RATE DISTRIBUTION BETWEEN MODEL AND SIGNAL

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ABSTRACT

Knowledge of a statistical model of the signal can be used to increase coding efficiency. A common approach is to use a fixed model structure with parameters that adapt to the signal. The model parameters and a signal representation that depends on the model are encoded. We show that, if the signal is divided into segments of a particular duration, and the model structure is fixed, then the optimal bit allocation for the model parameters does not vary with the overall rate. We discuss in detail the parameter rate for the autoregressive (AR) model. Our approach shows that the square error criterion in the signal domain is consistent with the commonly used root mean square log spectral error for the model parameters. Without using perceptual knowledge, we obtain a rate allocation for the model that is consistent with what is commonly used. This model rate is independent of overall coding rate. We provide experimental results for the application of the autoregressive model to speech that confirm the theory.

1. INTRODUCTION

It is common practice to use a model in the encoding of audio signals. The model provides a characterization of the statistical dependencies that exist between signal samples. Usage of the model allows more efficient encoding of the signal. In audio and speech coding, it is common to use adaptive models that describe the short-term statistics of the signal (statistics that are meaningful within signal segments of 5 to 30 ms). When a model is used, two sets of data must be transmitted: on the one hand the *model parameters* and on the other hand the *signal coefficients* that specify the signal given the model (we do not consider the case of backward adaptation in this paper).

Source coding is often formulated as a minimization of the bit rate required to transmit the signal at a given fidelity. If modeling is used, then it must be decided how to allocate the rate between model parameters and signal coefficients. The standard approach to rate allocation between model parameters and signal coefficients is based on experimental evidence. Optimization by experimentation is a laborious approach that is feasible only if it can be performed off-line. Thus, this approach to rate-allocation is natural only for coders that operate at a pre-determined rate.

Communication networks and applications of audio coding in general are becoming increasing heterogeneous. To facilitate usage in various environments, audio coders must be able to operate at a range of rates. This implies that off-line experiment-based optimization of the rate-distribution between model parameters and signal coefficients is not desirable. This motivates the work in this paper, which shows how a rate distribution between model parameters and signal coefficients can be derived theoretically based only on knowledge of certain signal properties. Our general approach towards rate distribution is based on that used in the context of the minimum description length (MDL) principle [1, 2]. The analytic relation for the bit rate allocation given in this paper provides a step towards source coding algorithms that can adapt in real-time to the allowed rate set by an external control mechanism.

We provide practical results for the rate distribution for the particular case of autoregressive (AR) modeling, also often referred to as linear-predictive modeling. AR modeling has long been used in speech coding and is becoming more common for audio coding, particularly in the context of a low delay constraint, e.g., [3]. Our results show that the rates commonly used for the AR model in speech coding, e.g., [4, 5], can be explained based only on coding efficiency and a squared error criterion operating directly on the speech. Importantly, this means that perceptual aspects play only a minor role in the bit allocation for the model.

To determine the rate allocation between model parameters and signal coefficients, we must define relations between rate and distortion for these variables. To this purpose we use a model of quantization that is accurate only in the asymptotic limit of high rate. Thus, our results are guaranteed only for high rates. However, experimental evidence indicates that the resulting principles hold over a wide range of rates, e.g., [6].

The remainder of this paper starts with a derivation of generic rate-allocation results in section 2. More detailed results are worked out for the AR case in section 3. To show that our results remain valid for practical rate allocations, we confirm the theory through experimental evidence in section 4.

2. RATE ALLOCATION

We consider a stochastic process (signal) X_i. To encode the signal we divide the signal into coding blocks of k samples. For each block, the k samples are encoded independently from the other blocks, using a signal model. Thus, we try to optimize the encoding of random vectors X^k using models that are specified by a set of random model parameters Θ.

We now compute the number of bits required to encode a particular data sequence x^k, using a model θ, when the coder operates at a mean distortion D. A particular data model θ corresponds to an assumed probability density of the data $p_{X^k|\Theta}(\cdot|\theta)$. We assume that the cells are identical in shape and write the relation between mean distortion and cell volume V as

$$D = CV^{\frac{2}{k}}, \tag{1}$$

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where C is the coefficient of quantization, a constant that depends only on the geometry of the quantization cell.

The data sequence (vector) x^k falls into a particular quantization cell, with index $i = i(x^k)$, $i \in \mathbb{N}$. The probability of this quantization index is, under the high-rate assumption,

$$p_I(i(x^k)) = V p_{X^k|\Theta}(x^k|\theta), \tag{2}$$

where, in general, $V = V(i)$ is a function of i. The source-coding theorem effectively states that the codeword length required for a particular index i is $-\log(p_I(i))$. (To facilitate notational brevity, we use nats as unit of codeword length.) If we constrain the average rate of the indices, $H(I) = -\sum_i p_I(i)\log(p_I(i))$, then we obtain a so-called *constrained-entropy* quantizer. In such quantizers, V does not depend on x^k under the high-rate assumption. Thus, the codeword length required to encode a particular x^k with the coder that operates at mean distortion D is

$$L_{X^k|\Theta}(x^k|\theta) = -\log\big(p_{X^k|\Theta}(x^k|\theta)\big(\frac{D}{C}\big)^{-\frac{2}{k}}\big). \tag{3}$$

The codeword length of (3) can be minimized by selecting the best model parameters for the particular sequence. This simply the parameter vector that maximizes the probability density $p_{X^k|\Theta}(x^k|\theta)$. Assuming a uniform prior distribution for Θ^k, the optimal parameter set has maximum likelihood parameter for the sequence x^k. We write the resulting maximum likelihood model as

$$\hat{\theta}(x^k) = \underset{\theta}{\operatorname{argmax}}\, p_{X^k|\Theta}(x^k|\theta). \tag{4}$$

While the maximum likelihood model minimizes the codeword length required for the signal vector x^k, the parameters $\hat{\theta}(x^k)$ of such a model can, in general, not be encoded at a finite rate.

To make the rate required for the parameters finite, we restrict the set of allowed parameter vectors to a countable set. The set $\{\bar{\theta}\}$ of admissable parameter vectors corresponds to the reconstruction vectors of a quantizer for the random parameter vector Θ. Let $p_{\bar{\Theta}}(\cdot)$ be the probability mass function for the countable set of allowed parameter vectors $\{\bar{\theta}\}$. Then the codeword length required to describe the model for $\bar{\theta}$ is

$$L_{\bar{\Theta}}(\bar{\theta}) = -\log(p_{\bar{\Theta}}(\bar{\theta})). \tag{5}$$

The rate required to encode x^k consists of the rate for the model and the rate for encoding the sequence x^k given the model:

$$\begin{aligned} L(x^k) =& L_{\bar{\Theta}}(\bar{\theta}(x^k)) + L_{X^k|\bar{\Theta}}(x^k|\bar{\theta}(x^k)) \\ =& -\log(p_{\bar{\Theta}}(\bar{\theta}(x^k))) - \log\big(p_{X^k|\bar{\Theta}}(x^k|\bar{\theta}(x^k))\big(\frac{D}{C}\big)^{-\frac{2}{k}}\big) \\ =& \psi(\bar{\theta}(x^k), \hat{\theta}(x^k), x^k) - \log\big(p_{X^k|\hat{\Theta}}(x^k|\hat{\theta}(x^k))\big(\frac{D}{C}\big)^{-\frac{2}{k}}\big) \end{aligned} \tag{6}$$

where

$$\psi(\bar{\theta}, \hat{\theta}, x^k) = -\log(p_{\bar{\Theta}}(\bar{\theta})) - \log\left(\frac{p_{X^k|\bar{\Theta}}(x^k|\bar{\theta})}{p_{X^k|\hat{\Theta}}(x^k|\hat{\theta})}\right) \tag{7}$$

is the *index of resolvability* [2]. The index of resolvability collects the terms of the overall rate that involve the quantized model parameters $\bar{\theta}$. The last term of (6) is the rate required to encode the signal vector x^k given the ideal (maximum-likelihood) model $\hat{\theta}$.

The index of resolvability (7) consists of two terms that have a clear interpretation. The first term represents the rate for the model parameters. The second term is the increase in the rate for the signal x^k resulting from using the non-optimal $\bar{\theta}$ instead of the optimal $\hat{\theta}$.

We are interested in the average performance for coding audio signal vectors X^k. Thus, we average (6) over the random vector X^k. Let $\mathrm{E}[\cdot]$ denote expectation over the ensemble of all audio signal vectors. The expected codeword length for X^k is then

$$\begin{aligned} \mathrm{E}[L(X^k)] =& -\mathrm{E}[\psi(\bar{\theta}(X^k), \hat{\theta}(X^k), X^k)] \\ & - \mathrm{E}[\log(p_{X^k|\hat{\Theta}}(X^k|\hat{\theta}(X^k)))] + \frac{2}{k}\log(\frac{D}{C}). \end{aligned} \tag{8}$$

The bit allocation for the model is determined by the mapping $\bar{\theta}(x^k)$. The optimal bit allocation for the model $\bar{\theta}$ is the result of a trade-off between the rate required for the model and the mean rate penalty resulting from using the quantized model if the same distortion must be attained. This trade-off involves only the mean index of resolvability (the first term of (8)). An important corrollary is that, under the assumptions of our derivation, *the optimal rate for the model parameters is unaffected by the mean signal distortion D*. The rate required for the model parameters depends on the structure of the model and the statistical properties of the data.

3. APPLICATION TO AUTOREGRESSIVE MODEL

Autoregressive (AR) models are commonly used in speech coding and are becoming more common for audio applications. This model class forms a natural first application for the theory. Our goal is to describe the index of resolvability for the constrained-entropy case in terms that are easily computed and interpreted. We assume that the random signal vector X^k has a Gaussian multivariate distribution

$$p_{X^k|\Theta}(x^k|\theta) = \frac{1}{\sqrt{(2\pi)^k \det(R_\theta)}} \exp\left(-\frac{1}{2} x^{kT} R_\theta^{-1} x^k\right), \tag{9}$$

where R_θ is the model covariance matrix for X^k corresponding to the AR model with parameters θ. To find the matrix R_θ, we model the random vector X^k as a segment of a stationary AR process. This implies that the matrix is Toeplitz, symmetric, and has as first column the autocovariance function of a signal generated with the AR model. The autocovariance function is the inverse discrete-time Fourier transform of the transfer function of AR filter, which means the first column of R_θ is

$$R_\theta(n, 0) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\sigma^2}{|A(\mathrm{e}^{j\omega})|^2} \mathrm{e}^{jn\omega} d\omega, \tag{10}$$

where σ^2 is the excitation signal variance (gain) and $A(z) = 1 + a_1 z^{-1} \cdots + a_m z^{-m}$ for an m'th order AR model. The model parameters are then $\theta = [\sigma^2, a_1, a_2, \cdots a_m]$.

As a first step towards obtaining the mean index of resolvability we determine an expression for $\log(p_{X^k|\Theta}(x^k|\theta))$. We note that the factor multiplying the exponential in (9) is, asymptotically with increasing k,

$$\begin{aligned} & -\frac{1}{2}\log((2\pi)^k \det(R_\theta)) \approx \\ & \quad -\frac{k}{2}\log(2\pi) - \frac{k}{2}\frac{1}{2\pi}\int_0^{2\pi} \log(R_\theta(\mathrm{e}^{j\omega}))d\omega = \\ & \quad -\frac{k}{2}\log(2\pi) - \frac{k}{2}\log(\sigma^2) \end{aligned} \tag{11}$$

Table 1: Bit rates of the AMR-WB coder [7].

Rate	6.6	8.85	12.65	14.25	15.85	18.25	19.85	23.05
AR model parameters	36	46	46	46	46	46	46	46
pitch-model parameter	23	26	30	30	30	30	30	30
excitation	48	80	144	176	208	256	288	352

where we used Szegö's theorem and that $R_\theta(z) = \sigma^2/|A(z)|^2$ and that since $A(z)$ is a monic minimum-phase polynomial $\int_0^{2\pi} \log(|A(e^{j\omega})|^2)d\omega = 0$. We then consider the argument of the exponential in (9). Let $R_{x^k}(e^{j\omega})$ be the Fourier transform of the auto-covariance function of the segment x^k. Then, again asymptotically with increasing k, we have

$$\frac{1}{2k} x^{kT} R_\theta^{-1} x^k \approx \frac{1}{4\pi}\int_0^{2\pi} \frac{R_{x^k}(e^{j\omega})}{R_\theta(e^{j\omega})} d\omega. \tag{12}$$

Thus, based on (11) and (12) we can approximate (9) as

$$\log(p_{X^k|\Theta}(x^k|\theta)) \approx -\frac{k}{2}\log(2\pi\sigma^2) - \frac{k}{4\pi}\int_0^{2\pi} \frac{R_{x^k}(e^{j\omega})}{R_\theta(e^{j\omega})} d\omega. \tag{13}$$

Ignoring scaling effects, the index of resolvability (7) is

$$\begin{aligned}\psi(\bar\theta, \hat\theta, x^k) \approx& -\log(p_{\bar\Theta}(\bar\theta)) + \\ & \frac{k}{4\pi}\int_0^{2\pi} \left(\frac{R_{x^k}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})} - \frac{R_{x^k}(e^{j\omega})}{R_{\hat\theta}(e^{j\omega})} \right) d\omega. \\ =& -\log(p_{\bar\Theta}(\bar\theta)) + \\ & \frac{k}{4\pi}\int_0^{2\pi} \frac{R_{x^k}(e^{j\omega})}{R_{\hat\theta}(e^{j\omega})} \left(\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})} - 1 \right) d\omega. \end{aligned} \tag{14}$$

Assuming that the effect of model quantization on the power spectrum is small, we use the expansion $u = 1 + \log(u) + \frac{1}{2}\log(u)^2 \cdots$:

$$\begin{aligned}\psi(\bar\theta, \hat\theta, x^k) \approx& -\log(p_{\bar\Theta}(\bar\theta)) + \frac{k}{4\pi}\int_0^{2\pi} \frac{R_{x^k}(e^{j\omega})}{R_{\hat\theta}(e^{j\omega})} \\ & \left(\log(\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})}) + \frac{1}{2}\log(\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})})^2 \right) d\omega. \end{aligned} \tag{15}$$

The ratios $\frac{R_{x^k}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})}$ and $\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})}$ represent the effect of modeling and the effect of quantization respectively. The modeling ratio averages to unity for maximum likelyhood gain. It is reasonable to assume that the effects of modeling and quantization are independent and we can approximate

$$\begin{aligned}\psi(\bar\theta, \hat\theta, x^k) \approx& -\log(p_{\bar\Theta}(\bar\theta)) + \\ \frac{k}{4\pi}\int_0^{2\pi} & \left(\log(\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})}) + \frac{1}{2}\log(\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})})^2 \right) d\omega. \end{aligned} \tag{16}$$

Neglecting the effect of gain quantization, (16) can be written as

$$\psi(\bar\theta, \hat\theta, x^k) \approx -\log(p_{\bar\Theta}(\bar\theta)) + \frac{k}{4\pi}\int_0^{2\pi} \frac{1}{2}\log(\frac{R_{\hat\theta}(e^{j\omega})}{R_{\bar\theta}(e^{j\omega})})^2 d\omega, \tag{17}$$

where the second term is the well-known *mean square log-spectral distortion* measure, which is commonly used to evaluate the performance of prediction coefficient quantizers, e.g., [8, 4].

By averaging the index of resolvability (17) over the ensemble of signal vectors, we obtain the equation that governs the rate allocation for the model parameters for the AR model

$$\mathrm{E}[\psi(\bar\theta, \hat\theta, x^k)] = R(\bar\Theta) + \frac{k}{4} D(\bar\Theta, \hat\Theta) \tag{18}$$

with

$$R(\bar\Theta) = -\mathrm{E}[\log(p_{\bar\Theta}(\bar\Theta))] \tag{19}$$

and

$$D(\bar\Theta, \hat\Theta) = \mathrm{E}\left[\frac{1}{2\pi}\int_0^{2\pi} \log(\frac{R_{\hat\Theta}(e^{j\omega})}{R_{\bar\Theta}(e^{j\omega})})^2 d\omega \right]. \tag{20}$$

We recognize in (18) a Lagrangian that minimizes the average rate for the parameter vector quantizer under a constraint on the mean log spectral squared error. The Lagrange multiplier is $\frac{k}{4}$. We can replace the mapping $x^k \to \bar\theta$ by the simpler two-stage mapping $x^k \to \hat\theta \to \bar\theta$.

The model parameters describe a manifold in the log power spectral domain with dimensionality $d \leq |\Theta|$. With increasing rate, the optimal constrained-entropy quantizer is asymptotically uniform on this manifold in the log-power-spectral domain. The performance of a high-rate constrained-entropy quantizer using the square error and lying on the manifold scales with rate as

$$D(\bar\Theta, \hat\Theta) = dC e^{-\frac{2}{d}(R(\bar\Theta) - h(\hat\Theta))}, \tag{21}$$

where C is the coefficient of quantization, $h(\hat\Theta)$ is the differential entropy of $\hat\Theta$ *as measured in the log power spectral domain*. The objective is to find the model rate $R(\bar\Theta)$ that minimizes

$$\mathrm{E}[\psi(\bar\theta, \hat\theta, x^k)] = R(\bar\Theta) + \frac{k}{4} dC e^{-\frac{2}{d}(R(\bar\Theta) - h(\hat\Theta))}, \tag{22}$$

which is solved by the optimal model rate allocation

$$R(\bar\Theta) = h(\hat\Theta) + \frac{d}{2}\log\left(\frac{k}{2}C\right). \tag{23}$$

4. RESULTS AND VERIFICATION

The principles introduced in this paper are of a general nature. We verify the principles with applications to the coding of speech. The motivation for the selection of the speech signal is that the AR model is commonly used in this context, which means reasonable model choices are well understood. More-over, existing results for standardized coders can provide a first indication of the principles derived here-in.

4.1. Corroborative Earlier Results

The present paper discusses the distribution of the bit rate for entropy-constrained coding, which is common in audio coding. The results are essentially identical for the case constrained-resolution coding. Practical results for constrained-resolution display the correct behavior. Table 1 shows bit allocations used in the adaptive-multirate wide-band (AMR-WB) speech coder [7]. It is seen that the bit allocation for the model parameters is independent of the rate of the codec, except at low rates. In contrast, the bit allocation for the excitation (the signal) increases rapidly with the overall rate. These bit allocations confirm our theoretical findings.

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4.2. Experimental Verification

The verification was performed for tenth-order AR modeling on 8 kHz sampled speech. We used the TIMIT database [9].

We first estimated the differential entropy and the manifold dimensionality of the random parameter vector θ, using the methods described in [10]. The dimensionality of the manifold was found as $d = 7.9$ and the differential entropy was 8.5 bits. From (23) it then follows that the optimal rate for the model is 19.0 bits for scalar quantization and 17.2 bits for vector quantization. The rates correspond to a root mean square log spectral distortion of 1.29 dB, close to the 1 dB commonly used on heuristic grounds [4].

The second step of our verification work is to confirm that the mean of the summation of model rate and signal rate is minimized using the measured rates for

$$\mathrm{E}[L(x^k)] = \mathrm{E}[L_{\bar{\Theta}}(\bar{\theta}(x^k))] + \mathrm{E}[L_{X^k|\bar{\Theta}}(x^k|\bar{\theta}(x^k))]. \qquad (24)$$

To confirm this, we measured the average rate that a coder operating on speech requires for the speech signal, for a given speech-signal distortion and with varying quantization accuracy for the model parameters. To this purpose, we extracted 10000 randomly located speech blocks of 160 samples (20 ms) from the 1680 utterance evaluation part of TIMIT. For each block we performed linear-predictive analysis (using a Hann window) to obtain a set of AR model parameters. To approximate optimal parameter quantization on the data manifold in the log spectral distortion domain, we converted the parameters to the line-spectral frequencies (LSFs) and computed the (diagonal) sensitivy matrix of the LSFs [11]. We then performed scalar quantization of the scaled LSFs. We used these parameters to estimate the bit allocation required for scalar quantization of the 40-dimensional speech vector x^k located in the center of the 160 sample block. The vector was first decorrelated using a model-based Karhunen-Loève transform, then scalar quantized. The rate was estimated using numerical integration of the probability density function over the cell. We multiplied by four to get the rate for a stationary block of 160 samples, approximating a common speech coder scenario.

In Fig. 1 we show the outcome of the experiments. It provides the overall rate as a function of the rate allocated to the model for a range of distortions for the signal. It is seen that at overall coding rates of about 2 to 5 bits per sample, the overall coding rate is minimized when the model rate is about 20 bits. It is seen that this rate is independent of the overall coding rate. As expected from heuristic reasoning, the actual optimal model rate decreases when the signal distortion is high and the overall rate falls below the range of rates where the theory is valid.

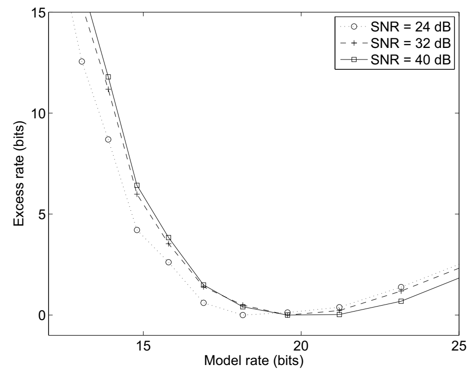

Figure 1: *The excess in overall rate (over its minimum value) for a block of 160 samples as a function of the model rate for different signal distortion levels. The data are for scalar quantization. The corresponding minimum rates are 340, 530, and 736 bits per block. The theory predicts a model rate of 19 bits.*

5. CONCLUSIONS

In this paper we considered the coding of signal segments with a model. We concluded that with increasing rate the rate allocation for the model becomes a constant and is independent of the overall rate allocation. An existing speech coding standard and our own experimental confirm the theoretical results. We conclude that our method can be used to predict the optimal model coding rate. For the AR model, our approach leads to the commonly used squared log spectral distortion measure for the prediction parameters. More-over, we can conclude that the required accuracy of the AR parameters is not a direct function of perceptual effects. Our results mean that audio (as well as other source) coders that adapt in real-time to changing network conditions can use a fixed quantizer for the signal model parameters.

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