THE IMPACT OF *A PRIORI* INFORMATION ON THE MAP EQUALIZER PERFORMANCE WITH M-PSK MODULATION

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ABSTRACT

In this paper, we consider a transmission of M-Phase Shift Keying (M-PSK) symbols with Gray mapping over a frequency selective channel. We propose to study analytically the impact of a priori information (provided for instance by a decoder in a turbo equalizer) on the maximum a posteriori (MAP) equalizer performance. We derive analytical expressions of the bit error probability at the output of the equalizer. Simulations show that the analytical expressions approximate well the bit error rate (BER) at the output of the MAP equalizer at high signal to noise ratio (SNR).

1. INTRODUCTION

The high data rate communication systems are impaired by intersymbol interference (ISI). To combat the effects of ISI, an equalizer has to be used. The optimal soft-input soft-output equalizer, that achieves minimum bit error rate (BER), is based on the maximum *a posteriori* (MAP) criterion. In this paper, we consider the case where the M-Phase Shift Keying (M-PSK) modulation (with $M = 2^q$, for $q \geq 1$) is used and the MAP equalizer has *a priori* information on the transmitted data. The *a priori* information are provided by another module in the receiver, for instance a decoder in a turbo-equalizer [1]. In a turbo-equalizer, the equalizer and the decoder exchange extrinsic information and use them as *a priori* in order to improve their performance.

In [2], the authors studied analytically the impact of *a priori* information on the MAP equalizer performance in the case of Binary Phase Shift Keying (BPSK) modulation ($M = 2$). The aim of our paper is to generalize the study of [2] to the case of M-PSK modulation. To do this, we derive analytical approximate expressions of the BER at the output of the MAP equalizer. Simulations show that these expressions approximate well the BER at the output of the MAP equalizer at high SNR.

This paper is organized as follows. In section 2, we give the system model. In section 3, we derive analytical expressions of the BER at the output of the MAP equalizer. In section 4, we give simulation results for 8-PSK and 16-PSK modulations. Throughout this paper matrices are upper case and vectors are underlined lower case. The operator $(.)^T$ denotes transposition, and $Re(.)$ represents the real value.

2. SYSTEM MODEL

We consider a data transmission system over a frequency selective channel. We assume that transmissions are organized into bursts of T symbols and the channel is invariant during one burst. The channel is spread over L symbols. The input information bit sequence $\underline{b} = (b_{q(1-L)}, ..., b_{(qT-1)})^T$, is mapped to a sequence of M-PSK symbols with Gray mapping where $M = 2^q$, and $q \geq 1$. The baseband signal received and sampled at the symbol rate at time k is:

$$y_k = \sum_{i=0}^{L-1} h_i x_{k-i} + n_k, \tag{1}$$

where h_i, represents the i^{th} complex channel tap gain, x_k, for $1-L \leq k \leq T-1$, are the transmitted symbols, n_k are modeled as independent random variables of a complex white Gaussian noise with zero mean and variance σ^2, with normal probability density function (pdf) $N_{\mathbb{C}}\left(0, \sigma^2\right)$ where $N_{\mathbb{C}}\left(\alpha, \sigma^2\right)$ denotes a complex Gaussian distribution with mean α and variance σ^2. Equation (1) can be rewritten as follows:

$$\underline{y} = H\underline{x} + \underline{n},$$

where $\underline{x} = (x_{1-L}, ..., x_{T-1})^T$ is the vector of transmitted symbols, $\underline{n} = (n_0, ..., n_{T-1})^T$ is the Gaussian noise vector, $\underline{y} = (y_0, ..., y_{T-1})^T$ is the vector of the output symbols and H is a $T \times (T+L-1)$ Toeplitz matrix defined as:

$$H = \begin{bmatrix} h_{L-1} & \dots & h_0 & 0 & \dots & 0 \\ 0 & h_{L-1} & \dots & h_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \dots & 0 & h_{L-1} & \dots & h_0 \end{bmatrix}. \tag{2}$$

We assume that the channel is normalized: $\sum_{i=0}^{L-1} |h_i|^2 = 1$.

In this section, we consider the case where no *a priori* information is provided to the equalizer. The data estimate according to the MAP sequence criterion (or to the Maximum Likelihood (ML) criterion, since there is no *a priori*) is given by:

$$\widehat{\underline{x}} = \arg\min_{\underline{u}} \left(\left\| \underline{y} - H\underline{u} \right\| : \underline{u} \in A^{(T+L-1)} \right),$$

where A is the symbol alphabet. An error occurs if the estimated sequence $\widehat{\underline{x}}$ is different from the true sequence $\underline{x}$. Let us denote $\underline{e} = \widehat{\underline{x}} - \underline{x}$ the resulting error vector. A subevent ξ_e of the error event is that "$\widehat{\underline{x}}$ is better than $\underline{x}$" in the sense of the ML metric:

$$\xi_e : \left\| \underline{y} - H\widehat{\underline{x}} \right\| \leq \left\| \underline{y} - H\underline{x} \right\|,$$

We now proceed to derive a lower bound on the BER of the equalizer. This can be done by computing the exact error probability of a genie aided equalizer that has some side information about the sent sequence. More precisely, we consider the case where the receiver has side information (from a genie) that one of the two sequences $\underline{x}$ or $\widehat{\underline{x}}$ was transmitted. When the genie aided equalizer has this side information, the pairwise error probability that it chooses $\widehat{\underline{x}}$ instead of $\underline{x}$ is given by [3]:

$$P_{\underline{x},\widehat{\underline{x}}} = Q\left(\frac{\|\varepsilon\|}{\sqrt{2}\sigma} \right), \tag{3}$$

where $\underline{\varepsilon} = H\underline{e}$, and $Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} \exp\left(-\frac{y^2}{2}\right) dy$.

Let us assume that the genie tells the receiver that it has to choose between the true sequence $\underline{x}$ and another sequence $\widehat{\underline{x}}$, such that (3) is maximal i.e. such that the distance $\|\underline{\varepsilon}\|$ is minimal. We now proceed to compute the exact error probability for this genie aided receiver. Let E_{min} be the set of all $\underline{e}$ achieving the minimum value of $\|\underline{\varepsilon}\|$. Let π_e be the probability that the input sequence $\underline{x}$ will be such that $\widehat{\underline{x}} = \underline{x} + \underline{e}$ is an allowable input sequence for this $\underline{e}$. π_e can be interpreted as the probability that the data sequence $\underline{x}$ is compatible with $\underline{e}$. Then, the probability that the genie aided equalizer makes an error is

$$\sum_{\underline{e} \in E_{min}} \pi_e Q\left(\frac{\min_{\underline{e}} \|\underline{\varepsilon}\|}{\sqrt{2}\sigma}\right)$$

In order to obtain a lower bound on the BER, we assume that $\underline{e}$ is made of $m(\underline{e})$ non-zero consecutive symbols, the other symbols being zero. Other saying, we assume that there exists an interval of size $m(\underline{e})$ such that all the symbols of $\widehat{\underline{x}}$ are different from the corresponding symbols of $\underline{x}$ while the preceding symbol and the following one are the same. $m(\underline{e})$ is referred to as the *weight* of the error sequence in the following. Moreover, we remark that there is at least one erroneous bit per erroneous symbol. Therefore, when the equalizer has no side information, the BER P_e is lower bounded by the probability achieved by the genie aided equalizer:

$$P_e \geq \sum_{\underline{e} \in E_{min}} m(\underline{e}) \pi_e Q\left(\frac{\min_{\underline{e}} \|\underline{\varepsilon}\|}{\sqrt{2}\sigma}\right)$$

As will be shown by simulations, this lower bound is a good approximation of the error probability at high SNR and for a Gray mapping. This can be explained by the fact that at high SNR and for a Gray mapping, the union bound (upper bound) and the above derived lower bound are equal.

Our goal is to find an approximation of the BER at the output of the MAP equalizer with *a priori* information.

3. PERFORMANCE ANALYSIS

In this section, we propose to evaluate the impact of the *a priori* information on the MAP equalizer performance when a M-PSK modulation with Gray mapping is used. The study will be done for the equalizer using the MAP sequence criterion. It holds for the MAP symbol equalizer using the BCJR algorithm [4] since the two equalizers have almost the same performance as observed in [5,page 814]. We assume that channels are perfectly known at the receiver. Moreover, we suppose that *a priori* observations at the input of the equalizer are modeled as outputs of an additive white Gaussian noise (AWGN) channel. This is a usual assumption in the analyses of iterative receivers [6]. These *a priori* observations on bits b_l, for $q(1-L) \leq l \leq qT-1$, are:

$$z_l = b_l + w_l,$$

where w_l are independent random variables of a real white Gaussian noise with zero mean and variance σ_a^2 with normal pdf $N(0, \sigma_a^2)$, where $N(\alpha, \sigma^2)$ denotes a real Gaussian distribution with mean α and variance σ^2. Then, the *a priori* Log Likelihood Ratios (LLRs) are:

$$LLR(b_l) = \log \frac{P(z_l | b_l = 1)}{P(z_l | b_l = -1)} = \frac{2}{\sigma_a^2} z_l.$$

Thus, these LLRs can be modeled as independent and identically distributed random variables with the conditional pdf $N\left(\frac{2b_l}{\sigma_a^2}, \frac{4}{\sigma_a^2}\right)$ (with b_l the transmitted bit).

Proposition 1 *Suppose we have a transmission of M-PSK symbols over a frequency selective channel defined by a Toeplitz matrix H and with a complex AWGN of variance σ^2. Consider that the MAP equalizer has side information (from a genie) that one of the two sequences $\underline{x}$ or $\widehat{\underline{x}}$ was transmitted and has a priori observations modeled as outputs of an AWGN channel with variance σ_a^2. Then, the pairwise error probability that it chooses $\widehat{\underline{x}}$ instead of $\underline{x}$ is:*

$$P_{\underline{x},\widehat{\underline{x}}} = Q\left(\frac{1}{\sqrt{2}\sigma}\sqrt{\|\underline{\varepsilon}\|^2 + 2m_b(\underline{e})\mu^2}\right), \qquad (4)$$

where $\underline{\varepsilon} = H\underline{e}$, $\underline{e} = \widehat{\underline{x}} - \underline{x}$ is the symbol error vector, $m(\underline{e})$ is the weight in symbols of $\underline{e}$, $m_b(\underline{e})$ is the weight in bits of $\underline{e}$ ($m(\underline{e}) \leq m_b(\underline{e}) \leq qm(\underline{e})$) and $\mu = \frac{\sigma}{\sigma_a}$.

The proof of proposition 1 is given in the Appendix. □

Here again we assume that the genie always chooses a sequence $\hat{x}$ that maximizes the pairwise error probability (4). Let E_{min} be the set of all $\underline{e}$ achieving the minimum value of $\sqrt{\|\underline{\varepsilon}\|^2 + 2m_b(\underline{e})\mu^2}$, π_e be the probability that the input sequence $\underline{x}$ will be such that $\widehat{\underline{x}} = \underline{x} + \underline{e}$ is an allowable input sequence. Then, the probability that the genie aided equalizer chooses an allowable sequence $\widehat{\underline{x}}$ instead of $\underline{x}$ is $\sum_{\underline{e} \in E_{min}} \pi_e Q\left(\frac{1}{\sqrt{2}\sigma}\left(\min_{\underline{e}} \sqrt{\|\underline{\varepsilon}\|^2 + 2m_b(\underline{e})\mu^2}\right)\right)$.

Corollary 1 *We consider a M-PSK modulation with Gray mapping. The BER at the output of the MAP equalizer without genie, using the a priori information can be approximated by:*

$$P_e \simeq \sum_{\underline{e} \in E_{min}} m(\underline{e}) \pi_e Q\left(\frac{1}{\sqrt{2}\sigma}\left(\min_{\underline{e}} \sqrt{\|\underline{\varepsilon}\|^2 + 2m(\underline{e})\mu^2}\right)\right), \quad (5)$$

where $m(\underline{e})$ is the number of (consecutive) non zero symbols in $\underline{e}$.

Proof of corollary 1: For the MAP equalizer without genie, the BER is lower bounded by the probability achieved by the genie aided equalizer:

$$\sum_{\underline{e} \in E_{min}} m_b(\underline{e}) \pi_e Q\left(\frac{1}{\sqrt{2}\sigma}\left(\min_{\underline{e}} \sqrt{\|\underline{\varepsilon}\|^2 + 2m_b(\underline{e})\mu^2}\right)\right).$$

In addition, since we use a Gray mapping, we further assume that there is one erroneous bit per erroneous symbol, s.t. $m_b(\underline{e}) \approx m(\underline{e})$. Simulations show that this assumption becomes true at high SNR. Moreover, at high SNR the union bound (upper bound) and the lower bound are equal. Therefore, the lower bound of the BER becomes an approximation as written in (5). □

In order to evaluate the expression (5), an exhaustive search has to be performed over all possible error sequences. We propose to reduce this exhaustive search and to restrict ourself to the case of error sequences of symbol weight 1 or 2. Simulations will show that this provides reliable approximations of the overall BER. In the following, we consider different cases according to the value of μ.

Corollary 2 *Let $d_1 = \min_{\underline{e}:\, m(\underline{e})=1} \|\underline{\varepsilon}\|$ and $d_2 = \min_{\underline{e}:\, m(\underline{e})=2} \|\underline{\varepsilon}\|$. If the a priori information is unreliable (i.e. $d_1 > d_2$ and $\mu \leq \mu_{lim} = \sqrt{\frac{d_1^2 - d_2^2}{2}}$), the BER at the output of the MAP equalizer can be approximated by:*

$$P_e \simeq 2\pi_{\underline{e}_2} Q\left(\frac{1}{\sqrt{2}\sigma}\sqrt{d_2^2 + 4\mu^2}\right), \qquad (6)$$

where $\pi_{\underline{e}_2}$ is the probability that we can find pair of sequences compatible with error sequences of (symbol) weight 2. More precisely $\pi_{\underline{e}_2} = \sum_{\underline{e} \in E_2} \pi_{\underline{e}}$ and E_2 is the set of (symbol) weight 2 sequences s.t. $\|\underline{\varepsilon}\| = d_2$.

On the other hand, if the information is reliable (i.e. $d_1 > d_2$ and $\mu > \mu_{lim}$ or if $d_1 \leq d_2$), the BER at the output of the MAP equalizer can be approximated by:

$$P_e \simeq Q\left(\frac{1}{\sqrt{2}\sigma}\sqrt{d_1^2 + 2\mu^2}\right), \qquad (7)$$

Proof of corollary 2: The value of μ_{lim} results from the equality between equations (6) and (7). When the *a priori* information at the input of the MAP equalizer is unreliable (i.e. $\mu \leq \mu_{lim}$), errors occur in bursts. Thus, we do not consider isolated errors and we look for the maximum of (5) for $m(\underline{e}) \geq 2$. Simulations show that the minimum value of $\sqrt{\|\underline{\varepsilon}\|^2 + 2m(\underline{e})\mu^2}$ is obtained for error sequence of weight 2 (i.e. $m(\underline{e}) = 2$ and $\|\underline{\varepsilon}\| = d_2$). Therefore the BER is approximated by (6). The values of $\pi_{\underline{e}_2}$ for the different modulations can be found by an exhaustive search and are given in Table 1.

In the case of reliable *a priori* information (i.e. $\mu > \mu_{lim}$), *a priori* observations have more influence on the detection than channel observations and most of the errors are isolated. Therefore, $m(\underline{e}) = 1$ and $\|\underline{\varepsilon}\| = d_1$. In this case for all modulations, $\pi_{\underline{e}_1} = \sum_{\{\underline{e}:\ m(\underline{e})=1 \text{ and } \|\underline{\varepsilon}\|=d_1\}} \pi_{\underline{e}}$ is equal to 1, which leads to (7).

Depending on μ, we get either (6) or (7). We determine a threshold μ_{lim} as the value of μ when (6) equals (7). It follows that $\mu \leq \mu_{lim}$ corresponds to the case with unreliable a priori and $\mu > \mu_{lim}$ to the case with reliable a priori. □

Table 1: $\pi_{\underline{e}_2}$ values when the *a priori* information at the input of the MAP equalizer are unreliable

	BPSK	QPSK	8-PSK	16-PSK
$\pi_{\underline{e}_2}$	$1/2$	$3/4$	$3/8$	$3/16$

4. SIMULATION RESULTS

In this section, we propose to validate the analysis by simulations. In the simulations, the modulation used is either a 8-PSK or a 16-PSK using Gray mapping and the channel is assumed to be constant. We provide the MAP equalizer with *a priori* information on the transmitted bits b_l modeled as independent and identically distributed random variables with the conditional pdf $N(\frac{2b_l}{\sigma_a^2}, \frac{4}{\sigma_a^2})$. We consider different channels.

Figures 1, 2 and 3 represent respectively the BER with respect to the SNR at the output of the MAP equalizer with 8-PSK modulation, for *channel*3, *channel*4 and *channel*3*c* with impulse responses: $channel3 = (0.5, 0.71, 0.5)$, $channel4 = (0.37, 0.6,\ 0.6, 0.37)$ and $channel3c = (0.6117 - 0.1223j, 0.4588 + 0.5505j, -0.3059 + 0.0612j)$.

Table 2 gives the values of d_1 and d_2 for each channel. For all these channels, we have $d_1 > d_2$. The limit values μ_{lim} are given in Table 3.

Table 2: d_1 and d_2 values for 8-PSK modulation

	*channel*3	*channel*4	*channel*3*c*
d_1	0.7653	0.7653	0.7653
d_2	0.5858	0.4730	0.6936

Figures 4 and 5 represent the BER with respect to the SNR at the output of the MAP equalizer with 16-PSK modulation respectively for *channel*3 and *channelcomp* with impulse responses: $channel3 = (0.5, 0.71, 0.5)$ and $channelcomp = (0.3127 - 0.4576j, 0.3813 + 0.5643j, -0.469 + 0.0953j)$.

Table 4 gives the values of d_1 and d_2 for each channel.

For all these channels, we have $d_1 > d_2$. The limit values μ_{lim} are given in Table 5.

Table 3: μ_{lim} values for 8-PSK modulation

	*channel*3	*channel*4	*channel*3*c*
μ_{lim}	0.3483	0.4255	0.2288

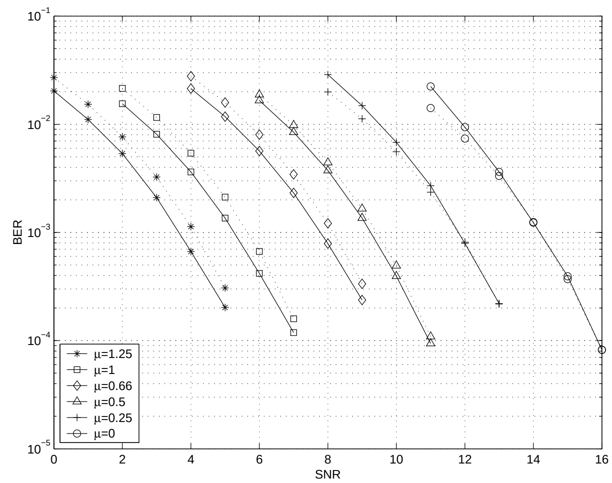

Figure 1: BER versus SNR: comparison of the equalizer performance (solid curves) and the theoretical performance (dotted curves) for *channel*3 and 8-PSK modulation

Table 4: d_1 and d_2 values for 16-PSK modulation

	*channel*3	*channelcomp*
d_1	0.3901	0.3901
d_2	0.2986	0.3005

In all figures, each curve is obtained while the ratio μ is kept constant. The solid lines indicate the equalizer performance given by simulations. The dotted lines are obtained by considering the analytical approximate expressions calculated in the previous section. We notice that the theoretical curves approximate well the BER. This approximation becomes better at high SNR (for low values of BER). We can also deduce that the approximation is more accurate for 8-PSK than for 16-PSK, since the assumption that there is one erroneous bit per erroneous symbol is better validated for a 8-PSK modulation.

5. CONCLUSION

In this paper, we considered a transmission of M-PSK symbols over a frequency selective channel. We proposed to study analytically the impact of *a priori* information on the MAP equalizer performance. We gave an approximation of the error probability at the output of the MAP equalizer. Simulation results showed that the analytical expressions give a good approximation of the equalizer performance. The aim of this work is to perform in the future the analytical convergence analysis of iterative receivers with MAP equalization.

6. APPENDIX: PROOF FOR PROPOSITION 1

We recall that the output of the channel during a burst is the $T \times 1$ complex vector $\underline{y} = (y_0, \ldots, y_{T-1})^T$ defined as:

$$\underline{y} = H\underline{x} + \underline{n},$$

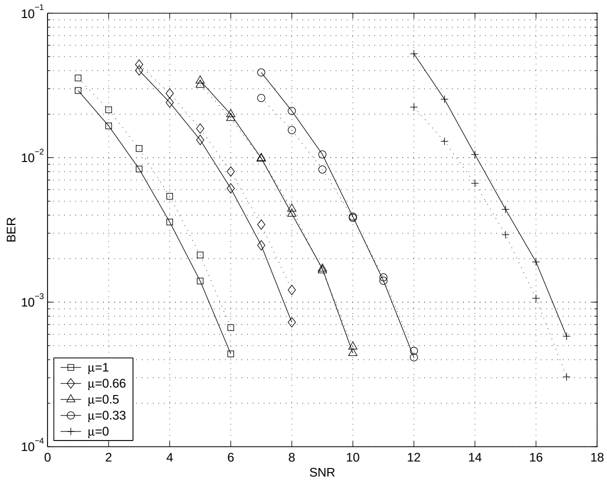

Figure 2: BER versus SNR: comparison of the equalizer performance (solid curves) and the theoretical performance (dotted curves) for *channel*4 and 8-PSK modulation

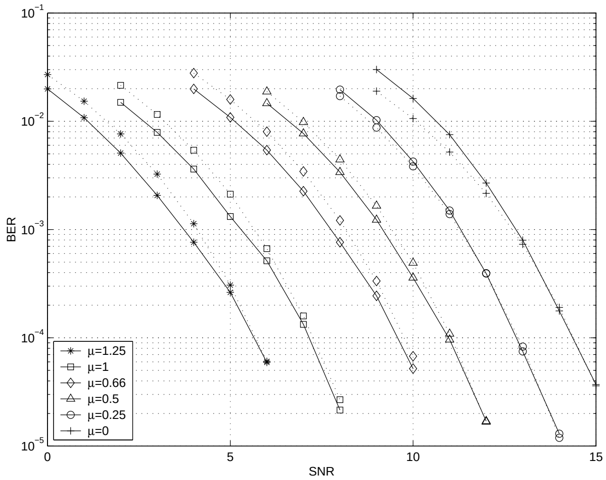

Figure 3: BER versus SNR: comparison of the equalizer performance (solid curves) and the theoretical performance (dotted curves) for *channel*3*c* and 8-PSK modulation

where $\underline{x} = (x_{1-L}, \ldots, x_{T-1})^T$ is the $(T+L-1)\times 1$ vector of transmitted symbols, $\underline{n} = (n_0, \ldots, n_{T-1})^T$ is a complex white Gaussian noise vector with zero mean and variance σ^2 and H is the $T\times(T+L-1)$ channel matrix given by (2).

The real vector of *a priori* observations $\underline{z} = (z_{q(1-L)}, \ldots, z_{(qT-1)})^T$ on the transmitted bits is defined as:

$$\underline{z} = \underline{b} + \underline{w},$$

where $\underline{w} = (w_{q(1-L)}, \ldots, w_{(qT-1)})^T$ is a real white Gaussian noise vector with zero mean and variance σ_a^2 and $\underline{b} = (b_{q(1-L)}, \ldots, b_{(qT-1)})^T$ is the vector of transmitted bits.

Taking into account the *a priori* information, the *a posteriori*

Table 5: μ_{lim} values for 16-PSK modulation

	*channel*3	*channelcomp*
μ_{lim}	0.1774	0.1758

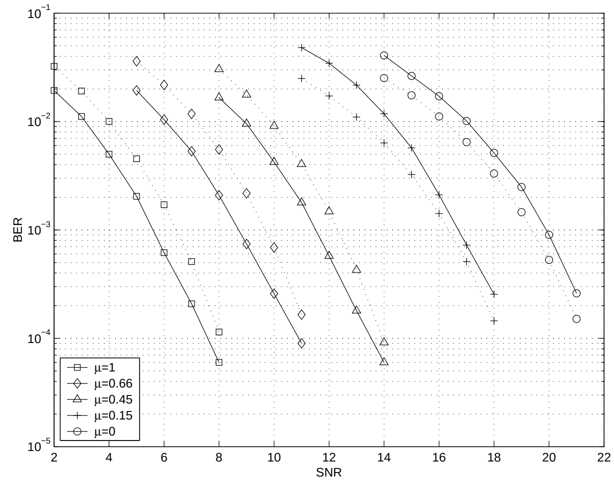

Figure 4: BER versus SNR: comparison of the equalizer performance (solid curves) and the theoretical performance (dotted curves) for *channel*3 and 16-PSK modulation

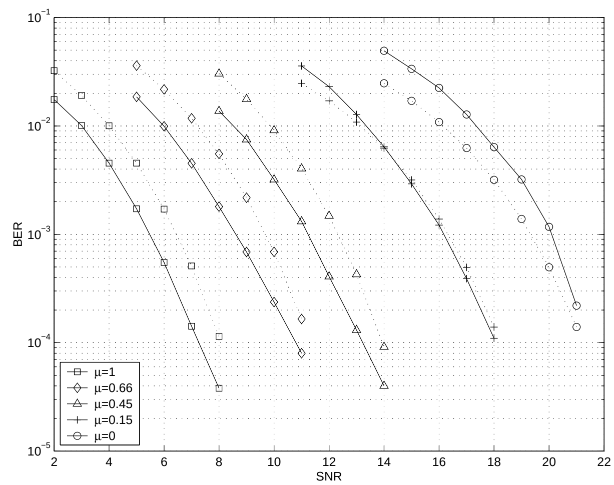

Figure 5: BER versus SNR: comparison of the equalizer performance (solid curves) and the theoretical performance (dotted curves) for *channelcomp* and 16-PSK modulation

probability of the sequence $\underline{x}$ is given by:

$$P\left(\underline{x}|\underline{y},\underline{z}\right) \propto \exp\left(-\frac{\left\|\underline{y}-H\underline{x}\right\|^2}{\sigma^2}\right)\exp\left(-\frac{\left\|\underline{z}-\underline{b}\right\|^2}{2\sigma_a^2}\right).$$

As in the case of no *a priori* information, we consider an error event ξ_e. This error event is that $\underline{\widehat{x}}$ is better than $\underline{x}$ in the sense of the MAP sequence metric:

$$\xi_e \quad : \quad \left\|\underline{y}-H\underline{\widehat{x}}\right\|^2 + \frac{\sigma^2}{2\sigma_a^2}\left\|\underline{z}-\underline{\widehat{b}}\right\|^2 \leq \left\|\underline{y}-H\underline{x}\right\|^2 + \frac{\sigma^2}{2\sigma_a^2}\left\|\underline{z}-\underline{b}\right\|^2, \qquad (8)$$

Let $\underline{e} = \underline{\widehat{x}} - \underline{x}$ and $\underline{a} = \underline{\widehat{b}} - \underline{b}$ be respectively the symbol error vector and the bit error vector. We have:

$$\begin{aligned} & \left\|\underline{y}-H\underline{\widehat{x}}\right\|^2-\left\|\underline{y}-H\underline{x}\right\|^2 \\ = & \left\|\underline{y}-H\underline{x}-H\underline{e}\right\|^2-\left\|\underline{y}-H\underline{x}\right\|^2 \\ = & \|H\underline{e}\|^2-2Re(\langle\underline{y}-H\underline{x},H\underline{e}\rangle) \\ = & \|\underline{\varepsilon}\|^2-2Re(\langle\underline{n},\underline{\varepsilon}\rangle), \end{aligned} \quad (9)$$

where $\underline{\varepsilon}=H\underline{e}$. In addition:

$$\begin{aligned} \left\|\underline{z}-\underline{\widehat{b}}\right\|^2-\|\underline{z}-\underline{b}\|^2 &= \|\underline{z}-\underline{b}-\underline{a}\|^2-\|\underline{z}-\underline{b}\|^2 \\ &= \|\underline{a}\|^2-2\langle\underline{z}-\underline{b},\underline{a}\rangle \\ &= \|\underline{a}\|^2-2\langle\underline{w},\underline{a}\rangle. \end{aligned} \quad (10)$$

We obtain using (8), (9) and (10):

$$\xi_e : \|\underline{\varepsilon}\|^2+\frac{\mu^2}{2}\|\underline{a}\|^2 \leq 2Re(\langle\underline{n},\underline{\varepsilon}\rangle)+\mu^2\langle\underline{w},\underline{a}\rangle,$$

where $\mu=\frac{\sigma}{\sigma_a}$. We note $\chi=2Re(\langle\underline{n},\underline{\varepsilon}\rangle)+\mu^2\langle\underline{w},\underline{a}\rangle$. We can write that $\chi\sim N\left(0,2\sigma^2\left(\|\underline{\varepsilon}\|^2+\frac{\mu^2}{2}\|\underline{a}\|^2\right)\right)$. Thus, when the genie aided equalizer is provided with *a priori* information, the pairwise error probability that it chooses $\underline{\widehat{x}}$ instead of $\underline{x}$ is given by:

$$P_{\underline{x},\underline{\widehat{x}}}=Q\left(\frac{1}{\sqrt{2}\sigma}\sqrt{\|\underline{\varepsilon}\|^2+\frac{\mu^2}{2}\|\underline{a}\|^2}\right).$$

Let $m_b(\underline{e})$ be the weight of the bit error vector, and $m(\underline{e})$ the weight of the symbol error vector with $m(\underline{e})\leq m_b(\underline{e})\leq qm(\underline{e})$ since the number of erroneous bit per erroneous symbol is between 1 and q. The vector $\underline{a}$ has $m_b(\underline{e})$ components equal to ±2 and the others equal to zero. Thus, we can replace $\|\underline{a}\|^2$ by $4m_b(\underline{e})$ and we obtain:

$$P_{\underline{x},\underline{\widehat{x}}}=Q\left(\frac{1}{\sqrt{2}\sigma}\sqrt{\|\underline{\varepsilon}\|^2+2m_b(\underline{e})\mu^2}\right).$$

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