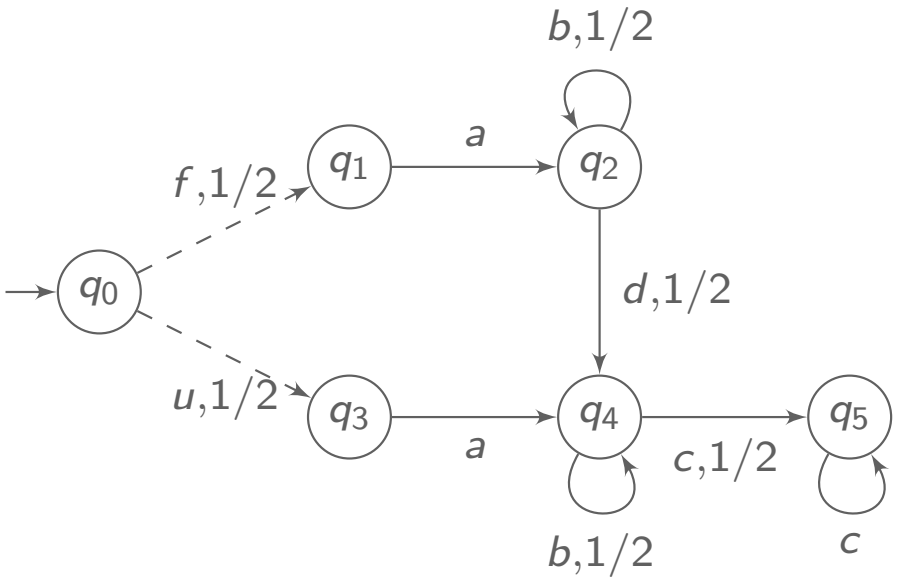

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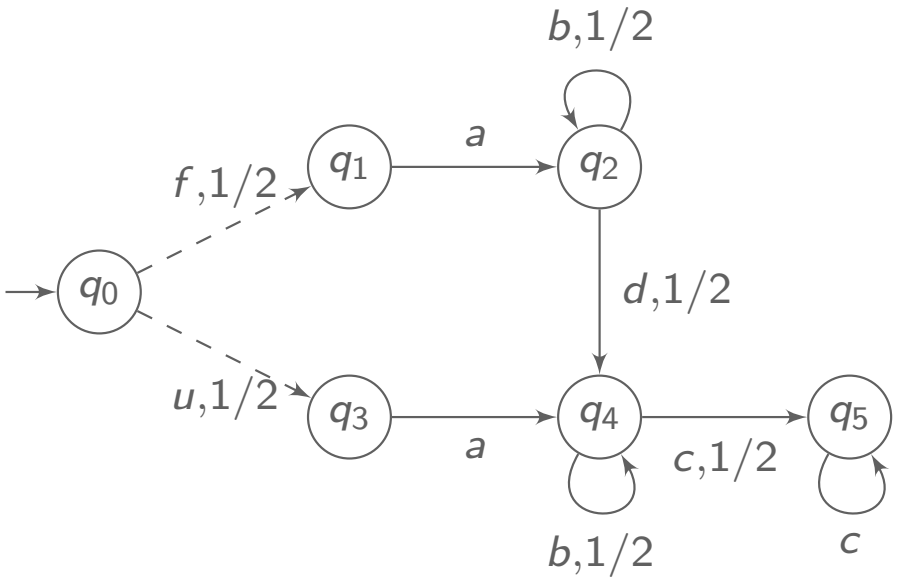

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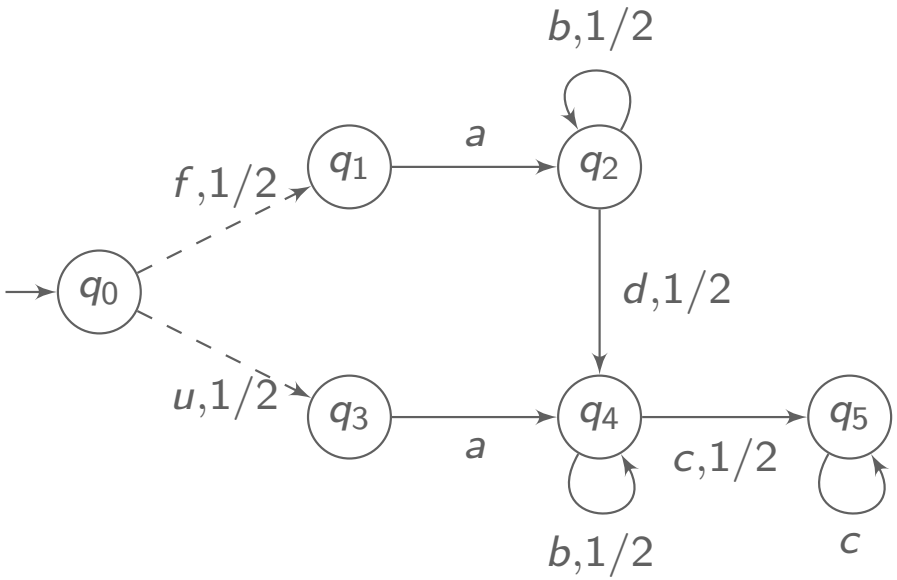

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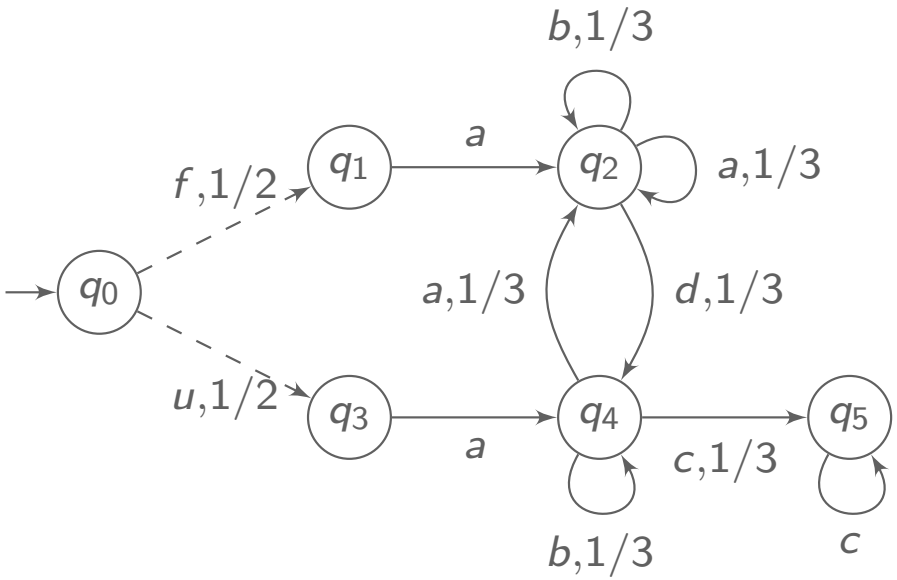
Almost-sure diagnosability: **almost all** runs have unambiguous observation

Active probabilistic diagnosis

Objective: control the system so that it is almost-surely diagnosable



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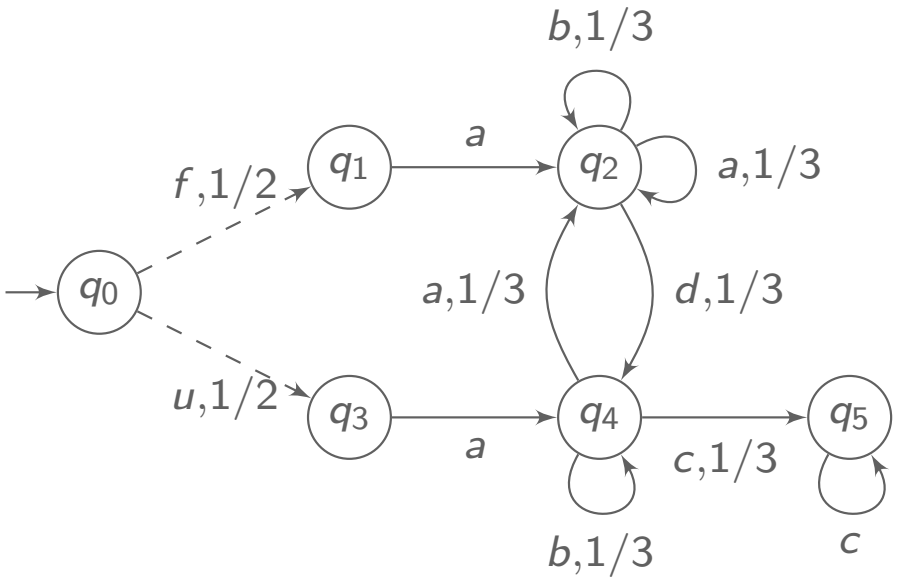

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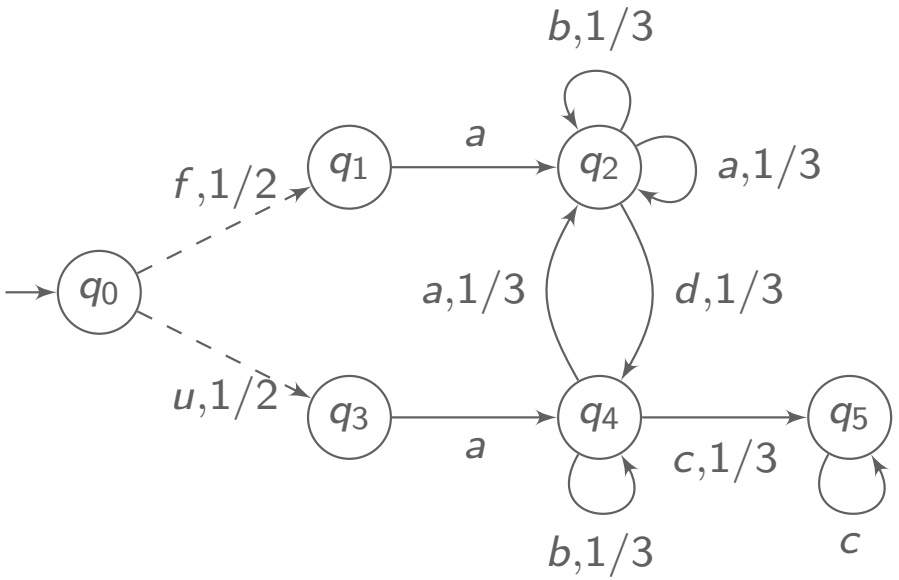

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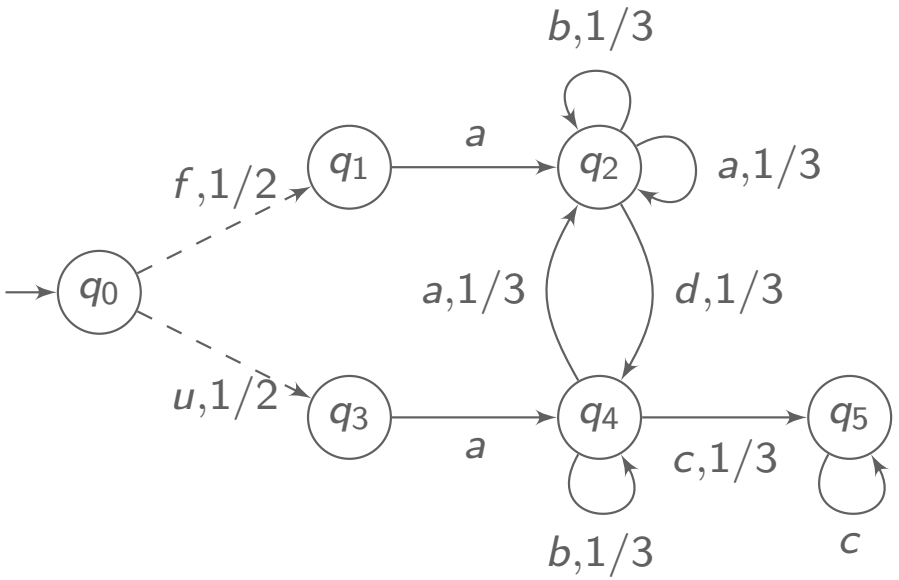

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Active probabilistic diagnosis problem
does there exist a controller such that the system is almost-surely diagnosable?

Active probabilistic diagnosis

The active probabilistic diagnosis problem is EXPTIME-complete.

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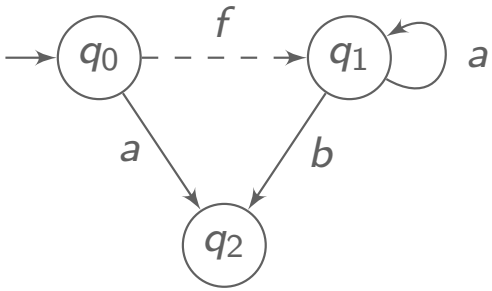
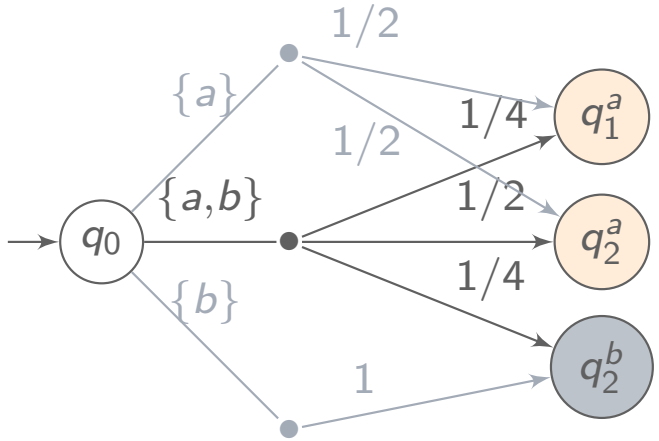
Proof idea (upper bound)

- characterize unambiguous sequences by deterministic Büchi automaton $\mathcal{B}$ [HHMS-fsttcs13]
- build the product of probabilistic LTS with $\mathcal{B}$
- view it as POMDP $\mathcal{P}$





- decide whether there is an almost-surely winning strategy for the Büchi condition on $\mathcal{P}$ [BBG-fossacs08,CDGH-mfcs10]

Safe active probabilistic diagnosis

Objective: avoid fault-provocative controllers

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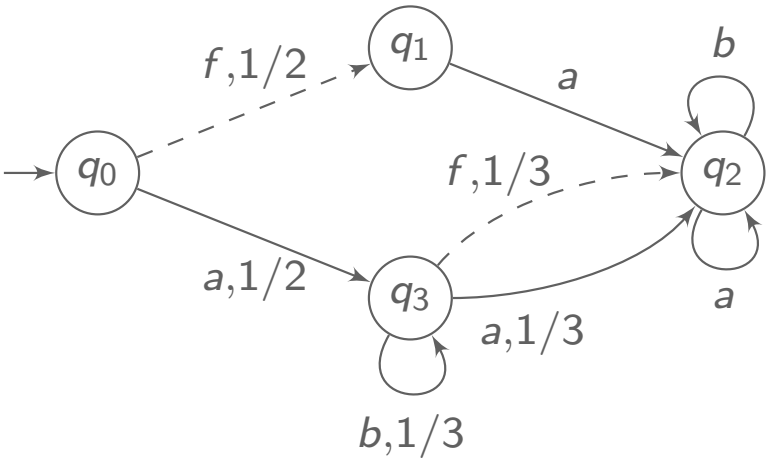

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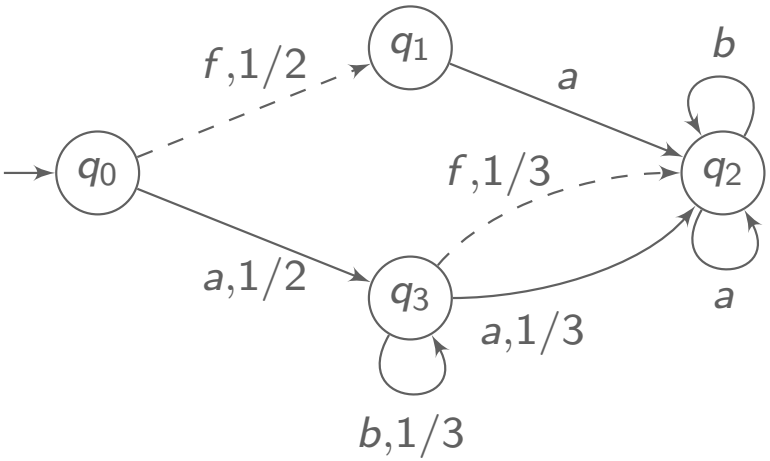

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forbid a after first a
$\Longrightarrow$ diagnosable...
but almost all sequences faulty!

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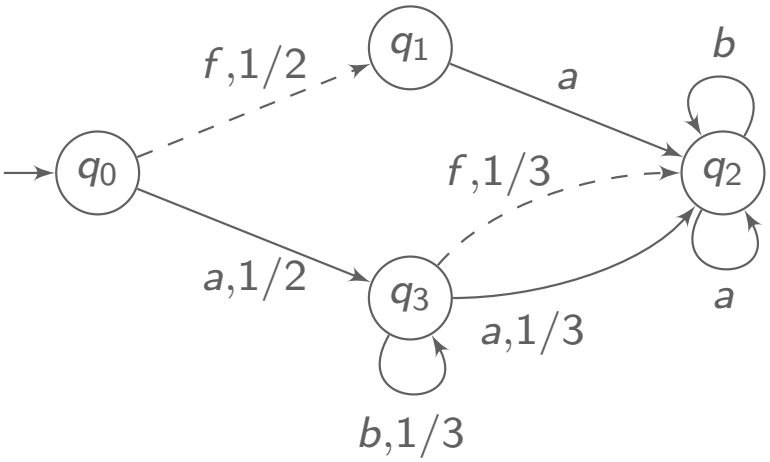

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Safe active probabilistic diagnosis
does there exist a controller such that the system is almost-surely diagnosable **and** correct runs have positive probability?

Safe active probabilistic diagnosis – beliefs are not enough!

Infinite memory is needed for safe probabilistic diagnosis.

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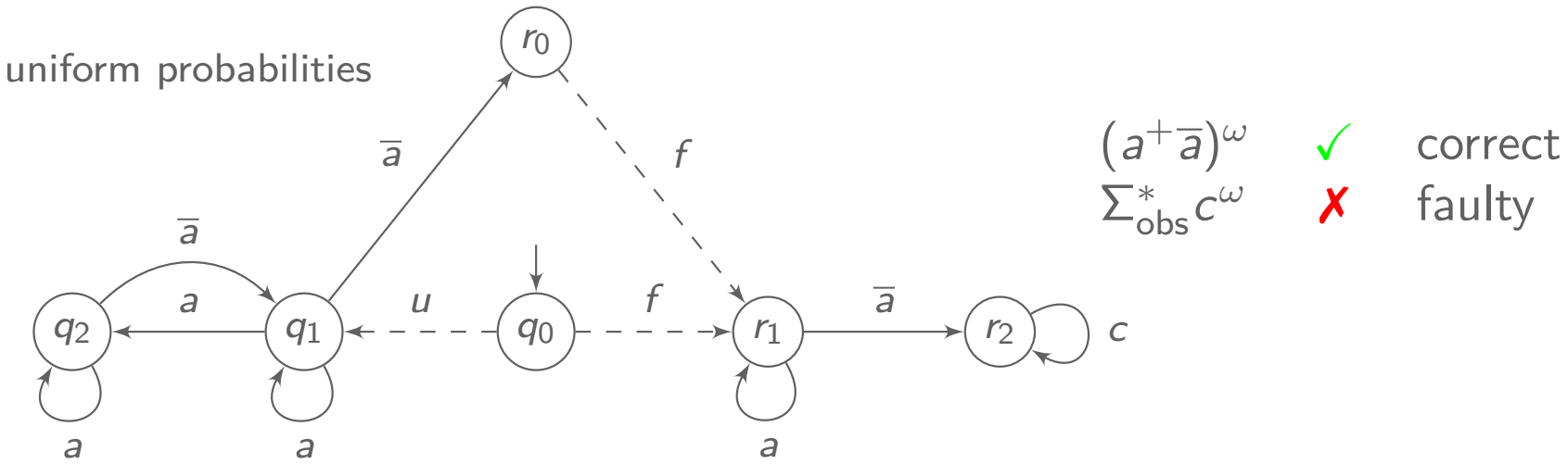
Infinite memory is needed for safe probabilistic diagnosis.



- Safe controller: infinitely many $\overline{a}$'s to diagnose all faults...
  but not too often, to have non-negligible correct runs
- Finite-memory controllers almost-surely force a fault.

Safe active probabilistic diagnosis

The safe active probabilistic diagnosis problem is undecidable.

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Proof idea

- reduction from the existence, in a blind POMDP, of a strategy ensuring a Büchi objective with positive probability
- mimicking example where infinite-memory is needed

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New result for POMDP

The existence of a strategy ensuring a Büchi objective almost-surely and a safety objective with positive probability is undecidable.

while independently, both problems are decidable

Conclusion

Summary

- (safe) active diagnosis problem for probabilistic systems
- partially observable Markov decision process framework
- active probabilistic diagnosis EXPTIME-complete
- safe active probabilistic diagnosis
  - undecidable in general
  - EXPTIME-complete for **finite memory** controllers (new result)

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Future work

- combinations of objectives for POMDP
- towards quantitative questions
- predictability for probabilistic systems

Thanks for your attention

Details for the undecidability proof