Knowledge and seeing

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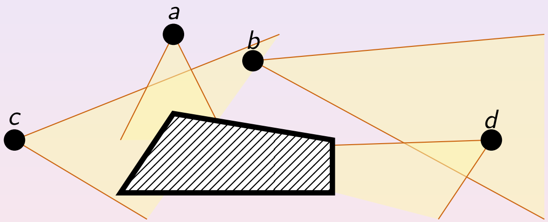
Outline

Scenario: agents equipped with vision devices, positioned in the plane / space.



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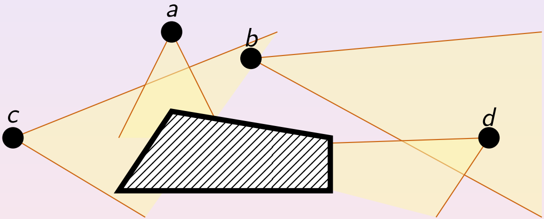

(E.g., robots that cooperate)

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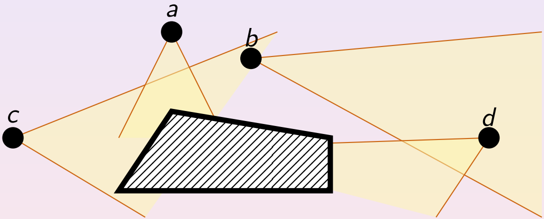

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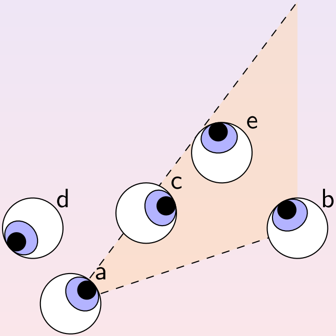
Aim:
To represent and compute visual-epistemic reasoning of the agents.

Outline

Modeling

Each agent has a sector (cone) of vision.



Assumptions (common knowledge):

- Agents are transparent points in the plane
- All objects of interest are agents
- Agents see infinite sectors
- Angles of vision are the same α
- No obstacles (yet)

Possible worlds

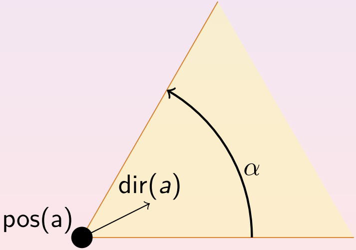
Let U be the set of unit vectors of $\mathbb{R}^2$.

Definition

A *geometrical possible world* is a tuple $w = (\text{pos}, \text{dir})$ where:

- $\text{pos} : \text{Agt} \to \mathbb{R}^2$
- $\text{dir} : \text{Agt} \to U$

$\text{dir}(a)$ is the bisector of the sector of vision with angle α:



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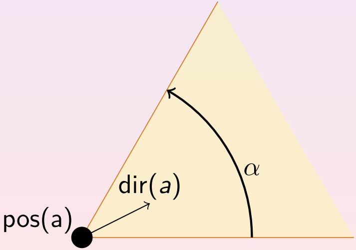
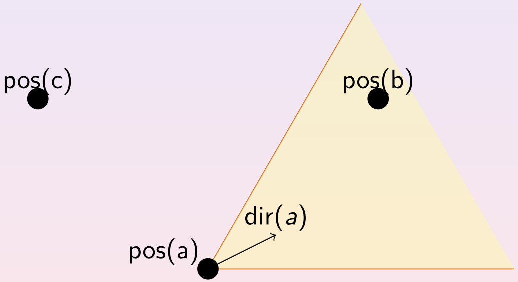


$C_{p,u,\alpha}$: the closed sector with vertex at the point p, angle α and bisector in direction u. The region seen by a is $C_{\text{pos}(a),\text{dir}(a),\alpha}$.

An agent sees another one

Definition

a sees b in $w = (\mathsf{pos}, \mathsf{dir})$ if $\mathsf{pos}(b) \in C_{\mathsf{pos}(a),\mathsf{dir}(a),\alpha}$.



Example

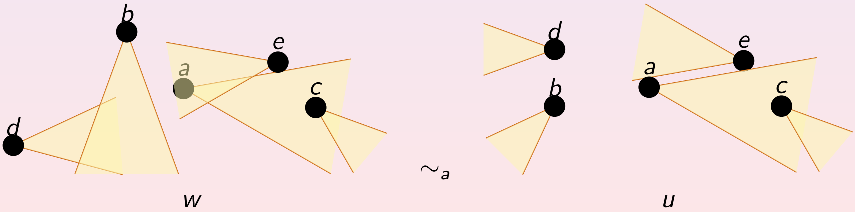
a sees a, a sees b.
a does not see c.

Epistemic model $\mathcal{M}_{\text{flatland}}$

Definition

$\mathcal{M}_{\text{flatland}} = (W, (\sim_a)_{a \in AGT}, V)$ with:

- W is the set of all geometrical possible worlds;
- $w \sim_a u$ if agents a see the same agents in both w and u and these agents have the same position and direction in both w and u;
- $V(w) = \{a \text{ sees } b \mid \text{agent } a \text{ sees } b \text{ in } w\}$.



In Hintikka's World: Flatland

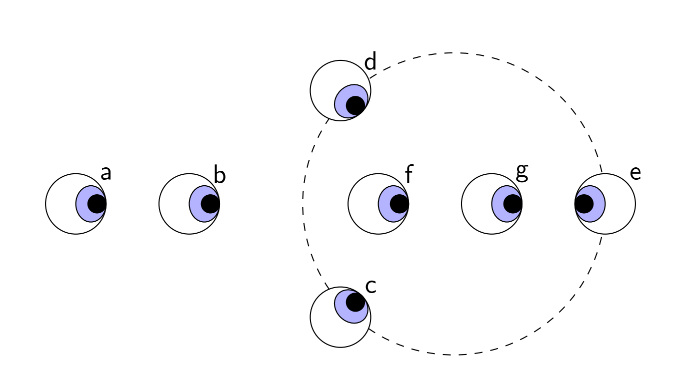
Outline

Disjunctive surprises!

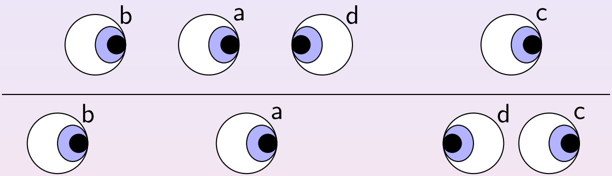
- $\models (K_a a \text{ sees } b) \vee (K_a a \cancel{\text{sees}}\, b)$;
- $\models K_a(b \text{ sees } c \vee d \text{ sees } e) \leftrightarrow K_a(b \text{ sees } c) \vee K_a(d \text{ sees } e)$;

Some formulas are... Boolean

$$K_a K_b CK_{c,d,e}(f \text{ sees } g)$$



In 1D, only qualitative positions matter



Expressivity

Qualitative positions are expressible in the language.

- $\text{sameDir}(a, b) := (a \text{ sees } b \leftrightarrow b \cancel{\text{ sees }} a)$
- a isBetween $b, c := (a \text{ sees } b \leftrightarrow a \cancel{\text{ sees }} c)$;

Abstraction of the Kripke model in 1D

Definition

$abs(w) = \{b \text{ sees } c \mid \mathcal{M}_{robots,1D}, w \models b \text{ sees } c\}$

$$
\begin{tikzcd}
w \arrow[r, "\text{abstraction}"] \arrow[d, "\sim_a"'] & abs(w) \arrow[d, "\sim_a^{abs}"] \\
u \arrow[r, "\text{abstraction}"'] & abs(u)
\end{tikzcd}
$$

Axiomatization in 1D

- Propositional tautologies;
- $(\text{sameDir}(a, b) \leftrightarrow \text{sameDir}(b, c)) \rightarrow \text{sameDir}(a, c)$;
- $\neg(a \text{ isBetween } b, c) \vee \neg(b \text{ isBetween } a, c)$;
- $(K_a a \,\text{sees}\, b) \vee (K_a a \,\cancel{\text{sees}}\, b)$;
- $a \,\text{sees}\, b \rightarrow ((K_a b \,\text{sees}\, c) \vee (K_a b \,\cancel{\text{sees}}\, c))$;
- $\chi \rightarrow \hat{K}_a\psi$ where χ and ψ are completely descriptions with $\chi \sim_a^{abs} \psi$;
- $K_a\varphi \rightarrow \varphi$.

[Balbiani et al. Agents that look at one another. Logic Journal of IGPL. 2012]

Definition

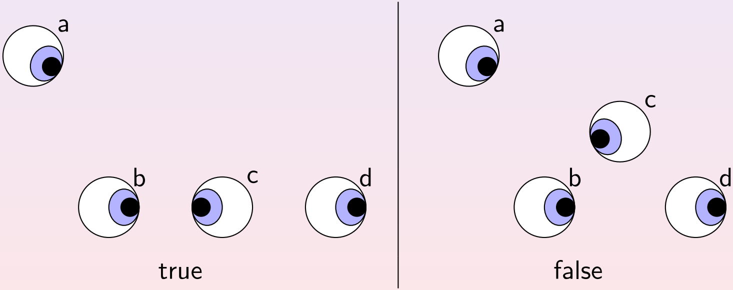
A complete decription is a conjunction that:

- contains $a \,\text{sees}\, b$ or $a \,\cancel{\text{sees}}\, b$ for all agents a, b;
- is satisfiable.

In 2D, the qualitative representation is a open issue

Example

$$K_b(a \text{ sees } b \wedge a \text{ sees } d \rightarrow a \text{ sees } c)$$



Abstraction of the Kripke model in 2D

Definition

$abs(w) = \{b \text{ sees } c \mid \mathcal{M}_{robots,2D}, w \models b \text{ sees } c\}$

$$\begin{tikzcd}
w \arrow[r, "\text{abstraction}"] \arrow[d, "\sim_a"'] & abs(w) \arrow[d, "\sim_a^{abs}"] \\
u \arrow[r, "\text{abstraction}"'] & abs(u)
\end{tikzcd}$$

Outline

Model checking

Input:

- a description of a world w

(and not a WHOLE Kripke model!);

- a formula φ.

Output:

- yes if $w \models \varphi$.

Complexity

lineland	flatland
PSPACE-complete	PSPACE-hard, in EXPSPACE
	translation to $\mathbb{R}$-FO-theory

Reduction to $\mathbb{R}$-FO-theory

Standard translation from modal logic to first-order logic

$K_a p$ rewrites in $\forall u, (wRu) \rightarrow p(u)$

[Blackburn et al., modal logic, 2001]

Adapted translation from modal logic with seeing to the $\mathbb{R}$-FO-theory

$K_a(b \text{ sees } c)$ rewrites in

$$\begin{aligned}&\forall pos'_a \forall pos'_b \ldots \forall dir'_a \forall dir'_b \ldots \\ &\quad \{\textstyle\bigwedge_{b \in AGT}[(\mathrm{pos}_b \in C_{\mathrm{pos}(a),\mathrm{dir}(a),\alpha}) \rightarrow (pos'_b = pos_b \wedge dir'_b = dir_b)] \wedge \\ &\qquad [(pos_b \notin C_{\mathrm{pos}(a),\mathrm{dir}(a),\alpha}) \rightarrow (pos'_b \notin C_{\mathrm{pos}(a),\mathrm{dir}(a),\alpha})\} \\ &\qquad\quad \rightarrow (pos'_c \notin C_{\mathrm{pos}(b),\mathrm{dir}(b),\alpha})\end{aligned}$$

Outline

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Agents are cameras

Cameras

- Can turn;
- Can not move.

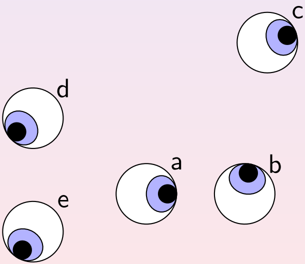
Common knowledge

- of the positions of agents;
- of the abilities of perception;

Semantics: restricted set of worlds

Set of worlds

Given a fixed $pos' : AGENTS \to \mathbb{R}^2$,
worlds are $w = (pos, dir)$ s. th. $pos = pos'$



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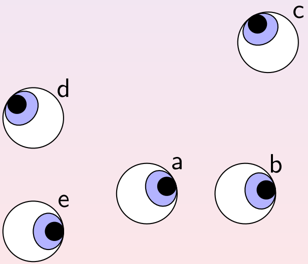

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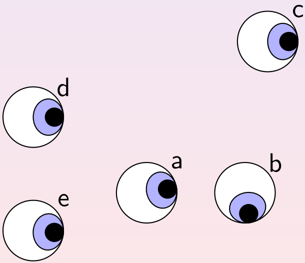

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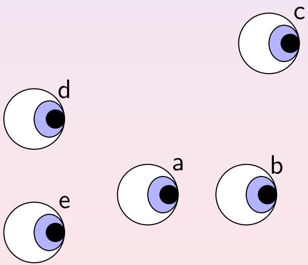
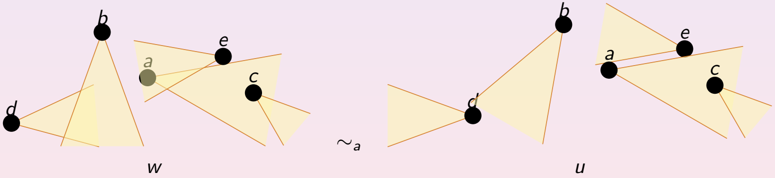


Semantics: $\mathcal{M}_{\text{cameras}}$

Definition

$\mathcal{M}_{\text{cameras}}$ is $\mathcal{M}_{\text{flatland}}$ where we publicly announced the current positions of the agents.



In Hintikka's World: Flatland with common knowledge of the positions

Outline

Abstraction of the Kripke model $\mathcal{M}_{\text{cameras}}$

Definition

$$abs(w) = \{ b \text{ sees } c \mid \mathcal{M}_{\text{cameras}}, w \models b \text{ sees } c \}$$

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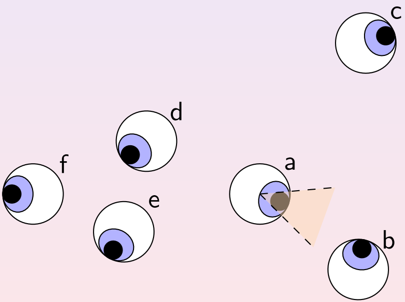
Spectrum of vision

Family of vision sets of agent a

$S_a = \{\{b\}, \emptyset, \{c\}, \{d\}, \{d, f\}, \{d, f, e\}, \{f, e\}, \{e\}\}.$



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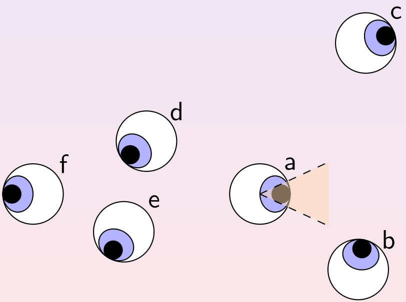

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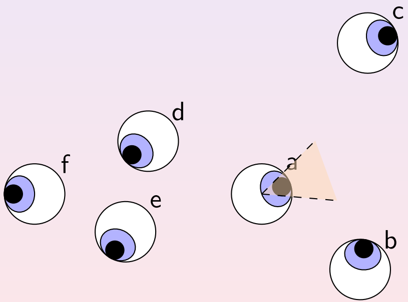

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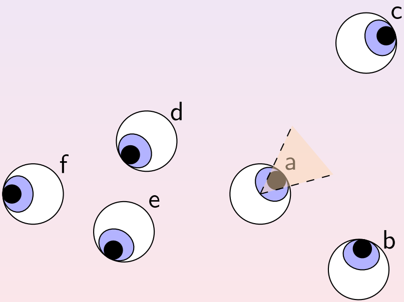

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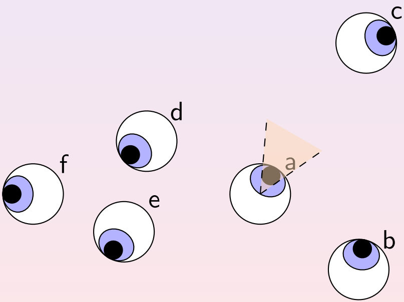

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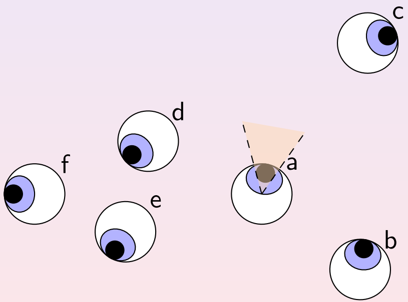

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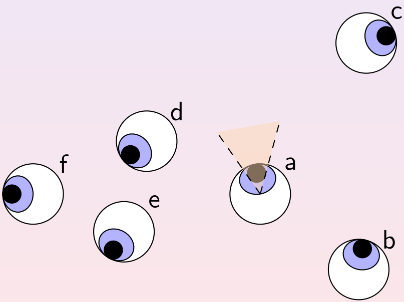

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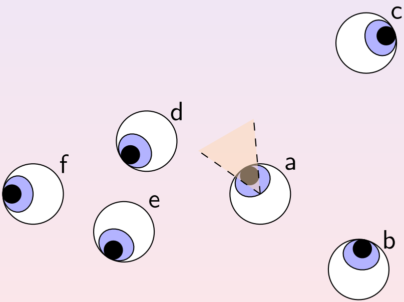

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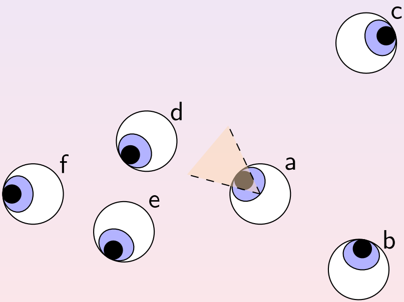

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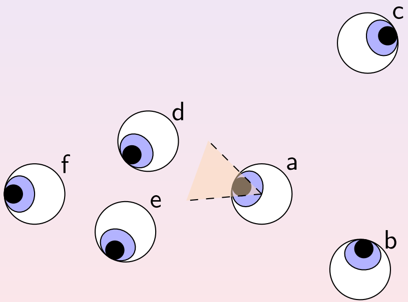

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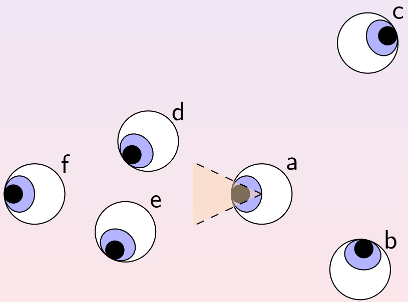

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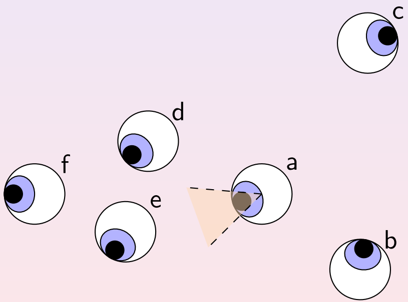

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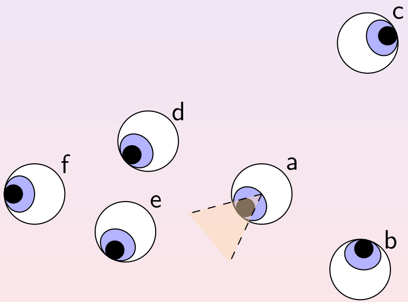

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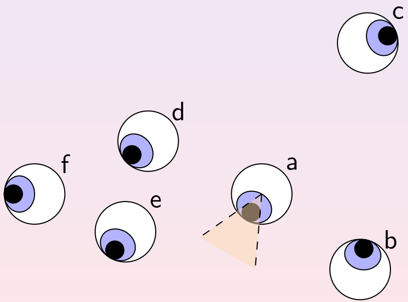

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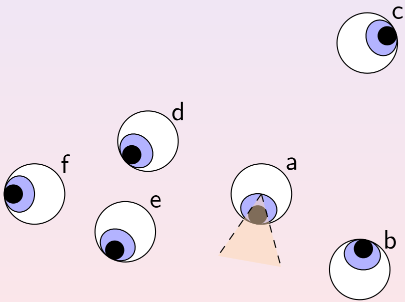

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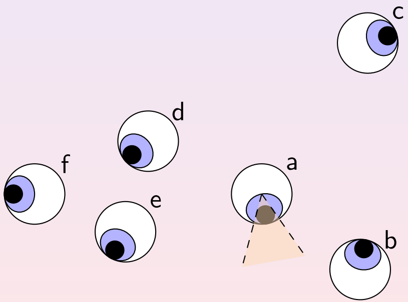

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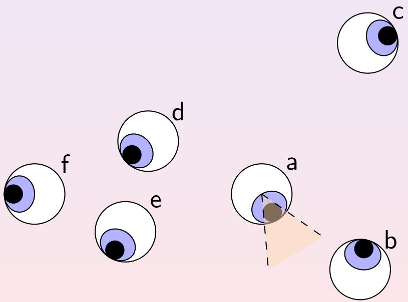

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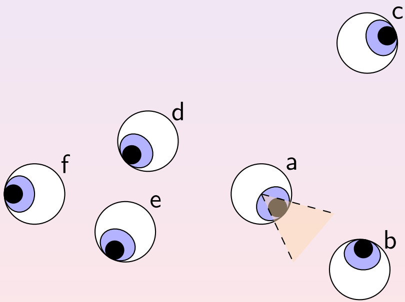

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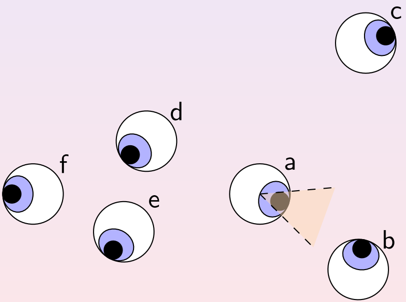

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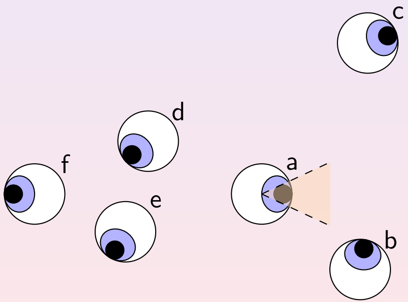

NB: each S_a is computed in $O(k \log k)$ steps, where $k = \#(\mathrm{Agt})$.

Outline

PDL Language

Grammar for formulas

$$\varphi, \psi, \ldots \quad ::= \quad a \text{ sees } b \quad | \quad \neg\varphi \quad | \quad \varphi \vee \psi \quad | \quad [\pi]\varphi$$

- $[\pi]\varphi$: after all executions of program π, φ holds.

Programs

Grammar for programs

$$\pi \ldots \quad ::= \quad \overset{\curvearrowright}{a} \quad | \quad \varphi? \quad | \quad \pi ; \pi' \quad | \quad \pi \cup \pi' \quad | \quad \pi^*$$

- $\overset{\curvearrowright}{a}$: a turns;
- φ?: the program succeeds when φ is true;

Translating epistemic operators in programs

K_a is simulated by:

$$\left[\underbrace{\Big(a\ \text{sees}\ b_1? \cup (a\ \cancel{\text{sees}}\ b_1?; \overset{\curvearrowright}{b_1})\Big); \ldots ; \Big(a\ \text{sees}\ b_n? \cup (a\ \cancel{\text{sees}}\ b_n?; \overset{\curvearrowright}{b_n})\Big)}_{\pi_a}\right]$$

Outline

Model checking

Theorem

Model checking of PDL for cameras is PSPACE-complete.

[Gasquet, Goranko, et al. Big Brother Logic: Logical modeling and reasoning about agents equipped with surveillance cameras in the plane, AAMAS 2014]
[JAAMAS2015]

Outline

Summary: Visual-epistemic reasoning of agents

- Epistemic language involving atomic propositions '*a sees b*'.
- Semantics in geometric and Kripke models.
- 1D case and 2D case with cameras (spectrum of vision):
  - Finite abstraction in the 1D case and in the 2D case with cameras (spectrum of vision).
  - Optimal PSPACE model checking.
- Open problem for the full 2D case: finite abstraction?

Future work

- Obstacles;
- Moving agents/cameras in the plane: mathematically more complex, finite abstractions may not work;
- Agents/cameras in the 3D space.