

# Bit-Plane Coding in Extractable Source Coding: optimality, modeling, and application to 360° data

Fangping Ye, Navid Mahmoudian Bidgoli, Elsa Dupraz, Aline Roumy, Karine Amis, Thomas Maugey

**Abstract**—In extractable source coding, multiple correlated sources are jointly compressed but can be individually accessed in the compressed domain. Performance is measured in terms of storage and transmission rates. This problem has multiple applications in interactive video compression such as Free Viewpoint Television or navigation in 360° videos. In this paper, we analyze and improve a practical coding scheme. We consider a binarized coding scheme, which insures a low decoding complexity. First, we show that binarization does not impact the transmission rate but only slightly the storage with respect to a symbol based approach. Second, we propose a  $Q$ -ary symmetric model to represent the pairwise joint distribution of the sources instead of the widely used Laplacian model. Third, we introduce a novel pre-estimation strategy, which allows to infer the symbols of some bit planes without any additional data and therefore permits to reduce the storage and transmission rates. In the context of 360° images, the proposed scheme allows to save 14% and 34% bitrate in storage and transmission rates respectively.

**Index Terms**—Source coding with side information, source modeling, bit plane coding, LDPC.

## I. INTRODUCTION

Extractable source coding (ExtSC) refers to the problem of compressing together multiple sources, while allowing each user to extract only some of them. This problem has numerous applications, particularly in video compression. In Free Viewpoint Television (FTV), the user can choose one view among the proposed ones and change it at anytime. In 360° images, the visual data is recorded in every direction and the user can choose any part of the image of arbitrary size and direction. Both examples are instances of ExtSC, where a data vector, generated by a source, gives a frame of a view in FTV or an image block in 360° images. The main advantage of ExtSC is that the sources can be compressed without the awareness of which sources will be requested, and still be extracted at the same compression rate (also called *transmission rate*) as if the encoder knew the identity of the requested sources [1], [2], [3]. Compressing without the request knowledge only slightly increases the overall *storage rate* of the complete set of sources.

Two characteristics are common to the above applications. First, low decoding computational complexity is required to allow smooth navigation. However, ExtSC, as an extension

of the Distributed Source Coding (DSC) problem [4], [5], is implemented by means of channel codes [6]. For non-binary sources, such as images, non-binary Low Density Parity Check (LDPC) codes can be used, but their decoding is highly complex [7]. To lower the computational complexity, one can binarize the symbols and use binary LDPC codes [8]. This binarization is optimal in the context of DSC [9], but no such result exists for ExtSC. Therefore, the first contribution of this paper consists in analyzing whether binarization remains optimal for the two compression rates (storage and transmission rates) involved in ExtSC.

The second characteristic that a practical implementation of ExtSC should satisfy is to reduce as much as possible the storage and transmission rates. For this, the implementation should adapt to the statistics of the source symbols to encode. To do so, a source model can be selected at the encoder and its parameters sent to the decoder. Laplacian model has been considered in DSC [10] or ExtSC [11]. Here, as a second contribution, we instead propose a  $Q$ -ary symmetric model to represent the pairwise joint distribution of the sources. The third contribution concerns the rate allocation among the bit planes. Indeed, we remarked that some bitplanes can be perfectly recovered from previously decoded bitplanes, without any need for additional data transmission. We propose a criterion to identify these bitplanes, and introduce this novel *pre-estimation* strategy into our implementation. Finally, the proposed coder is integrated into a 360° image extractable compression algorithm. Simulations show that the  $Q$ -ary symmetric model and the pre-estimation strategy significantly improve the PSNR compared to the conventional setup with the Laplacian model and without the pre-estimation strategy.

*Notation and definitions.* Throughout the paper,  $\llbracket 1, n \rrbracket$  denotes the set of integers from 1 to  $n$ . The discrete source  $X$  to be compressed has alphabet  $\mathcal{X}$ . It generates a length- $n$  random vector denoted  $\underline{X} = [X_1 X_2 \dots X_k \dots X_n]^T$ . We consider  $J$  different side informations  $Y^{(j)}$ ,  $j \in \llbracket 1, J \rrbracket$  and each  $Y^{(j)}$ , generates a length- $n$  vector  $\underline{Y}^{(j)} = [Y_1^{(j)}, Y_2^{(j)}, \dots, Y_n^{(j)}]^T$ . We assume that all side information symbols  $Y_k^{(j)}$  belong to the same discrete alphabet  $\mathcal{Y}$ .

## II. EXTSC AND OPTIMALITY OF THE BINARIZATION

### A. ExtSC scheme

The ExtSC coding scheme described in [2] consists of two steps. First, the encoder compresses a source vector  $\underline{X}$  knowing a set of side information vectors  $\{\underline{Y}^{(j)}, \forall j \in \llbracket 1, J \rrbracket\}$ , and outputs a *stored codeword*  $\underline{U}$  compressed at storage rate  $S$ . Second, with the knowledge that the decoder has access to

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side information  $\underline{Y}^{(j)}$ , the encoder transmits to the decoder a subpart  $\underline{U}^{(j)} \subseteq \underline{U}$  of the stored codeword, where  $\underline{U}^{(j)}$  is called the *transmitted codeword*. For this second step, the transmission rate  $R_{(j)}$  depends on the side information  $\underline{Y}^{(j)}$  available at the decoder. ExtSC is an extension of the compound conditional source coding (CCSC) problem [12]. However, in CCSC, the transmission rate and storage rate are equal, but for ExtSC, an extraction is allowed in the compressed domain to transmit only what is needed once the request is known.

With the above notation, the achievable transmission rates  $R_{(j)}$  and storage rate  $S$  for ExtSC are obtained from [2] as

$$\forall j \in \llbracket 1, J \rrbracket, R_{(j)} = H(\underline{X}|\underline{Y}^{(j)}) \quad (1)$$

$$S = \max_{j \in \llbracket 1, J \rrbracket} H(\underline{X}|\underline{Y}^{(j)}), \quad (2)$$

where  $H(\underline{X}|\underline{Y}^{(j)})$  is the conditional entropy of  $\underline{X}$  given  $\underline{Y}^{(j)}$ .

### B. Source binarization

We now describe how the source  $\underline{X}$  is transformed into bit planes prior to coding. In order to convert the symbols  $X_k$  into bits, we consider a sign-magnitude representation over  $L$  bits. For  $b \in \llbracket 0, L-1 \rrbracket$ , bit position  $b = 0$  gives the Least Significant Bit (LSB),  $b = L-1$  gives the Most Significant Bit (MSB), and  $b = L$  gives the sign bit. The  $b$ -th bit of  $X_k$  is denoted  $Q_k^{(b)}$ . As a result,  $X_k$  can be expressed as

$$X_k = \left(1 - 2Q_k^{(L)}\right) \sum_{b=1}^{L-1} Q_k^{(b)} 2^b. \quad (3)$$

When binarizing the whole vector  $\underline{X}$ , the  $b$ -th bit-plane  $\underline{Q}^{(b)}$  is defined as  $\underline{Q}^{(b)} = [Q_1^{(b)}, \dots, Q_n^{(b)}]^T$ . Note that in our scheme, there is no need to convert the vectors  $\underline{Y}^{(j)}$  into bits.

We now compare the bit-based and symbol-based models in terms of storage and transmission rates. The transmission rate achieved with the bit-based model is the sum of the rates achieved by encoding each bitplane  $\underline{Q}^{(b)}$  taking into account the side information  $\underline{Y}^{(j)}$  and all previously encoded bitplanes  $\underline{Q}^{(b+1)}, \dots, \underline{Q}^{(L)}$  through the conditional probability distribution  $P(\underline{Q}^{(b)} | \underline{Y}^{(j)}, \underline{Q}^{(b+1)}, \dots, \underline{Q}^{(L-1)})$ . By applying (1) to each bitplane, we get

$$\begin{aligned} R_{(j)}^{bin} &= \sum_{b=0}^{L-1} H(\underline{Q}^{(b)} | \underline{Y}^{(j)}, \underline{Q}^{(b+1)}, \dots, \underline{Q}^{(L-1)}) + H(\underline{Q}^{(L)} | \underline{Y}^{(j)}) \\ &= H(\underline{Q}^{(0)}, \dots, \underline{Q}^{(L)} | \underline{Y}^{(j)}) \end{aligned} \quad (4a)$$

$$= H(\underline{X} | \underline{Y}^{(j)}) \quad (4b)$$

where (4a) follows from applying the chain rule for entropy as in [9]. As a result, the bit-based model can achieve the same transmission rate as for the symbol-based model given in (1).

Bit-plane decoding will lead to the following storage rate

$$\sum_{b=0}^{L-1} \max_{j \in \llbracket 1, J \rrbracket} H(\underline{Q}^{(b)} | \underline{Y}^{(j)}, \underline{Q}^{(b+1)}, \dots, \underline{Q}^{(L)}) + \max_{j \in \llbracket 1, J \rrbracket} H(\underline{Q}^{(L)} | \underline{Y}^{(j)})$$

If the side information  $j$  that maximizes the conditional entropy  $H(\underline{Q}^{(b)} | \underline{Y}^{(j)}, \underline{Q}^{(b+1)}, \dots, \underline{Q}^{(L)})$  is different from bit-plane to bit-plane, then this storage rate is greater than the

symbol-based achievable storage rate given in (2). Therefore, bit-plane decomposition is optimal in terms of transmission rate, but may induce a loss in terms of storage rate.

## III. SOURCE MODELING

In FTV, as in standard video compression, the statistical relation between the source  $\underline{X}$  and the side information  $\underline{Y}^{(j)}$  varies a lot from frame to frame and from video to video [10]. An accurate statistical model between  $\underline{X}$  and  $\underline{Y}^{(j)}$  is required by the LDPC decoder used in our coding scheme.

In the following, we assume that an image transform exploits all the correlation inside a source and inside a side information as in [13]. Therefore, the source symbols  $X_k$  are assumed to be independent and identically distributed (i.i.d.) and for all  $j \in \llbracket 1, J \rrbracket$ , the side information symbols  $Y_k^{(j)}$  are also i.i.d. For simplicity, we further assume an additive model  $Y^{(j)} = X + Z^{(j)}$ , where  $X$  and  $Z^{(j)}$  are independent, as often considered in the literature [14].

Since in our scheme the model parameters are estimated at the encoder and transmitted to the decoder, we need to select a statistical model for  $Z^{(j)}$ , which minimizes the global cost of sending the model description and sending the data [15]. The Laplacian model is often considered in video coding to model the statistical relation between the source and the side information [16]. However, this model has two issues. First it is a continuous model while our data is discrete. Second, in case of strong correlation between the source and the side information, the Laplacian model fails to represent small values of  $Z^{(j)}$ . Indeed, when the variance  $\delta^2$  of  $Z^{(j)}$  is very small, the Laplacian density applied to values of  $Z_k \neq 0$  becomes numerically equal to 0. This is why, as an alternative, we propose to use a  $Q$ -ary symmetric model.

The probability mass function  $P_{Z^{(j)}}$  for a  $Q$ -ary symmetric model [17] is given by

$$P_{Z^{(j)}}(z) = \begin{cases} q_j & \text{if } z = 0 \\ \frac{1-q_j}{Z_{\max}^{(j)} - Z_{\min}^{(j)}} & \text{if } z \neq 0 \text{ and } Z_{\min}^{(j)} \leq z \leq Z_{\max}^{(j)} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where  $q_j \in [0, 1]$ , and  $Z_{\min}^{(j)}$ ,  $Z_{\max}^{(j)}$  are respectively the minimum and maximum values of  $Z^{(j)}$ . For a given vector  $\underline{Z}^{(j)}$ , the value of  $q_j$  can be estimated as  $\hat{q}_j = N_0^{(j)}/n$ , where  $N_0^{(j)}$  is the number of symbols  $Z_k^{(j)}$  equal to 0.

## IV. LOSSLESS CODING SCHEME

### A. Bit-plane coding

It was shown in [2] that practical ExtSC coding schemes can be constructed from channel codes such as LDPC codes. Non-binary LDPC codes show a very high decoding complexity [7] and this is why, here, we consider binary LDPC codes. Therefore, the information vector  $\underline{X}$  is encoded and decoded from bit plane to bit plane, starting with the sign bit with index  $b = L$ , and then processing from the MSB  $b = L-1$  to the LSB  $b = 0$ . For the encoding of the  $b$ -th bit plane  $\underline{Q}^{(b)}$ , we use an LDPC parity check matrix  $H_b$  with dimension  $n \times m_b$ , and we compute a stored codeword  $\underline{U}^{(b)}$  of length  $m_b$  as

$$\underline{U}^{(b)} = H_b^T \cdot \underline{Q}^{(b)}. \quad (6)$$

Next, in view of transmission, and for each side information  $\underline{Y}^{(j)}$ , the online encoder needs to extract codewords  $\underline{U}^{(b,j)} \subseteq \underline{U}^{(b)}$  from the stored codeword  $\underline{U}^{(b)}$ . Thus, we consider the rate-adaptive LDPC code construction of [18] which will provide incremental codewords. The transmitted codeword  $\underline{U}^{(b,j)}$  of length  $m_{(b,j)} \in \llbracket m_{\min}, m_b \rrbracket$  is chosen so as to ensure that  $\underline{Q}^{(b)}$  can be decoded without any error at the decoder from the side information  $\underline{Y}^{(j)}$  and from the previously decoded bit-planes (see next section for more details). This can be done since the encoder knows all the possible side information vectors  $\underline{Y}^{(j)}$  and can then simulate the decoders. The variable  $m_{\min}$  is the minimum possible length for the codeword, as defined in the rate-adaptive construction of [18].

Finally, the coder also computes and stores the estimated parameters  $\hat{q}_j$ ,  $Z_{\min}^{(j)}$ , and  $Z_{\max}^{(j)}$ , since these parameters are needed by the decoder in order to compute the probability mass function (5) of the  $Q$ -ary symmetric model. The parameter  $q_j$  is quantized with  $v_1$  bits, and the parameters  $Z_{\min}^{(j)}$  and  $Z_{\max}^{(j)}$  are quantized with  $v_2$  bits.

## B. Bit-plane decoding

We now consider that the decoder has access to side information  $\underline{Y}^{(j)}$ , and therefore receives the set of codewords  $\{\underline{U}^{(b,j)}, \forall b \in \llbracket 0, L \rrbracket\}$  as well as the model parameters  $\hat{q}_j$ ,  $Z_{\min}^{(j)}$ , and  $Z_{\max}^{(j)}$ . The bit plane  $\underline{Q}^{(L)}$  is decoded first. The decoding is realized with a standard Belief Propagation (BP) decoder as described in [19]. The BP decoder requires the bit probabilities. From [9], we get,  $\forall k \in \llbracket 1, n \rrbracket$ ,

$$P(Q_k^{(L)} = 0 | Y_k^{(j)} = y) = \sum_{x \geq 0} P_Z(x - y) = \sum_{i=0}^{2^L-1} P_Z(i - y)$$

$$P(Q_k^{(L)} = 1 | Y_k^{(j)} = y) = \sum_{x < 0} P_Z(x - y) = \sum_{i=-2^L+1}^{-1} P_Z(i - y)$$

where  $P_Z$  is given in (5).

Then, for all  $b \in \llbracket 0, L-1 \rrbracket$ , the previous decoded bit planes  $\hat{\underline{Q}}^{(b+1)} \dots \hat{\underline{Q}}^{(L)}$  will be used to decode the current bit plane  $\underline{Q}^{(b)}$ . The bit probabilities of the  $Q_k^{(b)}$  are required by the BP decoder. From [9], we get

$$P(Q_k^{(b)} = 0 | Y_k^{(j)} = y, \hat{Q}_k^{(b+1)}, \dots, \hat{Q}_k^{(L)})$$

$$= \alpha P(Q_k^{(b)} = 0, \hat{Q}_k^{(b+1)}, \dots, \hat{Q}_k^{(L)} | Y_k^{(j)} = y) \quad (7)$$

$$= \alpha \sum_{i=0}^{2^b-1} P_Z \left( \left(1 - 2\hat{Q}_k^{(L)}\right) \cdot \left(i + \sum_{b'=b+1}^{L-1} \hat{Q}_k^{(b')} \cdot 2^{b'}\right) - y \right) \quad (8)$$

and

$$P(Q_k^{(b)} = 1 | Y_k^{(j)} = y, \hat{Q}_k^{(b+1)}, \dots, \hat{Q}_k^{(L-1)})$$

$$= \alpha P(Q_k^{(b)} = 1, \hat{Q}_k^{(b+1)}, \dots, \hat{Q}_k^{(L-1)} | Y_k^{(j)} = y) \quad (9)$$

$$= \alpha \sum_{i=0}^{2^b-1} P_Z \left( \left(1 - 2\hat{Q}_k^{(L)}\right) \cdot \left(i + 2^b + \sum_{b'=b+1}^{L-1} \hat{Q}_k^{(b')} \cdot 2^{b'}\right) - y \right) \quad (10)$$

where  $P_Z$  is given in (5), and where  $\alpha$  is a normalization coefficient which can be calculated from the condition that the sum of (8) and (10) equals 1. In addition, the last equalities in (8) and (10) are deduced from (3).

In the above expressions, the probability of  $Q_k^{(b)}$  depends on previous decoded bits  $\hat{Q}_k^{(b+1)}, \dots, \hat{Q}_k^{(L)}$ . Note that, here we know that every bit is perfectly decoded, *i.e.*,  $\hat{Q}_k^{(b)} = Q_k^{(b)}$  since the decoder is simulated during the coding process, and the codeword length  $m_{(b,j)}$  is chosen so as to satisfy this condition.

Finally, a concurrent strategy would consist of using non-binary LDPC codes with alphabets of  $2^L$  symbols. But the optimal non-binary Belief Propagation LDPC decoder has a complexity proportional to  $nL \log_2(L)$  [20], while our solution with binary LDPC codes has a complexity proportional to  $nL$ . For instance, for  $L = 9$  quantization bits, our solution reduces the complexity by a factor 3 compared to using non-binary LDPC codes.

## C. Pre-estimation strategy

In the bit-plane coder, the minimum codeword length is given by a value  $m_{\min}$ , which is imposed by the rate-adaptive LDPC code construction method of [18]. This means that the minimum compression rate is given by  $m_{\min}/n$ . For instance, in the code construction we consider in our simulations,  $m_{\min}/n = 1/32$ . We propose the following pre-estimation strategy in order to further reduce this rate.

Sometimes, the bit plane  $\underline{Q}^{(b)}$  can be entirely deduced from previously decoded bit planes and from  $\underline{Y}^{(j)}$ . Therefore at the encoder, we first check whether estimating each symbol bit  $Q_k^{(b)}$  as

$$\hat{Q}_k^{(b)} = \max_{s \in \{0,1\}} P(Q_k^{(b)} = s | Y_k^{(j)} = y, \hat{Q}_j^{(b+1)}, \dots, \hat{Q}_j^{(L)}) \quad (11)$$

allows to perfectly reconstruct  $\hat{\underline{Q}}^{(b)}$  without any need for decoding. If this condition is verified, the transmitted codeword  $\underline{U}^{(b,j)}$  is simply set as 0 and the corresponding transmission rate is equal to 0. This pre-estimation step will allow to greatly reduce the transmission rate, especially in cases where  $\underline{X}$  is close to  $\underline{Y}^{(j)}$ . In order to indicate to the decoder whether the pre-estimation step is sufficient to reconstruct  $\hat{\underline{Q}}^{(b)}$ , we add a flag bit to the coded bitstream.

## D. Rate evaluation

The performance of our practical ExtSC bit-plane coding scheme can be evaluated by calculating the transmission and storage rates as follows. When side information  $\underline{Y}^{(j)}$  is available at the decoder, the transmission rate of the  $b$ -th bit plane  $R_b^{(j)}$  and the overall transmission rate  $R_{\text{pract}}^{(j)}$  are

$$R_b^{(j)} = \frac{m_b^{(j)} + 1 + v_1 + 2v_2}{n}, \quad (12)$$

$$R_{\text{pract}}^{(j)} = \sum_{b=0}^L R_b^{(j)}. \quad (13)$$

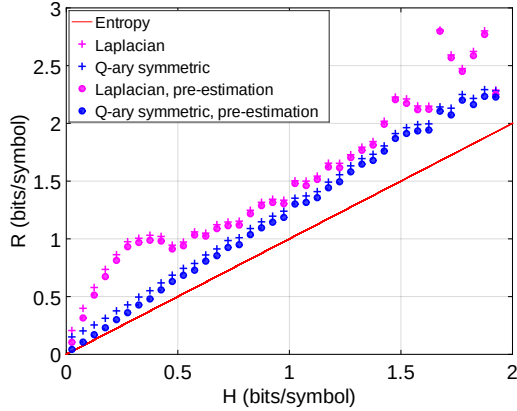


Fig. 1. Transmission rate comparison between Laplacian model and  $Q$ -ary symmetric model, with and without pre-estimation strategy

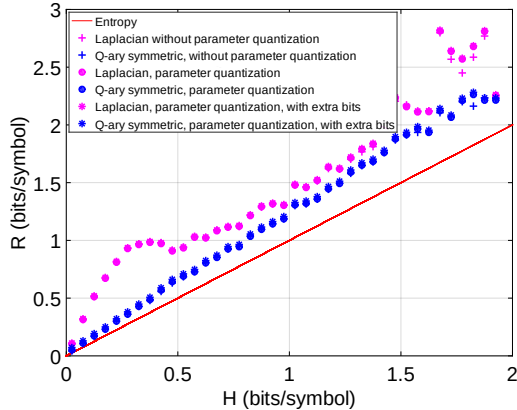


Fig. 2. Transmission rate comparison between Laplacian model and  $Q$ -ary symmetric model, with and without parameter quantization.

where in  $R_b^{(j)}$ , the value 1 comes from the flag bit due to pre-estimation, and  $v_1 + 2v_2$  gives the number of bits needed to transmit the  $Q$ -ary symmetric model parameters  $(q_j, Z_{\min}^{(j)}, Z_{\max}^{(j)})$ . In addition, the storage rate  $S_b$  of the  $b$ -th bit plane and the total storage rate  $S_{\text{pract}}$  are

$$S_b = \max_{j \in \llbracket 1, J \rrbracket} R_b^{(j)} \quad (14)$$

$$S_{\text{pract}} = \sum_{b=0}^L S_b. \quad (15)$$

Finally note that to achieve these rates, the proposed scheme needs to simulate the  $L$  LDPC decoders at the encoder part, in order to ensure that zero-error rate is achieved. Therefore, our method is especially dedicated to applications where the encoding can be performed offline, like in video streaming. In the following experimental section, we compare these practical rates to the theoretical rates provided in Section II.

## V. EXPERIMENTS

The following experiments have been conducted with the “pano\_aaaaajndugdzeh” image of the SUN360 database [21] processed with the practical ExtSC scheme presented in [13].

This scheme encodes a  $360^\circ$  image (in equirectangular format [22]) such that only a subpart of it (the one watched by a client) can be extracted and decoded. The  $360^\circ$  image is divided into small blocks ( $32 \times 32$  pixels), each of them corresponding to a source  $\underline{X}$ . Then, according to the current and previous requests of the user, some neighboring blocks are available at the decoder and provide an estimate  $\underline{Y}^{(j)}$  of the current block, see [13] for details about how the estimation is performed. This leads to one prediction per possible set of neighboring blocks. When a client observes the  $360^\circ$  image in a given direction, it requires a set of blocks in the image. The requested blocks are decoded in an optimized order which depends on client’s head position. In this causal decoding process, each block is predicted using the already decoded blocks, corresponding to one of the anticipated  $\underline{Y}^{(j)}$ . The server simply has to extract and transmit the proper codeword. We precise that the  $\underline{X}$  and  $\underline{Y}^{(j)}$  described above correspond to transformed and quantized versions of the original sources and predictions.

We insert the lossless coding method developed in this paper into the previous full coding scheme for  $360^\circ$  images. We then evaluate the proposed  $Q$ -ary symmetric model against the conventional Laplacian model, as well as the pre-estimation strategy. For this, we first perform a rate comparison over 1000 blocks  $\underline{X}$  taken from the  $360^\circ$  image, each of length  $n = 1024$  and with either  $J = 8$  or  $J = 12$  side information blocks  $\underline{Y}^{(j)}$ . For each considered block  $\underline{X}$ , we consider  $L = 9$  quantization bits, and we apply the incremental coding scheme developed in this paper, and measure the obtained transmission rate for each side information  $\underline{Y}^{(j)}$ .

For performance comparison, we consider two different setups. In Figure 1, we assume that the parameter values estimated at the encoder are known in full-precision at the decoder, and we compare the average transmission rates obtained under the  $Q$ -ary model and under the Laplacian model. In both cases, we also evaluate the transmission rate without and with the pre-estimation strategy, by taking into account extra-rate needed by the pre-estimation step. We first observe a clear gain at considering the  $Q$ -ary symmetric model, compared to the Laplacian model. We also see that pre-estimation improves the transmission rates by approximately 0.1 bits/symbol. Then, Figure 2 always considers pre-estimation, and evaluates the effect of parameter quantization on the transmission rate. In the  $Q$ -ary symmetric model, we consider that the value of  $q$  is quantized on  $v_1 = 3$  bits, and that the values of  $Z_{\min}$  and  $Z_{\max}$  are quantized on  $v_2 = 9$  bits. In the Laplacian model, we assume that the scaling parameter is quantized on 8 bits. We see that, even with parameter quantization, the  $Q$ -ary symmetric model is still the best, and we observe a negligible performance loss due to parameter quantization.

In order to confirm these results, we now perform a rate-distortion analysis over 6334 blocks  $\underline{X}$  of length  $n = 1024$ , each with either 8 or 12 side information blocks. Figure 3 (a) shows the distortion in PSNR with respect to the transmission rate in KB, and Figure 3 (b) represents the distortion with respect to the storage rate in MB. These two curves confirm the benefits of the proposed  $Q$ -ary model and pre-estimation strategy with respect to the conventional Laplacian model without pre-estimation. At low PSNR, though, the bitrate for

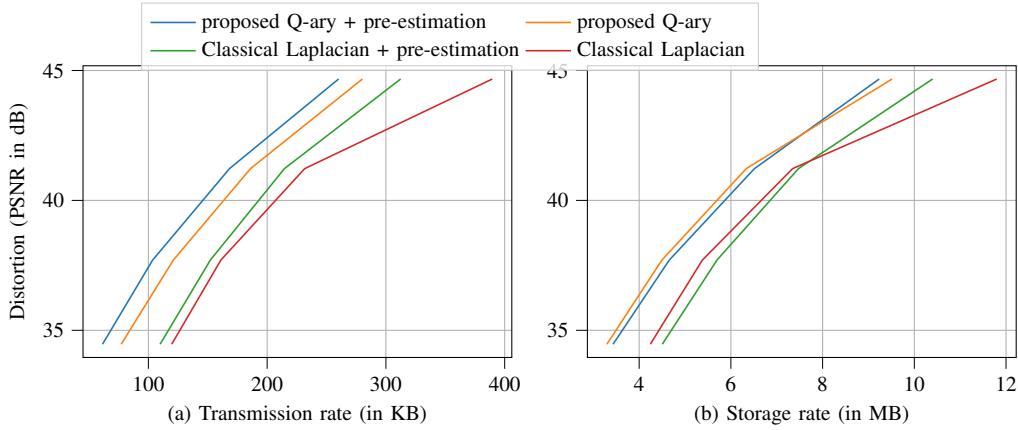


Fig. 3. Rate-distortion results for encoding spherical images with ExtSC. The user requests sub part of the whole image. (a) Transmission rate-distortion curve. (b) Storage-distortion curve.

TABLE I  
BD-R (TRANSMISSION RATE) AND BD-S (STORAGE) MEASURES (IN PERCENT) RELATIVE TO THE CLASSICAL LAPLACIAN SCHEME.

	Laplacian + pre-estimation	$Q$ -ary	$Q$ -ary + pre-estimation
BD-S	1.94	-16.48	-14.39
BD-R	-8.45	-24.67	-34.00

storage is slightly penalized by the pre-estimation strategy. In addition, the bitrate gains averaged over the whole PSNR range, called Bjøntegaard measures, are listed in Table I, where the classical Laplacian model with no pre-estimation is taken as a reference. Negative values represent compression gain. Results show that the two proposed strategies ( $Q$ -ary and pre-estimation) save 14% and 34% of storage and transmission rate respectively.

## VI. CONCLUSION

In this paper, we analyzed and improved an extractable coding scheme based on symbol binarization, which allows to reduce the decoding complexity. We then introduced a new statistical model, called  $Q$ -ary symmetric, which achieves a better tradeoff between the model description cost and the data representation cost, than the more common Laplacian model. Finally, we developed a pre-estimation strategy, which helps avoid the need to store/send any data for many bitplanes.

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