A theory of interfaces

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Introduction

- Agents, Groups. etc...
- How agents interact with each other and this is all that matters to reason about groups $\Rightarrow$ an interface between the agent and the external world.
- Agents can form groups, and groups are somehow (meta) agents.
- Given an interface (for say a group) can we combine agents to form "this" group?

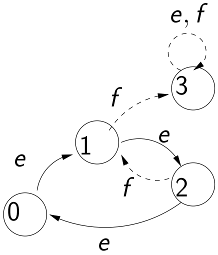
The formal framework

Fix an alphabet *Act* of *events*.

- Agents = Deterministic finite state machines
  $a = (Q_a, Act, q_a^0, \delta_a)$

  We use $a, b, c, a_1, \ldots$

- Interfaces over the alphabet $Act = \{e, f, g\}$



$\overset{e}{\dashrightarrow}$ "may e"

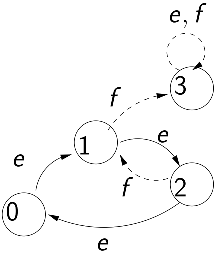
$\xrightarrow{e}$ "Must e"

We use $A, B, C, A_1, \ldots$

Interfaces

Fix an interface A (over Act).

- In each state q of A, we have
  - two subsets $may(A)(q), Must(A)(q) \subseteq Act$
  - the subset $maynot(A)(q) := Act \setminus may(A)(q)$

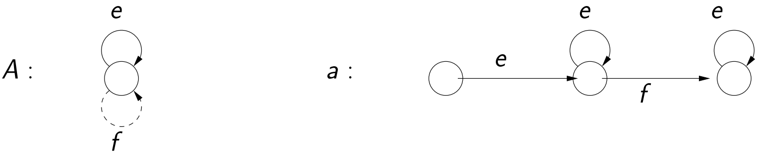


- If $Must(A)(q) \cap maynot(A)(q) \neq \emptyset$, state q is inconsistent, which we may write as $\perp$.

Agents and Interfaces

Assume for the moment that all states of A are consistent.

- Agent a satisfies interface A, written $a \models A$, or $a \in Mod(A)$, whenever a is obtained from A by cutting may-transitions or making them solid.



- Agents $\hookrightarrow$ Interfaces

$$a \hookrightarrow a^* \text{ in a natural way, ie } a \in Mod(a^*).$$

- $Mod(a^*)$ contains only a, up to bisimulation.

Refinement, as logical implication

$A \sqsubseteq B$ "A refines B", whenever there exists $\rho \subseteq Q_A \times Q_B$ such that

1. $(q_A^0, q_B^0) \in \rho$,
2. may-transitions in A are reflected in B, and
3. must-transitions in B are reflected in A.

Alternating simulation/refinement in games structures [AHKV98].

Proposition

$A \sqsubseteq B$ if, and only if, $Mod(A) \subseteq Mod(B)$.

In particular, $a \in Mod(A)$ can be rephrased as $a^* \sqsubseteq A$.

Proposition [AHKV98]

We can decide in PTIME whether $A \sqsubseteq B$.

Consistent interfaces, the conjunction

We want to define $A \wedge B$ so that

Proposition

$Mod(A \wedge B) = Mod(A) \cap Mod(B)$

As a corollary $A \wedge B \sqsubseteq A$ and it is the $\sqsubseteq$-greatest (lower bound)

Definition of $A \wedge B$

$A \wedge B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots)$ with

- $may(A \wedge B)(q_A, q_B) := may(A)(q_A) \cap may(B)(q_B)$
- $Must(A \wedge B)(q_A, q_B) := Must(A)(q_A) \cup Must(B)(q_B)$

Must increases whereas may decreases $\Rightarrow$ inconsistent states.

About consistency

Proposition

$Mod(A) \neq \emptyset$ if, and only if, no Must-path reaching $\bot$.

- You may prune your interface to remove inconsistent states.
- The empty interface $\bot$ corresponds to "**false**" in the underlying logic.
- The interface $\top$ for "**true**" is the one state + dashed flower structure.

Proposition

- Consistency can be decided in LOGSPACE (Reachability).
- If A is consistent it has a minimal and a maximal model.

Make agents work together

- Standard synchronous product of agents
  (= product of deterministic finite state machines)

$$a \times b := (Q_a \times Q_b, (q_a^0, q_b^0), \ldots)$$

- Abstract from which agents in particular

$$A \otimes B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots)$$

Proposition

$Mod(A) \otimes Mod(B) \subseteq Mod(A \otimes B)$ (strict inclusion in general).

Definition of $A \otimes B$

$A \otimes B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots)$ with

- $may(A \otimes B)(q_A, q_B) := may(A)(q_A) \cap may(B)(q_B)$
- $Must(A \otimes A)(q_A, q_B) := Must(A)(q_A) \cap Must(B)(q_B)$

$\otimes$	$\overset{e}{\dashrightarrow}$	$\xrightarrow{e}$	$\not\xrightarrow{e}$
$\overset{e}{\dashrightarrow}$	$\overset{e}{\dashrightarrow}$	$\overset{e}{\dashrightarrow}$	$\not\xrightarrow{e}$
$\xrightarrow{e}$	$\overset{e}{\dashrightarrow}$	$\xrightarrow{e}$	$\not\xrightarrow{e}$
$\not\xrightarrow{e}$	$\not\xrightarrow{e}$	$\not\xrightarrow{e}$	$\not\xrightarrow{e}$

Properties of $\otimes$

- $Mod(a^* \otimes b^*)$ contains only $a \times b$ (up to bisimulation).
- $\otimes$ is commutative and associative.
- Neutral element: one state + solid flower.
- $\otimes$ is monotonic: $A \sqsubseteq B$ implies $A \otimes C \sqsubseteq B \otimes C$

Realizing interfaces

Suppose we have to find $X_1, X_2, \ldots, X_k$ such that

$$X_1 \otimes X_2 \otimes \ldots \otimes X_k \sqsubseteq A$$

where the X_i's range over $\{A_1, A_2, \ldots, A_n\}$.

How can we proceed? We define a quotient $\oslash$ such that

$$A_1 \otimes A_2 \sqsubseteq A \text{ if, and only if } A_2 \sqsubseteq A \oslash A_1$$

Definition of $A \oslash B$

$$A \oslash B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots) \text{ with}$$

$\oslash$	$\overset{e}{\dashrightarrow}$	$\overset{e}{\longrightarrow}$	$\overset{e}{\not\longrightarrow}$
$\overset{e}{\dashrightarrow}$	$\overset{e}{\dashrightarrow}$	$\overset{e}{\dashrightarrow}$	$\overset{e}{\dashrightarrow} \top$
$\overset{e}{\longrightarrow}$	$\bot$	$\overset{e}{\longrightarrow}$	$\bot$
$\overset{e}{\not\longrightarrow}$	$\overset{e}{\not\longrightarrow}$	$\overset{e}{\not\longrightarrow}$	$\overset{e}{\dashrightarrow} \quad \top$

Proposition

$$(A \oslash A_1) \oslash A_2 \equiv A \oslash (A_1 \otimes A_2)$$

Achieving interfaces

There are many ways to think of it. I give here one example. From

$$X_1 \otimes X_2 \otimes \ldots \otimes X_k \sqsubseteq A$$

where each $X_i \in \mathcal{A} := \{A_1, A_2, \ldots, A_n\}$.

1. $C := A$
2. Select $A_{i_1} \in \mathcal{A}$ and compute $C := C \oslash A_{i_1}$;
3. Select $A_{i_2} \in \mathcal{A}$ and compute $C := C \oslash A_{i_2}$;
4. $\ldots$
5. Select $A_{i_k} \in \mathcal{A}$ and compute $C := C \oslash A_{i_k}$;

At the end, if $C \equiv \top$ then done,
otherwise C is the needed *mediator* interface.

The logical view

- Interfaces as a fragment of the μ-calculus [Kozen83]:
  - $\overset{e}{\dashrightarrow}$ is $[e]$ and $\xrightarrow{e}$ is $<e>$.
  - $+$ conjunction, greatest fixed-points and but no outermost negation.

  Interfaces $\hookrightarrow L_\mu$: $A \hookrightarrow \alpha$
- Quotient between mu-calculus formulas exists ([AVW03],...):
  - $P \models \varphi/\psi$ if and only if $\exists C \models \psi, P \times C \models \varphi$, contructive procedure on the tree automata of the formulas.
  - We can then compute $\alpha \oslash \beta$ as $\neg(\neg\alpha/\beta)$ but @exptime because of two complementations.

  However, our quotient is polynomial.

Discussion

- Slight extensions with "local" disjunctions, eg

$$\begin{aligned} Acc(A)(q) &= \{\{e, f\}, \{e, g\}\} \\ &\equiv (<\!e\!> \top \wedge <\!f\!> \top) XOR (<\!e\!> \top \wedge <\!g\!> \top) \end{aligned}$$

  It generalizes *may* and *Must* sets and the entire theory works well.

- Quotienting make things bigger.
- Selection criteria?
- Extension to capture, eg interdependency, reliability, prudence, ...

  Eg, $\overset{e}{\rightsquigarrow}$ for "I cannot do e but I can follow a companion".