

A Criterion of Nondisappearance of Invariant Sets Satisfying Krasovsky Property under C^0 Perturbations of Right Part of the System.

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Abstract

In general this work consists of investigation of autonomous systems of differential equations having Krasovsky property described in [1]. Briefly speaking a system satisfies Krasovsky property in the neighborhood of a bounded invariant set if all the solutions of this system leave this neighborhood with increasing or decreasing of time. Also, as it is derived from [1], Krasovsky property is equivalent to existing of a continuous function (so called Lyapunov - Krasovsky function) in this neighborhood, which changes monotonously along the solutions of the system, and as a consequence this property is kept under small C^0 perturbations of right part of the system.

It was noted that for such a system there is a topological structure depending on behavior of the solutions in the neighborhood of invariant set only. So, first of all, it was an idea to define an index of the invariant set, something similar to rotation of vector field for example. Unfortunately it was not managed, i. e., it was not succeeded to define an index with “good” properties.

Finally, this investigation has been pointed to the development of a criterion of nondisappearance of invariant sets satisfying Krasovsky property under C^0 perturbations of the right part of the system. In other words, it could happen that the invariant set disappears (i. e. becomes empty) for the perturbed system. At the same time, the Krasovsky property is still held for small C^0 perturbations. The developed criterion is based on structure of Lyapunov - Krasovsky function and allows to determine, if such a disappearance could happen.

After the presentation of this master thesis some lecturers have noted that this work intersects in a certain way with so called “Index Conley Theory”. An introduction in this theory can be found in [2] (Conley itself called this index as “Morse index”) and [3]. The Conley Index is defined for an *isolated invariant set* of a *flow* and it is a topological space (or more precisely as an equivalence class of homotopic topological spaces).

The Conley Index Theory was studied. It was shown that Krasovsky property in the neighborhood of an invariant set of an autonomous system is necessary and sufficient for this invariant set to be isolated with respect to flow generated by system. Finally, the same criterion of nondisappearance has been derived by means of Conley Index Theory.

References

- [1] Reysin L. E. *Lyapunov functions and distinction problems*. (in Russian). Riga. Zinatie., 1986.
- [2] C. Conley. Isolated invariant sets and the Morse index. *CBMS Regional Conf*, 38, 1976.
- [3] Konstantin Mischaikow. Conley Index Theory. *Lectures Notes in Mathematics*, 1995.