Properties of pedestrians walking in line: Stepping behavior

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In human crowds, interactions among individuals give rise to a variety of self-organized collective motions that help the group to effectively solve the problem of coordination. However, it is still not known how exactly are the humans adjusting their behavior locally, nor what are the direct consequences on the emergent organization. One of the underlying mechanisms of adjusting individual motions is the stepping dynamics. In this paper, we present first quantitative analysis on the stepping behavior in a one-dimensional pedestrian flow studied under controlled laboratory conditions. We find that the step length is proportional to the velocity of the pedestrian, and is directly related to the space available in front of him, while the variations of the step duration are much weaker. Furthermore, we study the phenomena of synchronization –walking in lock-steps– and show its dependence on the flow densities. We show that the synchronization of steps is particularly important at high densities, which has direct impact on the studies of optimizing pedestrians flow in congested situations. However, some small synchronization and antisynchronization effects are found even at very low densities, for which no steric constraints exist between successive pedestrians, showing the natural tendency to synchronize according to the perceived visual signals.

I. INTRODUCTION

As in many biological systems such as fish schools, flocks of birds, or ant colonies, the dynamics of large pedestrian groups are governed by local interactions between individuals which give rise to a variety of collective motions occurring on a macroscopic scale [1]. Such self-organized behaviors of large pedestrian groups are studied for practical and safety reasons –improving pedestrians facilities and preventing accidents in emergency regimes– but also as an intriguing problem of out of equilibrium physics. Indeed, many emerging collective phenomena and pattern formation still can not be completely explained by the standard models describing the dynamics of pedestrians [2]. Furthermore, flux-density and velocity-density relations, known also as fundamental diagrams, are studied in different situations [3], since no general characteristics can be given. Besides, it is of crucial importance to understand in which situations the emerging organization allows the group to solve effectively coordination problems, for avoiding future accidents and easing the high congestion situations in pedestrian facilities. Having this idea in mind, in this paper, we analyze the stepping behavior in a simple situation of a pedestrian flow, in order to understand in which way on an individual level the pedestrians are adjusting their behavior according to the individuals surrounding them.

Interactions between pedestrians usually take place in a two-dimensional space and produce velocity changes in direction and modulus. However, there are some situations where interactions are mostly longitudinal, for example, when people are walking along a narrow corridor. It is also known that counterflows –pedestrian flows of opposite direction– induce lane formation [4, 5], and within these lanes longitudinal interactions should dominate. Indeed, some experiments have suggested that the adaptations of velocity in angle and in modulus could be decoupled to some extent [6]. This decoupling has already been used in some models [6, 7].

As one-dimensional pedestrian flows involve purely longitudinal interactions which induce only changes in the velocity modulus, a lot of interest has been shown recently in studying how pedestrians follow each other in such settings. From the point of view of modeling, one-dimensional flows can serve as a simple test of the following behavior in models. The settings become even easier if boundary effects can be avoided by having pedestrians walking on a closed line. Such experiments have been already conducted in recent years. Seyfried et al. [8–10] have performed experiments with pedestrians following an oval trajectory. Similar studies have been reported in references [11, 12]. These experiments have been performed either by using video analysis of the individual trajectories along one straight portion of the set-up [10], or by measuring times at which each participant passes a given measuring point [12].

We have reported in [13] new experiments performed on circular trajectories, with two different radii. Pedestrians were tracked with a high precision motion capture devices. As a result, we were able to have instantaneous measurements of the three-dimensional trajectories and velocities during the whole duration of experiment. Be-

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ing able to cover a larger range of densities than in previous experiments, we found that the velocity-spatial headway relation is exhibiting two transitions in the pedestrian following behavior [13, 14]. Moreover, tracking of all the individual trajectories during the whole duration of the experiment, allowed us access to other features of the following dynamics (e.g. forming of jams and stepping dynamics) which were not accessible in previous experiments.

In this paper, we focus on the stepping dynamics in the one-dimensional pedestrian flow experiment presented in [13]. A high precision motion capture system was crucial for obtaining data allowing for the analysis of steps. Indeed, we were able to extract steps from the radial oscillations of the trajectory, originating from the oscillating body movement during stepping. This enabled us to perform the first quantitative analysis of the stepping dynamics in large pedestrians groups We show that the size of steps is directly related to the space available in front of the pedestrian, and that the step frequency is far less sensitive to the local density. Furthermore, we examined the effect of the flow densities on the synchronization of steps among the consecutive pairs of pedestrians.

When pedestrians are separated by a small distance, they cannot walk freely. It is an open question how they adapt to large densities. Indeed, it is known that large density pedestrian flows give rise to increasing fluctuations in the individual motions, that can eventually lead to the so-called "crowd turbulence" [15] known to be responsible for crowd disasters. A careful analysis of the behavior of pedestrians at relatively high densities can give some information on the individual behaviors that could lead to such transitions.

When pedestrians are very close to each other, the distance between them can become of the same order as the longitudinal displacement due to steps. Besides, accelerations and decelerations occur on time scales similar to the stepping period. Thus, steps cannot be ignored when dealing with high density flows. In fact, it was observed in the videos of the experiments reported in [8] that at high densities, people were walking in lock-steps (which we term "synchronization") in order to optimize the use of the available space. Here we have studied systematically on a large number of data the importance of step synchronization. Moreover, at lower densities we also also found occurrence of the "antisynchronization" phenomenon, i.e. when the consecutive pair of pedestrians is synchronized in such a way that when one of them is stepping with the left leg, the other is stepping with the right leg, and vice versa. While synchronization occurs mostly at the highest densities, both synchronization and antisynchronization occur at lower densities. This natural tendency to synchronize could actually be exploited to improve the flow in a congested situation, for example using the effect of rhythm and music on the stepping behavior [12, 16].

First we shall summarize the experimental protocol (section II). In section III we present results showing that step length and duration obey simple laws. Section IV is devoted to the study of step synchronization.

II. THE EXPERIMENT

The aim of the experiment was to study the longitudinal interactions between pedestrians walking in line, without overpassing each other. Here we focus on the extraction of the stepping behavior in the various dynamic regimes (free flow, jammed, etc.). The average global density was varied from 0.31 to 1.86 ped/m by varying the number of participants involved, and the length (or the radius) of the circular trajectory. Between 8 and 28 participants have been walking on two circles of radii 2.4 and 4.1 meters. More experimental details are given in [13].

Each participant was equipped with 4 markers (one on the left shoulder, two on the right shoulder, and one on top of head). Motion was tracked by 12 infra-red cameras (VICON MX-40 motion capture system). The raw data are turned into 3D trajectories using the reconstruction software VICON IQ, with a frequency of 120 frames per second [17]. The main position for each participant is calculated as an average from the positions of the 4 markers. We take particular care on the number of markers that are visible at each time step for every participant, since some markers may be hidden by the walls or participants' bodies at certain time steps.

In our previous paper on these experimental data [13], we were interested in properties like fundamental diagrams and velocity-spatial headway relations, and therefore we used filtering in order to eliminate the oscillations of the trajectories due to steps. Here, however, we focus on these oscillations. Steps are best observed on the radial coordinate which we will use in order to extract information on their length, duration, and also to analyze the synchronization phenomena.

III. STEPPING LAWS

We shall explore in this section the characteristics of steps. In our data, steps are clearly visible on the radial coordinate of each participant as in Fig. 1. The oscillations around an average radius of a circle along which the pedestrian is walking are due to the lateral body movement at consecutive steps.

We define step duration as time Δt_s passed between two consecutive local extrema in the radial coordinate (consecutive local minimum and maximum of oscillations). The step length is then the distance that a participant has traveled along the circle during this time. It is defined as $l_s = \Delta\theta_s \left\langle R \right\rangle_s$, where $\Delta\theta_s$ is the angle covered during time Δt_s, and $\left\langle R \right\rangle_s$ the average of the radius of a circular trajectory along which pedestrian is walking during step duration Δt_s. We performed analysis on the set of all steps made by each participant during all of our

52 experiments, each of the duration of about 1 minute. On average, step duration is estimated to be of the order of 1 s, meaning each participant made approximately 60 steps during one experiment. We kept only data with at least 2 visible markers at each time frame between two steps (two local extrema), the rejection ratio being around 10%. In the following analysis, we will use the remaining data obtained both along the inner and outer circle.

In figure 2 we show the results for the dependence of the step length and duration on the instantaneous velocity (top) and density (bottom). The values of velocity and density were obtained as averages of the instantaneous velocity and density during the duration of a given step. For two successive pedestrians, instantaneous density for the pedestrian in the back is obtained as the inverse of the distance that is available in front of him, i.e. to his predecessor.

The most striking feature is that the step length is overall proportional to the velocity, up to velocities very close to zero (see Fig. 2 (a)). Of course, the step length decreases when the velocity decreases. Having a vanishing step length for vanishing velocities means that, within a jammed regime, when pedestrians are forced to come almost to a stop, they continue to oscillate from one leg to the other without moving forward. A linear fit for velocities between 0.2 to 1.1 m/s gives

$$l_s = 0.065m + 0.724v. \quad (1)$$

This differs from the expression given in [18] and used in [19] ($l_s = 0.235m + 0.302v$) for which a residual length of 0.235 m was found at vanishing velocity. However the details of the measurements were not given in these references.

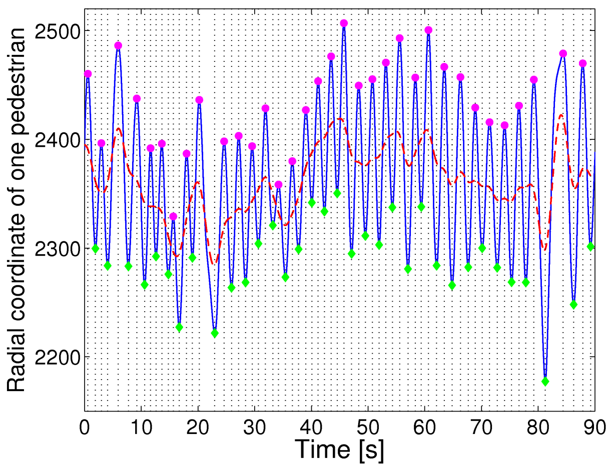

FIG. 1: The full (blue) line is the non-filtered radial coordinate of one participant (in mm) with clearly visible oscillations due to the consecutive steps. The dashed (red) line is the filtered radial coordinate. Thin vertical lines (..) mark the local extrema –local maxima (magenta circles) and minima (green diamonds)– which are used in our definitions of step characteristics.

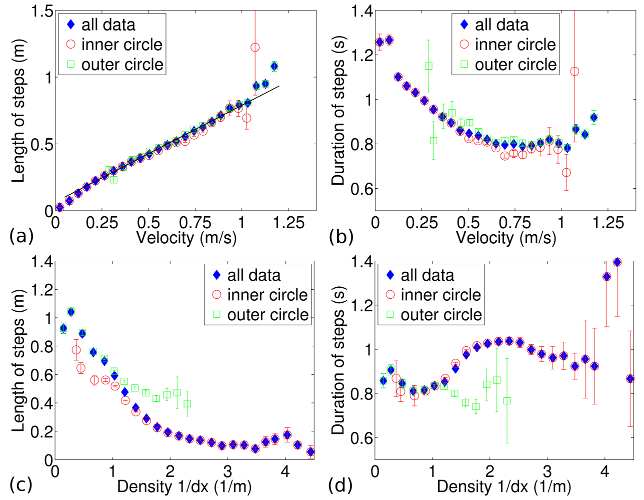

FIG. 2: Dependence of the step length and duration as a function of the instantaneous velocity (top) and density (bottom).

As the velocity of a pedestrian is mechanically produced by the steps, one assumes that the relation $v = l_s/\Delta t_s$ holds at least in a mean-field approximation. In a lowest order approximation, we have seen in figure 2 (a) that the velocity is proportional to the step length, and, as a consequence, the step duration should be constant - at least in a first approximation. This is indeed what we find in Fig. 2 (b) when the velocity is large enough (larger than 0.6 m/s): the step duration is then mostly constant and takes a value around 0.8 s. For velocities below 0.6 m/s, corrections to this lowest order approximation are seen as the step duration increases when the velocity of the participants decreases.

At first sight, our results could seem in contradiction with those obtained in the field of locomotion studies: Inman's law [20] states that the step length [21, 22] or step frequency [23] both vary as the square root of the velocity. This law has been indeed verified in locomotion experiments [23, 24]. Note that Inman's law requires to normalize data either by the hip joint height, or total height of the pedestrian, an information that is not available in our case, but that could be measured in future experiments. However, this renormalization cannot explain the difference with our result.

In fact, it must be noted that locomotion experiments are always performed with *isolated* pedestrians. The pedestrian makes a conscious decision to walk slowly, and knows that he will keep this slow pace for a while. In our experiment, participants walk slowly only because they are prevented to walk faster by other participants. Besides, they expect to be able to walk faster in a near future. As a consequence of these features, our pedestrians keep a constant pace (to enable a quick restart), and rather adopt small step lengths (to comply with steric constraints).

As a conclusion, we observe that in a constrained environment, pedestrians rather adapt their velocity through their step length rather than step frequency. At high densities, when there are stop-and-go waves, pedestrians tend to keep a rather high step frequency, whatever the surrounding density. This could be a sign that pedestrians do not like rapid modifications of their stepping pace. At low densities, the step frequency saturates towards a limit value. Again, this could indicate that increasing the step frequency beyond a certain value is not comfortable for pedestrians.

Finally, in Figs. 2 (a) and (b), we find that the data obtained both along the inner and outer circle fall on top of each other. This confirms that there is no influence of geometry on the stepping behavior.

The behavior of the step length as a function of density (see Fig. 2 (c)), is exactly of the same form as the fundamental diagram found in [13]. Indeed, this is a direct consequence of the above result that the step length is being proportional to velocity. As the average velocity converges towards a finite non zero value when the density becomes large, the average step length saturates around $0.1m$ at large densities. By contrast, the step duration changes much less within different walking regimes. The step duration varies only for around 20% as a function of density (see Fig. 2 (d)).

We had previously found in [13] that for density fluctuations far away from the average global density the velocity could be quite different from the average behavior. We recover here (Figs. 2 (c) and (d)) these atypical behaviors. Indeed, global densities in experiments performed on the outer (inner) circle are always below (above) 1 ped/m, and thus the tails ('inner and outer circle data') diverging from the average behavior ('all data') in Figs. 2 (c) and (d) correspond to large fluctuations. Still, for the average behavior, no discontinuity is seen when going from the inner to the outer circle data.

IV. STEP SYNCHRONIZATION

It was already noticed in [8] that at high densities, as pedestrians do not have much space to walk, they tend to synchronize their steps, so as to squeeze the front leg into the hole left by the front leg of the predecessor. The authors refer to this phenomenon as walking in lock-steps. In the following, we want to see whether this tendency is confirmed in our experiments, and to quantify this effect.

First we have to define synchronization. If all pedestrians were walking very regularly, so that their radial coordinate would be a perfect sinusoid with the same frequency for each pedestrian, it would be quite easy to measure the phase between two successive pedestrians, which indicates the amount of synchronization. However, in the experiments, and especially at high densities, the shape of the oscillations can be quite far from a sinusoid, and the oscillation frequencies can vary from one pedestrian to another.

Thus, our measurements included two stages that we will detail below. In a first stage, we have analyzed the data to select pairs of pedestrians for which the stepping frequencies were not too different. Then, for this subset, we have measured the phase shift between close minima (maxima) of the stepping cycles of the two successive pedestrians. More precisely, for each detected minimum (maximum) on the radial coordinate of the predecessor (occurring at time t_0) our analysis consists of the following steps:

- We measure the 'individual and local' frequency of the leader over 3 periods, defined from the two maxima (minima) just before and after the given minimum (maximum). Let T be the average period over these three cycles.

- We determine whether there is a minimum (maximum) in the radial coordinate of the follower within the time range $[t_0 - T/4, t_0 + 3T/4]$. This minimum occurs at time t_1.

- We evaluate the 'individual and local' frequency $1/T'$ for the follower, exactly as it was done for the leader.

- If T' and T differ less than 25%, we select the current steps of this pair of pedestrians. The rejection ratio due to too large difference in frequencies is up to 20%.

- Then we measure the phase
$$\phi = 2\pi(t_1 - t_0)/T. \tag{2}$$
In a similar way, we also measure the phase ψ separating a minimum (maximum) in the radial coordinate of the leader, from a maximum (minimum) in the radial coordinate of the follower. In this way, we expect to obtain more precise measurements for the antisynchronization phenomena.

- Finally, we measure the instantaneous density. It is defined as the inverse of the distance between the centers of mass of the two pedestrians. As this distance oscillates with the steps, we found more relevant to evaluate it on the filtered data. However, the results presented in Fig. 3 are similar when non-filtered data are used.

Figure 3 shows the histograms of ϕ obtained for various local density ranges. At large densities (beyond 1.25 ped/m), we observe a peak around phase $\phi = 0$, that clearly indicates existence of the synchronization phenomenon. This must correspond to the pedestrians walking in lock-step, as for these densities the steric constraints become important.

When the density is lower, the peak around zero is still observed, though it is weaker than for higher densities. Surprisingly, another peak appears around $\phi = \pi$. This second peak corresponds to antisynchronization, i.e. walking in phase with the opposite legs. When antisynchronization occurs at high densities, we could expect

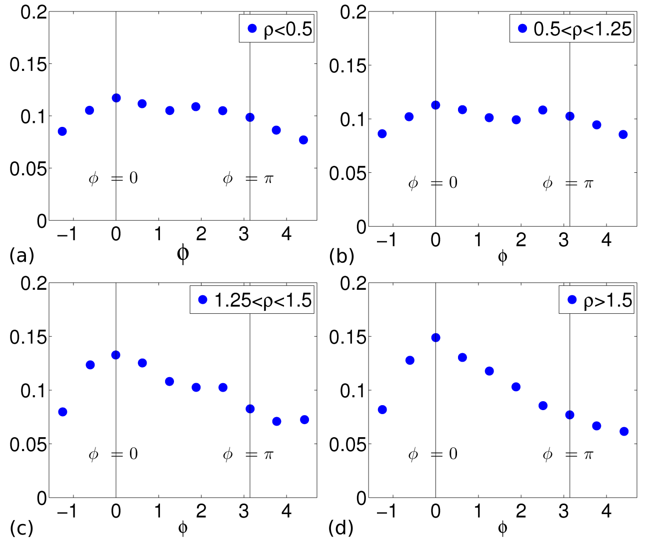

FIG. 3: Normalized distributions of ϕ, for various density ranges. Synchronization corresponds to $\phi = 0$.

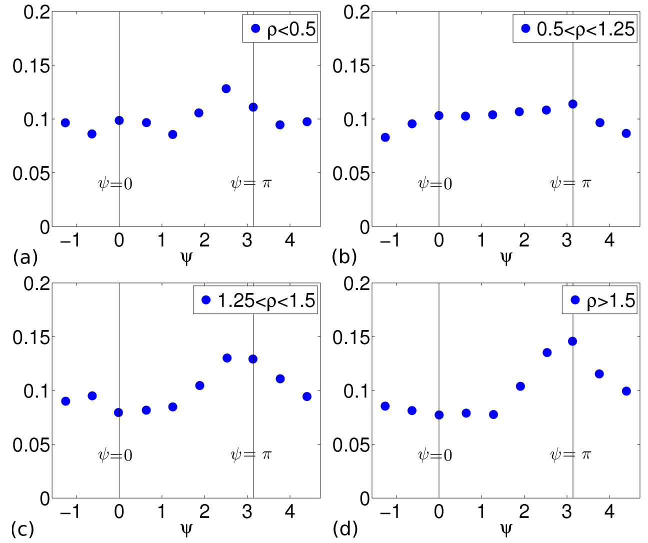

FIG. 4: Normalized distributions of ψ, for various density ranges. Antisynchronization corresponds to $\psi = 0$.

that pedestrians would be located at different distances from the wall, so that the left leg of one pedestrian is more or less aligned with the right leg of the other. However, we did not observe any visible effect of this type in the data. Besides, antisynchronization mostly disappears when the density is large, while it survives at densities as large as $\rho < 0.5$ ped/m, for which there are clearly no steric constraints between the pedestrians.

As the pedestrians may not have exactly the same frequency, in Fig. 4 we checked if the antisynchronization is also visible when we measure the phase shift ψ between the local extrema of the opposite kind (minimum and maximum) in the radial coordinate. This corresponds to the steps made by the left leg of one and the right leg of the other of the two consecutive pedestrians. In this case, antisynchronization should appear as a peak around zero – and there is indeed a second peak located around $\psi = 0$ for the weakest densities as seen in Fig. 4. On the other hand, synchronization can be seen again, but this time as a peak around $\psi = \pi$.

These results, showing both synchronization and antisynchronization at low enough densities where the pedestrians are not bound by the steric constraints, suggest that pedestrians are sensitive to the stepping oscillations that they perceive visually when watching their predecessor, and that they naturally synchronize. It is still an open question whether they are more sensitive to the oscillations of the height rather than of the radial coordinate.

V. DISCUSSION AND CONCLUSION

In this paper, we have presented new experimental results about steps characteristics of pedestrians following each other along a one-dimensional trajectory. We have obtained for a large range of velocities several simple laws for the steps length and duration, namely that the step length is proportional to the velocity, while the variations on the step duration are much weaker.

This result is in contrast with the hypothesis used in [25] that at high densities, when it is no longer possible to take normal steps, pedestrians would rather completely stop until they gain enough space to make a step. Our observation is that pedestrians continue to oscillate from one foot to the other even when they cannot move forward at all, and keep their stepping pace even at very high densities. In fact, they are not reluctant at performing steps with very small amplitude.

Our results also highlight that the stepping behaviors in a crowd can be quite different from those measured in locomotion experiments with isolated pedestrians [20], though for the same range of walking speeds.

This raises some questions that would be interesting to tackle in the future. Indeed, several effects can be responsible for the change of walking behavior in a crowded environment. Walking very close to other pedestrians induces physical constraints that obviously have to be taken into account in the stepping strategy. However, at high densities, another effect comes from the presence of stop-and-go waves: pedestrians may have different steps characteristics depending on whether they are accelerating, decelerating, or walking at constant pace. Even the anticipation that the pedestrian will have to accelerate in the near future could modify his behavior. It would be interesting to design new experiments to discriminate between these various effects, taking also into account the relative height of interacting pedestrians, and distinguishing the inter- and intra-pedestrian variations.

Another question that we have addressed in this paper is whether pedestrians walk in lock-steps at high densities [8]. Indeed, at high densities, beyond 1.25 ped/m, we have observed some synchronization between the step cycles of successive pedestrians. This can be easily explained by the strong steric constraints that occur at these densities. Surprisingly, synchronization is still observed –though less frequently– at much lower densities. It seems that even when pedestrians are more than 2 m apart, they still have some tendency to synchronize their rhythm, probably as a consequence of the visual stimulus given by the pedestrian ahead. Besides, in the absence of steric constraint, we observed that synchronization and antisynchronization are both observed at such low densities.

Interest in the synchronization phenomena also stems from the observations that music can induce particular stepping behaviors. In [16], experiments in which pedestrians were asked to synchronize their steps with the indicated rhythm were reported. It was shown that it was more efficient to indicate the rhythm with music than with a simple metronome [16]. In Ref. [12], it was found in an experiment that when pedestrians walking in line are asked to walk with a rhythm (given by a metronome) slower than the natural pace of pedestrians, the flow in the congested regime is improved, as a result of the synchronization of steps. It would be interesting to investigate this further, and in particular to determine the relation between the improvement of the flow and the ratio of the pedestrians ‘synchronized’ with the rhythm.

There would be a practical interest in knowing whether such synchronization would occur when the music is just used as a background, without any special assignment, and also what would be the consequence on the macroscopic characteristics of the flow. Further investigations are needed. It would be necessary to perform new experiments with tracking methods such as the one described in this paper, to measure in particular the amount of step synchronization between pairs of successive pedestrians. If it was shown that a musical background can improve the flow, this could be used in particular as a strategy to improve evacuation.

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