Subspace Gradient Domain Mesh Deformation

Presented by Xinguo Liu (刘新国)
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To Appear in ACM SIGGRAPH 2006

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Brief Introduction to Some Sample Works
Sample works

- High quality real time rendering
  - 1 EGSR (2004)

- Geometric Processing
  - 1 ACM SPM (2005)
  - 1 EG Point Based Graphics (2005)
  - 1 J. CAVW (2006)
Rendering

- Synthesizing Bidirectional Texture Functions for Real-world Surfaces. [ACM SIGGRAPH 2001]
  - Xinguo Liu, et al.

- Biscale Radiance Transfer [SIGGRAPH 2003]
  - Peter-Pike Sloan, Xinguo Liu, et al.

- Synthesis and Rendering of Bidirectional Texture Functions on Arbitrary Surfaces. [IEEE TVCG 10(3), 2004]
  - Xinguo Liu et al.
Rendering

- All-Frequency Precomputed Radiance Transfer for Glossy Objects [Eurographics Symposium on Rendering 2004]
  - Xinguo Liu et al.

- Real-time Soft Shadows in Dynamic Scenes using Spherical Harmonic Exponentiation. [SIGGRAPH 2006]
  - Zhong Ren, Rui Wang, John Snyder, Kun Zhou, Xinguo Liu, et al.
Biscale Radiance Transfer
[SIGGRAPH 2003]

Lighting
Biscale Radiance Transfer

SIGGRAPH 2003

Global Lighting

Local Lighting

Macro-Scale Radiance Transfer by PRT

Meso-Scale Radiance Transfer by RTT
Biscale Radiance Transfer
[SIGGRAPH 2003]
All-Frequency Precomputed Radiance Transfer for Glossy Objects [GSR04]
All-Frequency Precomputed Radiance Transfer for Glossy Objects [EGSR04]
Real-time soft shadows in dynamic scenes using spherical harmonic exponentiation

[SIGGRAPH 2006]
Geometric Preprocessing

- Directional Histogram Model for Three-Dimensional Shape Similarity. [IEEE CVPR 2003]
  - Xinguo Liu, et al.

- Computing Variation Modes for Point Set Surfaces. [Eurographics Symposium on Point-Based Graphics 2005]
  - Lanfang Miao, Jin Huang, Xinguo Liu et al.
Geometric Processing

- Large mesh deformation using the volumetric graph Laplacian. [SIGGRAPH 2005]
  - Kun Zhou, Jin Huang, John Snyder, Xinguo Liu, et al.

- Clustering method for fast deformation with constraints. [ACM SPM 2005]
  - Jin Huang, Xinguo Liu, et al.
**Directional Histogram Model for Three-Dimensional Shape Similarity**

[IEEE CVPR 2003]

<table>
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<th>Rabbit</th>
<th>Horse</th>
<th>Elephant</th>
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Large mesh deformation using the volumetric graph Laplacian

[SIGGRAPH 2005]
An Efficient Large Deformation Method using Domain Decomposition [Computer & Graphics, 2006]
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CAD&CG     MSRA     Boston Univ.
Deformation

Original          twisting          bending
Mesh Deformation

- Scanned Models
- Preserve surface details
Mesh Deformation

- Scanned Model
  - Dense samples
  - High quality
  - Arbitrarily shape
  - In triangular meshes
Related Work

- Mesh Deformation
  - Traditional method (blend, interpolation)
    - Skeleton skinning
    - FFD: Free form deformation
    - Multiresolution mesh editing
  - Gradient domain method (linear least square)
    - Poisson Mesh Editing
    - Laplacian Mesh Editing
    - Volumetric Graph Laplacian
Gradient Domain Method

- Differential coordinates (Laplacian)

\[ \delta_i = \sum (\cot \alpha_j + \cot \beta_j)(x_i - x_{i,j}) \]
Gradient Domain Method (Cont.)

- Differential coordinates in local frame
  - get the local transformation through interpolation/diffusing/fitting ...

\[
\hat{\delta}_i = T_i \delta_i
\]

- Linear least square reconstruction

\[
\mathcal{L} X = \hat{\delta}
\]
Gradient Domain Method

- Local Transforms
  - User rotate some handles
  - Other vertices obtain their transformation by
    - Transformation Propagation
    - Harmonic Guidance Field
    - Linear Rotation Invariant

- Need users’ input
Gradient Domain Method

- Local transformations must be consistent with deformation.
Our Method

- Non-linear least square energy
  - Get rid of transformation (of user input)
    - Nontrivial to handle
  - useful constraints
    - skeleton
    - projection
    - volume
- Subspace solver
  - Stable, fast convergence
Our Method (Cont.)

- Video
Encoding Differential Coordinates

We need the consistent direction of differential coordinates.

Express it by neighbor vertices:

\[ \delta_i = \sum_{j=1}^{n_i} \mu_{ij} \left( (x_{i,j-1} - x_i) \otimes (x_{i,j} - x_i) \right) \]
Encoding Differential Coordinates (Cont.)

- Calculate the direction:
  \[ d_i(X) = \sum_j \mu_{ij} ((x_{i,j-1} - x_i) \otimes (x_{i,j} - x_i)) \]

- Rescale back to original length:
  \[ \hat{\delta}_i(X) = \frac{\hat{\gamma}_i}{\gamma_i} d_i(X) \]
Non-linear Linear Square

- For preserving surface detail (differential coordinates), we have:

\[ \min_X \| \mathcal{L}X - \hat{\delta}(X) \|^2 \]

- Add soft position constraints:

\[ \min_X \left( \| \mathcal{L}X - \hat{\delta}(X) \|^2 + \| \Phi X - \hat{V} \|^2 \right) \]
Quasi-Linear

- **Object energy:**
  \[ \min_x \| f(x) \|^2 \]

- **Linear:**
  \[ f(x) = 2x \]

- **Non-linear:**
  \[ f(x) = 2x + 5 \sin(x + 1.5) \]

- **Quasi-linear:**
  \[ f(x) = 2x + \sin(x + 1.5) \]
Inexact Gauss-Newton

\[ f(X) \equiv LX - b(X) \]

- **Gauss-Newton**

\[ f(X + h) \approx l(h) \equiv f(X) + (L - J_b(X))h \]

- **Inexact Gauss-Newton**

\[ l(h) \equiv f(X) + (L - J_b(X))h \approx f(X) + Lh \]
Video

- Walking Dinosaur
Constraints

- Skeleton
  - Soft
- Projection
  - Hard
- Volume
  - Hard
Skeleton Constraints

- Extremely useful when deforming articulated figures
Skeleton Constraints (Cont.)

- Create skeleton by stroking
Skeleton Constraints (Cont.)

- Video
Skeleton Constraints (Cont.)

- Keeping straightness and length:
  \[
  \begin{aligned}
  (s_i - s_{i-1}) - (b - a)/r &= 0 \\
  i &= 1, 2, \ldots, r, \\
  \|b - a\| &= \hat{\rho}.
  \end{aligned}
  \]

- Reform to ...
  \[
  \begin{aligned}
  \Gamma X &= 0 \\
  \|\Theta X\| &= \hat{\rho}
  \end{aligned}
  \]

- Make it quasi-linear:
  \[
  \Theta X = \hat{\rho}(\Theta X/\|\Theta X\|)
  \]
Projection Constraints

- Manipulate the object interactively in only one view
Projection Constraints

- Video
Volume Constraints

- Keep/control the object's volume exactly

\[ \psi(X) = \frac{1}{6} \sum_{T_{ijk}} (x_i \otimes x_j) \cdot x_k \]

(a) 60% volume  (b) 160% volume  (c) 160% volume + edge length
Volume Constraints

- Video
Solve Hard Constraints

- Projection and volume constraints are hard constrains.

\[
\text{minimize } \frac{1}{2} \| l(h) \|^2 \quad \text{subject to } g(X + h) = 0
\]

- Apply Lagrange multipliers at each iteration:

\[
g(X + h) \approx g(X) + J_g(X)h
\]
Instability

- Video
Deformation Space

- The deformation space can be spanned by a lower dimensional linear space.
  - low frequency deformation is what we want.
  - high frequency deformation means surface detail changes
Deformation Space (Cont.)

- Subspace can be constructed using the eigen vectors of the Laplacian matrix
  - SVD is costly for models with large number of vertices.
Deformation Space (Cont.)

- Create a coarse control mesh
  - Using 3D mean value interpolation
Subspace Solver

- Can be viewed as a simple variable replacement:

\[ X = WP \]

- Then

\[
\begin{align*}
\text{minimize} & \quad \| (LW)P - b(WP) \|^2 \\
\text{subject to} & \quad g(WP) = 0.
\end{align*}
\]
Subspace Solver

- Fast and stable
  - Much better condition number.
  - Smooth out non-linear factor.
Subspace Solver

- Video
Objects with multiple components

- Components are binded together in the subspace

original + control meshes

deformation 1

deformation 2
Multi-part Object

- Video
Comparison with FFD

FFD                Ours
More Results
More Results
Future Work

- Subspace construction
  - Better methods, e.g. based on user specified deformation examples
- Local support basis for subspace
  - lower cost for more DOFs
  - local surface editing
- Multigrid
  - hierachical constraints
  - remove the residua for better results
Thanks!

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