Ray Tracing
Texture Mapping

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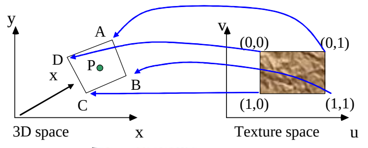
Texture Mapping

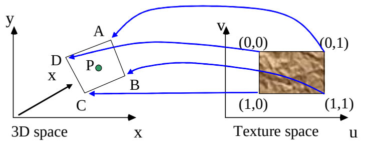
- Texture = 2D Image (we do not consider 3D textures)
- Texture : represented by a 2D array of RGB triplets
- Triplet: Color, Normal or another thing
- Normalized texture space: (u,v), u et v ranging from 0 to 1
- For an intersection point P: compute its texture coordinates either directly or by interpolation



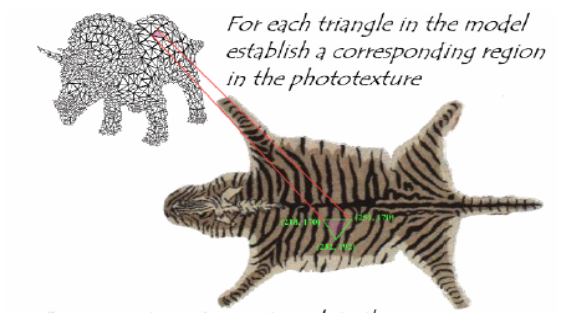
Texture Mapping

Polygon: Triangle or quadrilateral

- Each vertex of the 3D polygon is assigned texture coordinates
- For an intersection point P: compute its texture by bilinear interpolation of the texture coordinates of the polygon's vertices

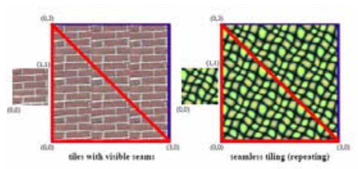


Texture Mapping


For each triangle in the model
establish a corresponding region
in the phototexture

Texture Mapping

- Specify a texture coordinate (u,v) at each vertex
- Canonical texture coordinates (0,0) → (1,1)



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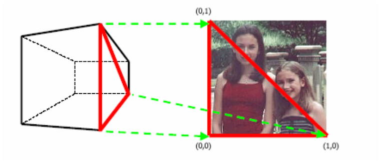
Texture Mapping: Interpolation

- Specify a texture coordinate (u,v) at each vertex
- Interpolate the texture values of intersection points lying on the polygon using those of its vertices



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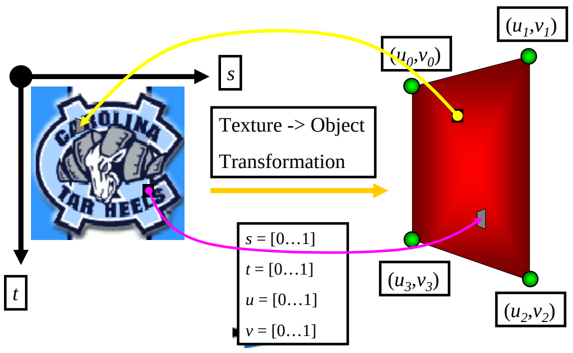
Texture Mapping: Interpolation



Texture Mapping

- Common texture coordinate mapping
  - planar
  - Cylindrical
  - Spherical

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Texture Mapping

- Planar: Triangle
  - Barycentric coordinates: a way to parametrize a triangle with P_0, P_1 and P_2 as vertices
  - Let P be a point on the triangle and $\beta_0, \beta_1, \beta_2$ its barycentric coordinates

  -Thus: $P = \beta_0 P_0 + \beta_1 P_1 + \beta_2 P_2$

  $P = (1 - \beta_1 - \beta_2) P_0 + \beta_1 P_1 + \beta_2 P_2$

  Since: $\beta_0 + \beta_1 + \beta_2 = 1$

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Texture Mapping

- Planar: Triangle
  - Given an intersection P lying on a triangle
  - Compute its texture coordinates (s,t) by solving the following linear system:

$$P = (1 - \beta_1 - \beta_2) P_0 + \beta_1 P_1 + \beta_2 P_2$$

$$P - P_0 = (P_2 - P_1\ ,\ P_1 - P_0) \begin{pmatrix} \beta_2 \\ \beta_1 \end{pmatrix}$$

  - Unknowns: β_1 and β_2

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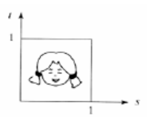
Texture Mapping

Cylinder

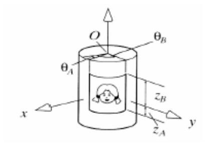
- P(x,y,z): point on the cylinder
- (s,t): texture coordinates



$$\theta \in [0, 2\pi],\ \ z \in [z_A, z_B]$$

$$x = r\cos\theta,\ y = r\sin\theta,\ z \in [z_A, z_B]$$

$$s = \frac{\theta - \theta_A}{\theta - \theta_B} \qquad t = \frac{z - z_A}{z_B - z_A}$$



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Texture Mapping

Cylinder

- How to computeTexture coordinates
- Given an intersection P lying on a cylinder
- Compute its texture coordinates (s,t) as:

$$s = (a\cos(x/r) - \theta_A)/(\theta_B - \theta_A)$$

$$t = (z - z_A)/(z_B - z_A)$$

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Texture Mapping

Sphere

- P=(x,y,z) on the sphere
- (s,t): texture coordinates

$$x = r\sin\theta\cos\varphi,\ \ y = r\sin\theta\sin\varphi,\ \ z = r\cos\theta$$

$$\theta\in[0,\pi], \varphi\in[0,2\pi] \qquad s=\frac{\theta}{\pi},\ t=\frac{\varphi}{2\pi}$$

Texture Mapping

Sphere

- How to compute Texture coordinates
- Given an intersection P lying on a sphere
- Compute its texture coordinates (s,t) as:

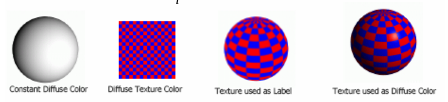
$$s = a\cos(z/r)/\pi$$

$$t = a\cos(\mathrm{x}/(r\sin(\pi\ \mathrm{s})))/2\pi$$

Texture Mapping & Illumination

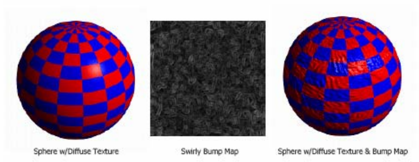
- Texture mapping can be used to alter some or all of the constants in the illumination equation:
  - pixel color, diffuse color, alter the normal, ….
- Classical texturing: diffuse color changes over a surface and is given by a 2D texture which is an image

$$I_{local} = \sum_{i=0}^{nbLum} I_i \times \frac{vis(i)}{d_i^{\,2}} \times \left(k_d\left(\vec{N}\cdot\vec{L_i}\right) + k_s\left(\vec{R_i}\cdot\vec{V}\right)^n\right)$$


Constant Diffuse Color — Diffuse Texture Color — Texture used as Label — Texture used as Diffuse Color

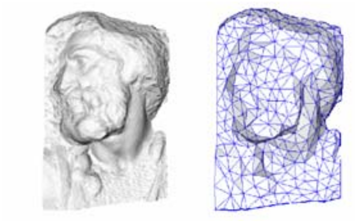
Bum Mapping

- Use textures to alter the surface normal
  - Does not change the actual shape of the surface
  - Just shaded as if it were a different shape

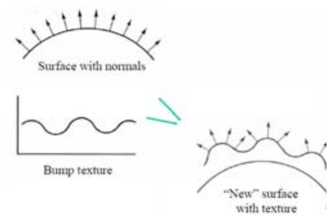

Sphere w/Diffuse Texture — Swirly Bump Map — Sphere w/Diffuse Texture & Bump Map

Bum Mapping

- Add more realism to synthetic images without adding a lot of geometry



Bum Mapping



Bum Mapping

- Normal of bumped surface, so-called *perturbed* normal:
- Derivation can be found in "Simulation of Wrinkled Surfaces"

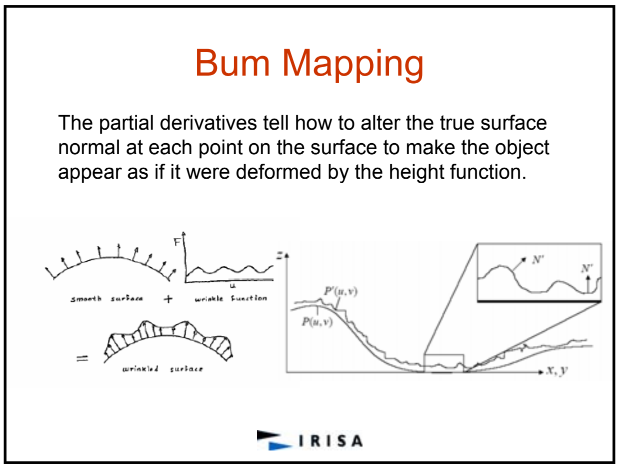
  James F. Blinn

  SIGGRAPH '78 Proceedings, pp. 286-292, 1978

  (Pioneering paper...)
- (Mis-)use texture to store either:
  - perturbed normal map
  - bump–map itself

Bum Mapping

- The light at each point depends on the normal at that point.
- Take a smooth surface and perturb it with a function B.
- But we don't really perturb that surface (that is not displacement mapping).
- We modify the normals with the function B(u,v), measuring the displacement of the irregular surface compared to the ideal one.
- we are only shading it as if it were a different shape! This technique is called bump mapping.
- The texture map is treated as a single-valued height function.
- The value of the function is not actually used, just its partial derivatives.



Bum Mapping

The partial derivatives tell how to alter the true surface normal at each point on the surface to make the object appear as if it were deformed by the height function.

F
u
smooth surface
+
wrinkle function
=
wrinkled surface
z
P'(u,v)
P(u,v)
N'
N'
x, y

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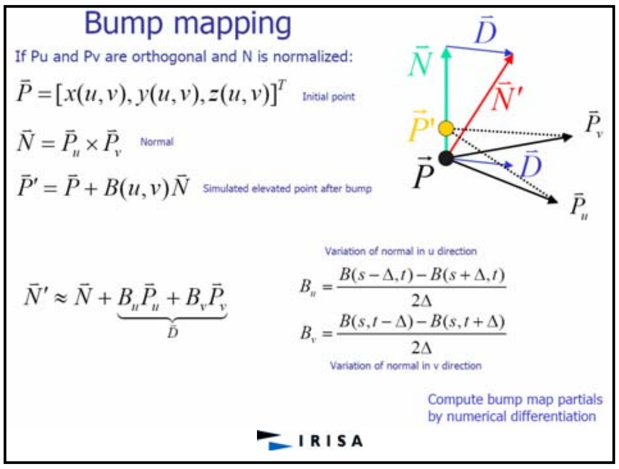

Bump mapping

If Pu and Pv are orthogonal and N is normalized:

$\vec{P} = [x(u,v), y(u,v), z(u,v)]^T$ Initial point

$\vec{N} = \vec{P}_u \times \vec{P}_v$ Normal

$\vec{P}' = \vec{P} + B(u,v)\vec{N}$ Simulated elevated point after bump

$$\vec{N}' \approx \vec{N} + \underbrace{B_u \vec{P}_u + B_v \vec{P}_v}_{\vec{D}}$$

Variation of normal in u direction

$$B_u = \frac{B(s-\Delta,t) - B(s+\Delta,t)}{2\Delta}$$

$$B_v = \frac{B(s,t-\Delta) - B(s,t+\Delta)}{2\Delta}$$

Variation of normal in v direction

Compute bump map partials by numerical differentiation

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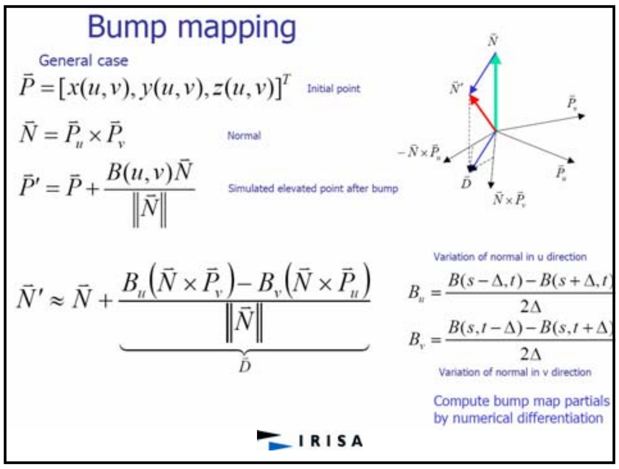

Bump mapping

General case

$\vec{P} = [x(u,v), y(u,v), z(u,v)]^T$ Initial point

$\vec{N} = \vec{P}_u \times \vec{P}_v$ Normal

$\vec{P}' = \vec{P} + \dfrac{B(u,v)\vec{N}}{\|\vec{N}\|}$ Simulated elevated point after bump

$$\vec{N}' \approx \vec{N} + \underbrace{\frac{B_u(\vec{N} \times \vec{P}_v) - B_v(\vec{N} \times \vec{P}_u)}{\|\vec{N}\|}}_{\vec{D}}$$

Variation of normal in u direction

$$B_u = \frac{B(s-\Delta,t) - B(s+\Delta,t)}{2\Delta}$$

$$B_v = \frac{B(s,t-\Delta) - B(s,t+\Delta)}{2\Delta}$$

Variation of normal in v direction

Compute bump map partials by numerical differentiation

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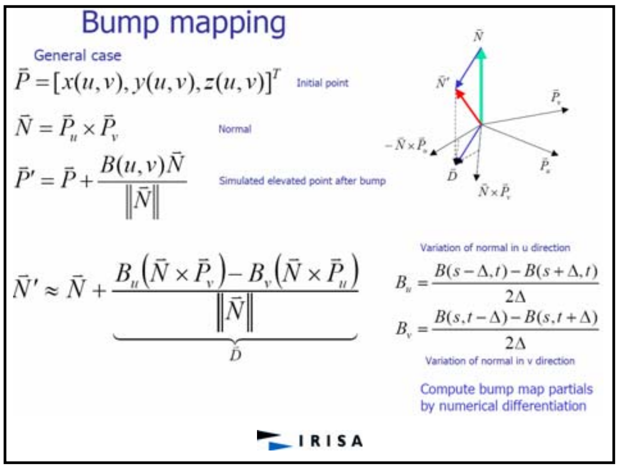

Bump mapping

General case

$\vec{P} = [x(u,v), y(u,v), z(u,v)]^T$ Initial point

$\vec{N} = \vec{P}_u \times \vec{P}_v$ Normal

$\vec{P}' = \vec{P} + \dfrac{B(u,v)\vec{N}}{\|\vec{N}\|}$ Simulated elevated point after bump

$$\vec{N}' \approx \vec{N} + \underbrace{\frac{B_u(\vec{N} \times \vec{P}_v) - B_v(\vec{N} \times \vec{P}_u)}{\|\vec{N}\|}}_{\vec{D}}$$

Variation of normal in u direction

$$B_u = \frac{B(s-\Delta,t) - B(s+\Delta,t)}{2\Delta}$$

$$B_v = \frac{B(s,t-\Delta) - B(s,t+\Delta)}{2\Delta}$$

Variation of normal in v direction

Compute bump map partials by numerical differentiation

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Bump mapping derivation

$$\bar{P}' = \bar{P} + \frac{B(u,v)\bar{N}}{\|\bar{N}\|}$$

$$\bar{P}'_u = \bar{P}_u + \frac{B_u\bar{N}}{\|\bar{N}\|} + \underbrace{\cancel{\frac{B\bar{N}_u}{\|\bar{N}\|}}}_{\approx 0}$$

Assume B is very small...

$$\bar{P}'_v = \bar{P}_v + \frac{B_v\bar{N}}{\|\bar{N}\|} + \underbrace{\cancel{\frac{B\bar{N}_v}{\|\bar{N}\|}}}_{\approx 0}$$

$$\bar{N}' = \bar{P}'_u \times \bar{P}'_v$$

$$\bar{N}' \approx \bar{P}_u \times \bar{P}_v + \frac{B_u(\bar{N}\times\bar{P}_v)}{\|\bar{N}\|} + \frac{B_v(\bar{P}_u\times\bar{N})}{\|\bar{N}\|} + \frac{B_u B_v(\bar{N}\times\bar{N})}{\|\bar{N}\|^2}$$

But $\bar{P}_u \times \bar{P}_v = \bar{N}$, $\bar{P}_u \times \mathrm{N} = -\bar{N}\times\bar{P}_u$ and $\bar{N}\times\bar{N} = 0$ so

$$\bar{N}' \approx \bar{N} + \frac{B_u(\bar{N}\times\bar{P}_v)}{\|\bar{N}\|} - \frac{B_v(\bar{N}\times\bar{P}_u)}{\|\bar{N}\|}$$

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Bum Mapping

Choice of F fonction

- Blinn has proposed various techniques:
- F defined analytically as a polynomial with 2 variables or a Fourier serie (very expensive approach)
- F defined by 2-entry table (poor results, requires large memory)
- F defined by 2-entry table smaller and an interpolation is performed to find inbetween values

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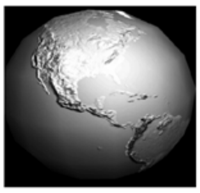
Bum Mapping

- Treat the texture as a single- valued height function
- Compute the normal from the partial derivatives in the texture



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Bump Mapping

- There are no bumps on the silhouette of a

  bump-mapped object
- Bump maps don’t allow self-occlusion or self-shadowing
- Problem solved with Displacement Mapping

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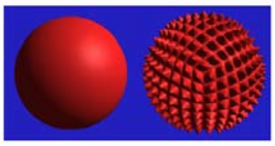
Displacement Mapping

- Use the texture map to actually move the surface point along the normal to the intersected point.
- The geometry must be displaced before visibility is determined, which is different from bump mapping



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Displacement Mapping

- Image from:
  Geometry Caching for
  Ray-Tracing Displacement Maps
- by Matt Pharr and Pat Hanrahan.
- *note the detailed shadows cast by the stones*

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Displacement Mapping

- Bump Mapping combined with texture

Schweppes
Black
Cola

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