Introduction to Radiometry and Photometry

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Radiometry

• The goal of a global illumination algorithm is to compute a steady-state distribution of light in a scene
• To compute this distribution, we need an understanding of the physical quantities that represent light energy
• *Radiometry* is the basic terminology used to describe light
Photons

• The basic quantity in lighting is the photon

• The energy (in Joule) of a photon with wavelength $\lambda$ is: $q_\lambda = \frac{hc}{\lambda}$
  – $c$ is the speed of light
    • In vacuum, $c = 299,792,458$ m/s
  – $h \approx 6.63 \times 10^{-34}$ Js is Planck’s constant
(Spectral) Radiant Energy

• The *spectral radiant energy*, $Q_\lambda$, in $n_\lambda$ photons with wavelength $\lambda$ is

$$Q_\lambda = n_\lambda q_\lambda$$

• The *radiant energy*, $Q$, is the energy of a collection of photons, and is given as the integral of $Q_\lambda$ over all possible wavelengths:

$$Q = \int_0^\infty Q_\lambda d\lambda$$
Radiant Power or Radiant Flux

- *Radiant flux*, also called *radiant power*, is the time rate flow of radiant energy

  \[ \Phi = \frac{dQ}{dt} \]

- Flux expresses how much energy (Watts = Joule/s) flows to/through/from an (imaginary) surface per unit time

- For wavelength dependence, *spectral radiant flux* is defined as

  \[ \Phi_\lambda = \frac{dQ_\lambda}{dt} \]
Radiant Flux Area Density

- The *radiant flux area density* is defined as the differential flux per differential area $d\Phi/dA$
  - In English: The energy arriving at or leaving a surface over a short interval of time
- Traditionally, radiant flux area density is separated into *irradiance*, $E$, which is flux arriving at a surface and *radiant exitance*, $M$, which is flux leaving a surface
  - Radiant exitance is also known as *radiosity*, denoted $B$
Radiance

- Probably, the most important quantity in global illumination is *radiance*
- Radiance is defined as emitted flux per unit projected area per unit solid angle \(W/(\text{steradian} \times \text{m}^2)\)
- Intuitively, radiance tells us how much energy leaves a small area per unit time in a given direction

\[
L = \frac{d^2 \Phi}{d\omega \, dA \cos \theta}
\]

- Hemisphere of radius 1
- Surface of area A
Solid Angle

- **Solid angle** is the measure for ‘angles’ in 3D
  - The unit for solid angle is steradians, $\omega \in [0, 4\pi]$

- The solid angle subtended by an object is defined as the area of the object projected onto a sphere of radius 1 centered at the viewpoint

- The ‘size’ of a differential solid angle in spherical coordinates is $d\omega = \sin\theta d\theta d\phi$
Solid Angle

hemisphere of radius 1

Surface of area $A$

$d\omega$
Back To Radiance

- Radiance is defined as flux per unit projected area per unit solid angle \((W/(\text{steradian}\cdot\text{m}^2))\)

\[
L = \frac{d^2\Phi}{d\omega dA \cos \theta}
\]

- An important property of radiance is that, in vacuum, it is constant along a line of sight
Scattering of Light

• When light reaches a surface, it is either scattered or absorbed
  – We assume that the light is scattered immediately after reaching the surface
    • Thus, we ignore fluorescence effects
  – We also assume that light incident at some point also exits at that same point
    • This effectively means no subsurface scattering
BRDF

- A ray of light hits a surface:
  - arriving from a direction $k_i$,
  - and reflected in the direction $k_o$
- How much of this light is reflected in the direction $k_o$?
- This question is answered by the bidirectional reflectance distribution function, BRDF
The BRDF is a 4 dimensional function defined as:

\[ f_r(x, k_i, k_o) = \frac{dL_s(x, k_o)}{dE(x, k_i)} = \frac{dL_s(x, k_o)}{L_i(x, k_i) \cos \theta_i d\omega_i} \]

- BRDF could change over a surface (texture)
- \( L_s \) is the outgoing radiance
- \( L_i \) is the incoming radiance
- \( d\omega_i \) is the differential solid angle associated with the incident direction
BRDF Properties

• A brdf can take on any positive value
  \[ f_r(x, k_i, k_o) \in [0; \infty[ \]
• The value of a brdf remains unchanged if the incident exitant directions are interchanged
  \[ f_r(x, k_i, k_o) = f_r(x, k_o, k_i) \]
• A physically plausible brdf conserves energy, that is:
  \[ \forall k_i : \int_{all k_o} f_r(x, k_i, k_o) \cos \theta_o d\omega \leq 1 \]
Directional Hemispherical Reflectance

• Related to the BRDF, we may wish to know exactly how much light is reflected due to light coming from a fixed direction $k_i$

• This is answered by the *directional hemispherical reflectance*, $R(k_i)$, given as:

$$R(x, k_i) = \int_{\text{all } k_o} f_r(x, k_i, k_o) \cos \theta_o \, d\omega$$
Example

• A Lambertian surface is an idealized diffuse surface with a constant BRDF, \( f_r = c \)

\[
R(x, k_i) = \int_{\text{all } k_i} c \cos \theta_o d\omega
\]

\[
= \int_0^{2\pi} \int_0^{\pi/2} c \cos \theta \sin \theta d\theta d\phi
\]

\[
= \pi c
\]

• So, for a perfectly reflecting Lambertian surface, we have \( f_r = 1/\pi \), and if \( R(x, k_i) = r \), \( f_r = r/\pi \)
The Rendering Equation

• Consider again the BRDF: \( f_r(x, k_i, k_o) = \frac{dL_s(x, k_o)}{L_i(x, k_i) \cos \theta_i d\omega_i} \)

• Rearranging the terms, we get

\[
dL_s(x, k_o) = f_r(x, k_i, k_o) L_i(x, k_i) \cos \theta_i d\omega_i
\]

• Integrating over the entire hemisphere, we get the reflected radiance

\[
L_s(x, k_o) = \int_{\Omega} f_r(x, k_i, k_o) L_i(x, k_i) \cos \theta_i d\omega_i
\]

– This is known as the rendering equation

– For translucent objects, we need the lower hemisphere as well
Alternate Transport Equation

• The rendering equation describes the reflected radiance due to incident radiance on the entire hemisphere.

• Sometimes we’ll need the transport equation in terms of surface radiance only.
  – Because radiance is constant along a straight line, the field radiance \( L_i(x, k_i) \) is equal to the surface radiance from some surface: \( L_i(x, k_i) = L_i(x', -k_i) \).
  – The solid angle subtended by a surface is
  
  \[
  d\omega = \frac{dA \cos \theta'}{|x - x'|^2}
  \]
Alternate Transport Equation

• Putting this together, we get

\[ L_s(x, k_o) = \int_{\text{all } x_i} \frac{f_r(x, k_i, k_o) L_s(x', x - x') v(x, x') \cos \theta_i \cos \theta' dA}{|x - x'|^2} \]

– Where \( v(x, x') \) is a visibility term, equal to 1 if \( x \) and \( x' \) are mutually visible and 0 otherwise

– \( K_i = x'x \)

• Integral equation: to be solved