

Knowledge-Based Policies for Qualitative Decentralized POMDPs



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Automation of complex tasks



Building surveillance

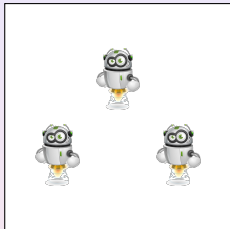


Nuclear decommissioning



Intelligent farming

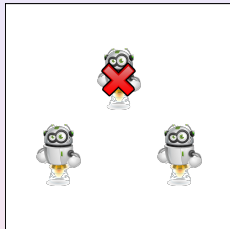
Multiple robots



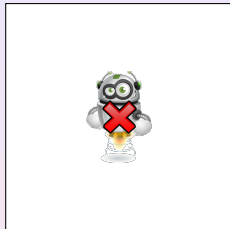
more robust/efficient than



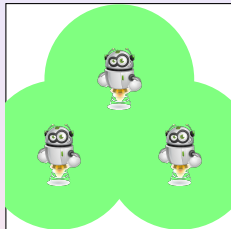
Multiple robots



more **robust**/efficient than



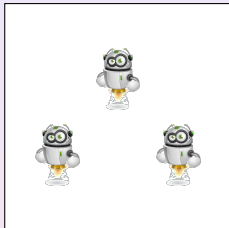
Multiple robots



more robust/**efficient** than



Multiple robots



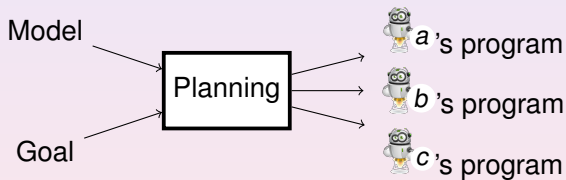
more robust/efficient than



Settings

- Cooperative agents;
- Common goal;
- Imperfect information;
- Decentralized execution.

Methodology



Need: understandable system

Motivation

- Legal issues in case of failure
- Interaction with humans

```
1  #include "fixed.h"
2  #include "fixed_private.h"
3
4  int16_T error;
5  int16_T torque_request;
6  D_Work DWork;
7  void fixed_step(void)
8  {
9      int16_T FilterCoefficient_m;
10     FilterCoefficient_m = (int16_T)((int32_T)((int16_T)(5403L * (int32_T)error >>
11     13U) - DWork.Filter_DSTATE) << 4U) * 17893L >> 14);
12     torque_request = (((int16_T)(12475L * (int32_T)error >> 14U) >> 1) +
13     (DWork.Integrator_DSTATE >> 2)) + (FilterCoefficient_m >> 1);
14     DWork.Integrator_DSTATE = (int16_T)((int32_T)((int32_T)error >> 13U) * 5243L >>
15     19U) + DWork.Integrator_DSTATE;
16     DWork.Filter_DSTATE = (int16_T)(5243L * (int32_T)FilterCoefficient_m >> 16U) +
17     DWork.Filter_DSTATE;
18 }
19
20 void fixed_initialize(void)
21 {
22     torque_request = 0;
23     (void) memset((void *)&DWork, 0,
24     sizeof(D_Work));
25     error = 0;
26 }
27
```



Our contribution: use of knowledge-based programs

KBP for agent a

listenRadio

if a **knows** *strike*

| toStation

else

| toAirport

KBP for agent b

readNewsPaper

if b **knows** *strike*

| toStation

else

| toAirport



- Operational Semantics for Knowledge-based programs;
- (Un)decidability/complexity and succinctness.

Extends: 📄 [Lang, Zanuttini, ECAI2012, TARK2013]

Outline

- 1 Knowledge-based programs
 - Epistemic formulas
 - Program constructions
- 2 Semantics
- 3 Mathematical properties
- 4 Conclusion

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Properties expressed in epistemic logic

Language constructions

room 43 is safe door 12 is locked ...

not ...

(... **or** ...)

(... **and** ...)

(... \rightarrow ...)

(... **knows** ...)

(... **knowswhether** ...)

Example

(**a knows** door 12 is locked) **and not** (**c knows** door 12 is locked)

a knowswhether (**c knows** door 12 is locked)

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Program constructions

Language constructions

turn left

stay

broadcast temperature

...; ...

if φ **then** ...**else** ...

while φ **do** ...

Example (knowledge-based program for agent a)

if a **knows** (door 12 is locked **and** *justobserved*(🔥)) **then**

turn left

broadcast temperature

else

stay

Outline

- 1 Knowledge-based programs
- 2 **Semantics**
 - Models: QdecPOMDP
 - Interlude: semantics of epistemic formulas
 - Operational semantics of KBPs
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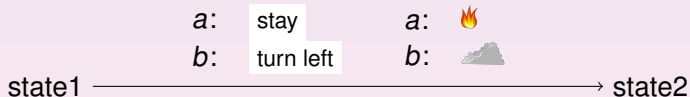
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QdecPOMDP

Qualitative decentralized Partially Observable Markov Decision Processes
= Concurrent game structures with observations.

Transitions of the form:



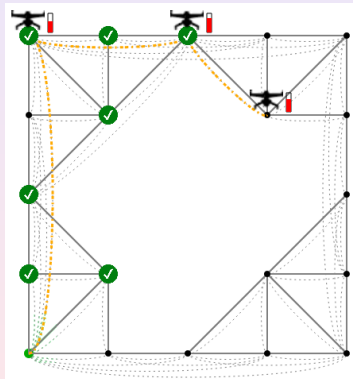
A non-empty set of possible initial states;

A set of goal states.

States

Typically, a state describes:

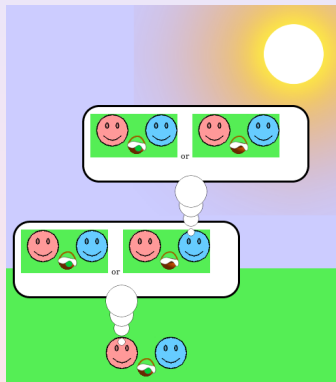
- positions of agents;
- battery levels;
- etc.



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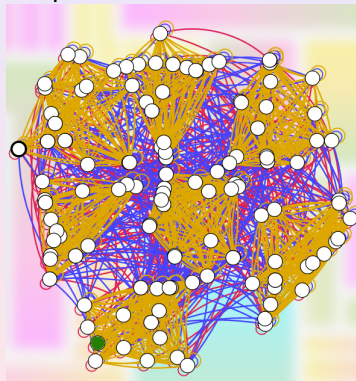
Prototype



<http://people.irisa.fr/Francois.Schwarzentruber/hintikkasworld/>

Semantics of epistemic formulas

Epistemic structure \mathcal{S}, w

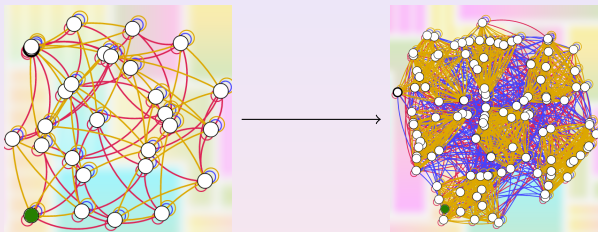


$\mathcal{S}, w \models a \text{ knows } \varphi$ iff for all u , $w \sim_a u$ implies $\mathcal{S}, u \models \varphi$.

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Operational semantics



Epistemic structure

Higher-order knowledge about:

- the current state of the QdecPOMDP;
- the current program counters in KBPs.

Assumptions

Common knowledge of:

- the QdecPOMDP;
- the KBPs;
- synchronicity of the system;
 - tests last 0 unit of time;
 - actions last 1 unit of time.

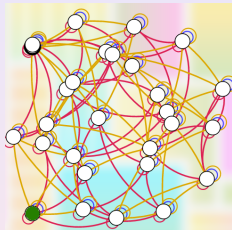
KBP for agent a

```
listenRadio
if  $a$  knows strike
|      toStation
else
|      toAirport
```

KBP for agent b

```
readNewsPaper
if  $b$  knows strike
|      toStation
else
|      toAirport
```


Epistemic structures at time T : worlds



Worlds = consistent histories of the form
 (wait few slides)

$$s^0 \vec{pc}^0 \boxed{\vec{obs}^1 s^1 \vec{pc}^1} \dots \boxed{\vec{obs}^T s^T \vec{pc}^T}$$

where

\vec{obs}^t	vector of observations at time t
s^t	state at time t
\vec{pc}^t	vector of program counters at time t



Epistemic structures at time t : indistinguishability relations

Agent a confuses two histories iff she has received the same observations.

$$\begin{array}{l}
 s^0 \vec{pc}^0 \boxed{\vec{obs}^1 s^1 \vec{pc}^1} \dots \boxed{\vec{obs}^T s^T \vec{pc}^T} \\
 \sim_a \\
 s'^0 \vec{pc}'^0 \boxed{\vec{obs}'^1 s'^1 \vec{pc}'^1} \dots \boxed{\vec{obs}'^T s'^T \vec{pc}'^T}
 \end{array}
 \quad \text{iff} \quad
 \begin{array}{l}
 \text{for all } t \in \{1, \dots, T\}, \\
 \vec{obs}_a^t = \vec{obs}'_a^t
 \end{array}$$

Program counters

Definition (Program counter)

(guard, action just executed, continuation)

● listenRadio
■ if $K_a strike$ then
| toStation
else
| toAirport
▲

(\top , start, ●)

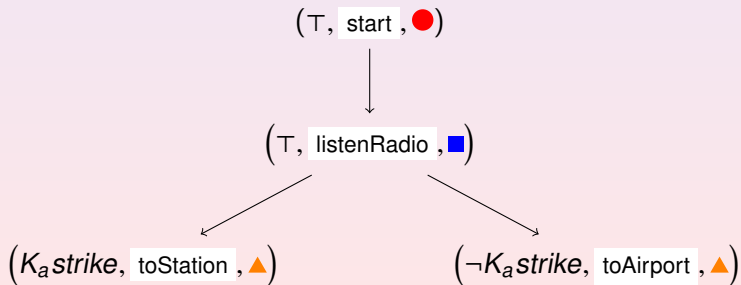
(\top , listenRadio, ■)

($K_a strike$, toStation, ▲)

($\neg K_a strike$, toAirport, ▲)

Control-flow graph

● listenRadio
■ if K_a strike then
| toStation
else
| toAirport
▲

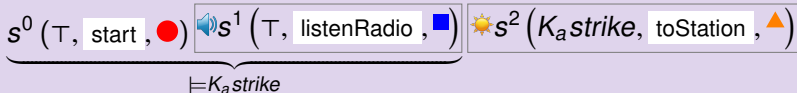
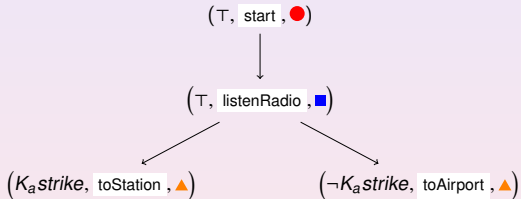


Consistent histories (explained with one agent)

In the QdecPOMDP:



KBP control-flow graph



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 - Verification
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 - Succinctness
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Verification problem

Input:

- A QdecPOMDP model;
- Knowledge-based programs for each agent;

Output: yes if all executions of the KBPs lead to a goal state.

Verification problem for while-free KBPs

Theorem

The verification problem for while-free KBPs is PSPACE-complete.

PROOF IDEA.

- **Upper bound:** on-the-fly model checking;
- **Lower bound:** reduction from TQBF.



value of
 p_1



value of
 p_2



value of
 p_3

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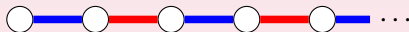
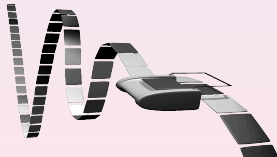
value of
 p_3

Verification problem for general KBPs

Theorem

The verification problem for general KBPs is undecidable.

PROOF IDEA. Reduction from the halting problem of a Turing machine on input ϵ .



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Execution problem

Input:

- an agent a ;
- a QdecPOMDP model;
- policies (e.g. KBPs), one for each agent;
- a local view of the history for agent a .

Output: the action act agent a should take.

Execution problem

Input:

- an agent a ;
- a QdecPOMDP model;
- policies (e.g. KBPs), one for each agent;
- a local view of the history for agent a ;
- an action act .

Output: yes, if the next action of agent a is act ; no otherwise.

Reactive policy representation

Definition (reactive policy representation)

A class of policy representations is reactive
iff its corresponding execution problem is in P.

Example (Tree policies are reactive policy representation)

if justobserved(🔥) **then** turn left **else** stay

Unless $P = PSPACE$, KBPs are not reactive. Indeed:

Proposition

The execution problem for KBPs is PSPACE-complete.

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Modal depth

Modal depth = number of nested '**... knows**' operators.

Formulas	Modal depths
<i>justobserved</i> (🔥)	0
<i>a</i> knows <i>p</i>	1
<i>a</i> knows (<i>b</i> knows <i>p</i>)	2

Succinctness

Theorem (📄 [Lang, Zanuttini, 2012] for $d = 1$; 📄 [AAAI2018], for $d > 1$)

Let $d \geq 1$.

There is a $\text{poly}(n)$ -size QdecPOMDP family $(\mathcal{M}_{n,d})_{n \in \mathbb{N}}$ for which:

- 1 there is a d -modal depth $\text{poly}(n)$ -size valid KBP family;
- 2 no $(d - 1)$ -modal depth valid KBP family;
- 3 assuming $NP \not\subseteq P/\text{poly}$, for any reactive policy representations, no $\text{poly}(n)$ -size valid policy family.

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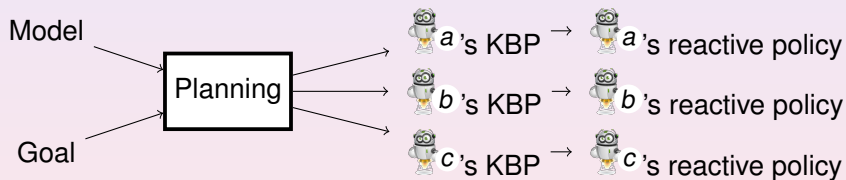
PROOF IDEA. $\mathcal{M}_{n,d}$:

- run a $\text{poly}(n)$ -time protocol revealing a $\text{poly}(n)$ -size 3-CNF β ;
- β satisfiable iff a d -md non $d - 1$ -md expressible epistemic property holds.

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Perspectives

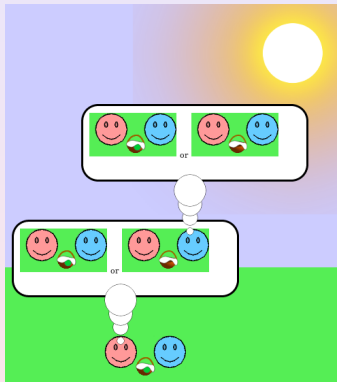
- Implementation of the verification problem;
- Heuristics for the planning problem;
- More tractable fragments;

- decPOMDP (with probabilities);
- Temporal properties;
- Strategic reasoning;
- Develop proof systems for KBPs. Use of Coq?

Coming soon... New graphics for Hintikka's world...



Trugarez bras. Merci. Thank you. Dank u wel.



Feel free to use it!

[http://people.irisa.fr/Francois.Schwarzentruber/
hintikkasworld/](http://people.irisa.fr/Francois.Schwarzentruber/hintikkasworld/)