# ENTROPY-CONSTRAINED HIGH-RESOLUTION LATTICE VECTOR QUANTIZATION USING A PERCEPTUALLY RELEVANT DISTORTION MEASURE

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## ABSTRACT

In this paper we study high-resolution entropy-constrained coding using multidimensional companding. To account for auditory perception, we introduce a perceptual relevant distortion measure. We will derive a multidimensional companding function which is asymptotically optimal in the sense that the rate loss introduced by the compander will vanish with increasing vector dimension. We compare the companding scheme to a scheme which is based on a perceptual weighting of the source, thereby transforming the perceptual distortion measure into a mean-squared error distortion measure. Experimental results show that even at low vector dimension, the rate loss introduced by the compander is low (less than 0.05 bit per dimension in case of two-dimensional vectors).

## 1. INTRODUCTION

The most commonly used fidelity criteria in source coding is the mean-squared error (MSE) fidelity criterion. The MSE criterion is mainly used because of its mathematical tractability. In application involving a human observer, however, it has been noted that distortion measures which include some aspects of human perception generally perform better than the MSE. For example, most state-of-the-art audio coding schemes exploit the phenomenon of spectro-temporal masking to discard perceptually irrelevant information.

To date, the rate-distortion function of stationary sources using the MSE criterion (and variations of it like the weighted squared-error criterion [1]) is known analytically only in situations involving Gaussian and some special non-Gaussian sources. Other examples for which the rate-distortion function can be computed include the locally quadratic distortion measures for fixed-rate vector quantizers under high-resolution assumptions [2], results which were extended to variable-rate (entropy-constrained) vector quantizers in [3].

In this work we use recent results on high-resolution source coding for non-difference distortion measures [4] in order to numerically evaluate the rate-distortion function of sources under a perceptually relevant distortion measure. We will investigate entropy-coded multidimensional companding vector quantization. Multidimensional companding is a type of structured vector quantization of low complexity, where a $k$-dimensional source vector $X$ is first "compressed" by an invertible mapping $F$ (the compressor function), next quantized by a lattice vector quantizer $Q_\Lambda$ and finally inversely mapped by $F^{-1}$ to obtain the reconstruction $Y$ of $X$. That is, the companding scheme is

$$X \to F(\cdot) \to Q_\Lambda(\cdot) \to F^{-1}(\cdot) \to Y.$$

It has been shown that for high dimensions and low distortions, an entropy-constrained companding scheme with an optimal compressor (if exists) can approach the rate-distortion limit arbitrarily close [4].

This paper is organized as follows. In Section 2 we introduce the perceptual distortion measure we will consider here. Next, in Section 3, we discuss multidimensional companding and introduce a compander function which becomes optimal for large vector dimensions. In Section 4 we consider the rate-distortion performance of the proposed companding scheme and compare the results to schemes based on perceptual weighting. Finally, in Section 5, we draw some conclusions.

## 2. PERCEPTUAL DISTORTION MEASURE

Let $x_i = (x_{i1}, \ldots, x_{ik})^t \in \mathbb{R}^k, y_i = (y_{i1}, \ldots, y_{ik})^t \in \mathbb{R}^k$, where the superscript $^t$ denotes matrix transposition, and let $x = (x_1, \ldots, x_n)^t \in \mathbb{R}^{nk}, y = (y_1, \ldots, y_n)^t \in \mathbb{R}^{nk}$. In this work we will consider the distortion measure given by

$$d(x, y) = \sum_{i=1}^{n} \frac{\|x_i - y_i\|^2}{\|x_i\|^2}. \tag{1}$$

Each term $\|x_i - y_i\|^2$ in the summation is normalized by $\|x_i\|^2$, reflecting Weber's law which states that the just noticeable difference in signal level is a constant percentage of the level (0.5-1 dB for pure tones) [5, 6]. The index $i$ could, for example, refer to a particular time frame consisting of $k$ samples. In case $k = 1$, (1) reduces to the well-known single-letter magnitude-normalized distortion measure. Alternatively, in case $x$ and $y$ are frequency representations, the index $i$ could refer to a particular frequency band with $k$ the number of frequency bins within the band[1]. An example is the spectral distortion measure presented in [7], which was derived from the monaural masking model introduced in [8]. The model computes the detectability of distortions by combining the information at the output of the auditory filters, which is in

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[1]In this paper we will assume that $k$ does not depend on $i$. However, it is straightforward to extend the results to variable $k$ values.

line with recent findings within the field of psychoacoustics which show that the human auditory system is able to integrate distortions that are present within a range of auditory filters [9]. Thus, if we set $x_i$ and $y_i$ to be the (band-pass filtered) input signal and its reconstruction, respectively, present at the output of the $i$th auditory filter and ignore the detectability of signals near the absolute hearing threshold (threshold in quiet), the distortion measure presented in [7] reduces to (1) (up to a proper scaling).

## 3. MULTIDIMENSIONAL COMPANDING

The classical approach for coding a source under the distortion measure (1) is to normalize the input, thereby transforming it into a MSE distortion measure,

$$d(x,y) = \sum_i \frac{\|x_i - y_i\|^2}{\|x_i\|^2} = \sum_i \|x_i' - y_i'\|^2, \qquad (2)$$

where $x_i' = x_i/\|x_i\|$ and $y_i' = y_i/\|x_i\|$. Similar normalizations are found in sinusoidal coding applications [10, 11] and MDCT audio coding [12, 13], where the data is normalized by weights which are related to the masking threshold. The advantage of working with the MSE distortion measure is that, under high resolution, lattice vector quantization is optimal, independent of the source distribution [14]. There are, however, two problems with the normalization approach. First of all, in order to reconstruct the source, the weights have to be known at the decoder. As a consequence, these weights have to be transmitted as side information to the decoder, in addition to the source data itself. Secondly, the process of normalization and separately transmitting the weights introduces a rate loss which we will quantify in Section 4.

It can be shown that the distortion (1) is a locally quadratic distortion measure, so that, under high-resolution assumptions, it can be approximated by [2]

$$\begin{aligned} d(x,y) &\approx (x-y)^t M(x)(x-y) \\ &= \sum_{i=1}^{n} (x_i - y_i)^t M_i(x)(x_i - y_i) \qquad (3) \end{aligned}$$

where $M_i(x)$ is the $k \times k$ matrix whose elements are given by

$$(M_i)_{lm}(x) = \frac{1}{2} \left. \frac{\partial^2 d(x,y)}{\partial y_{il} \partial y_{im}} \right|_{y=x}.$$

The matrix $M(x)$ was named the ***sensitivity matrix*** in [2], where it was first pointed out that certain useful distortion measures can be represented by (3).

Let the expected distortion in quantizing the $nk$-dimensional random vector $X$ denoted by

$$D = Ed(X,Y),$$

and assume the expectation is finite. Let $F(x)$ denote the compressor function and let $Q_{D,F}$ denote the compander vector quantizer using $F$ resulting in an expected distortion $D$. The rate of encoding the source will be measured by the Shannon entropy in bit per dimension, and denoted by $H(Q_{D,F})$. We have the following theorem [4]:

**Theorem 3.1.** Suppose $d(x,y)$ and $X$ satisfy some technical conditions, *cf. [4]*. Then

$$\lim_{D\to 0} \left( H(Q_{D,F}) + \frac{1}{2}\log_2(D) \right) = \bar{h}(X) + \frac{1}{nk} E \log_2 |\det F'(X)| + \frac{1}{2}\log_2 \left( G(\Lambda) E \left( \operatorname{tr}(\Gamma(X)) \right) \right),$$

where $G(\Lambda)$ is the normalized second moment of inertia [15] of the lattice quantizer, $\bar{h}(X)$ the dimension normalized differential entropy, $\operatorname{tr}(\Gamma(x))$ denotes the trace of

$$\Gamma(x) = F'(x)^{-t} M(x) F'(x)^{-1},$$

and $F'(x)$ is the derivative of $F(x)$.

The optimal compressor then satisfies [4]

$$F'(x)^t F'(x) = cM(x) \quad \text{a.e.}, \qquad (4)$$

where $c > 0$ is a scalar constant. Note that the optimal compander does not depend on the source or the quantizer used. If (4) has been satisfied and we choose $c = 1$, the result of Theorem 3.1 reduces to

$$\begin{aligned} &\lim_{D\to 0} \left( H(Q_{D,F}) + \frac{1}{2}\log_2(D) \right) \\ &\quad = \bar{h}(X) + \frac{1}{2}\log_2(nkG(\Lambda)) + \frac{1}{2nk} E \log_2 (\det M(X)). \end{aligned}$$

Clearly, since (4) is expressed in terms of partial derivatives, the optimal compander does in general not exist, an observation made earlier in [14]. An exception forms the case $k = 1$ for which any nonuniform (scalar) quantizer can be implemented using a compressor function and uniform quantizer, a result first proved in [16]. The asymptotic rate redundancy (or rate loss) introduced by a suboptimal compander, say $\tilde{F}$, then follows from Theorem 3.1 and (4) (see also [4])

$$\begin{aligned} &\lim_{D\to 0} \left( H(Q_{D,\tilde{F}}) - H(Q_{D,F}) \right) \\ &\quad = \frac{1}{2nk} E \log_2 \left( \frac{\det(\tilde{M}(X))}{\det(M(X))} \right) \\ &\qquad + \frac{1}{2}\log_2 \left( E \operatorname{tr} \left( \frac{\tilde{M}(X)^{-1} M(X)}{nk} \right) \right), \qquad (5) \end{aligned}$$

where $\tilde{M}(x) = \tilde{F}'(x)^t \tilde{F}'(x)$.

Let us consider the distortion measure (1). In this case the sensitivity matrix is given by

$$M(x) = \operatorname{diag}\left( \|x_1\|^{-2} I_k, \ldots, \|x_n\|^{-2} I_k \right), \qquad (6)$$

where $I_k$ is the identity operator in $\mathbb{R}^k$. Clearly, since $M(x)$ is diagonal with diagonal elements depending on all entries of the vectors $x_i$, the optimal compander does not exist. An exception forms the case $k = 1$ where the optimal compander satisfies

$$F'(x) = \operatorname{diag}\left( |x_1|^{-1}, \ldots, |x_n|^{-1} \right),$$

and thus $F(x) = (F_1(x_1), \ldots, F_n(x_n))^t$, where, for $x_i > 0$,

$$F_i(x) = \int_1^{x_i} |t|^{-1} dt = \ln(x_i),$$

the optimal scalar compander function for the magnitude-normalized distortion measure.

In order to compute a (suboptimal) companding function for the distortion measure (1) in case $k > 1$, we note that

$$\|x_i\| = (x_{i1}^2 + \cdots + x_{ik}^2)^{\frac{1}{2}} = (x_{ij}^2 + z_{ij}^2)^{\frac{1}{2}},$$

where $z_{ij}^2 = \sum_{l,l\neq j} x_{il}^2$. We consider the compander function given by

$$\begin{aligned}\tilde{F}_{ij}(x) &= \int_{c_{ij}}^{x_{ij}} (t^2 + z_{ij}^2)^{-\frac{1}{2}}\, dt \\ &= \ln\left(x_{ij} + \|x_i\|\right) - \ln\left(c_{ij} + (c_{ij}^2 + z_{ij}^2)^{\frac{1}{2}}\right), \qquad (7)\end{aligned}$$

where $x_{ij}, c_{ij} > 0$ and the constant $c_{ij} = EX_{ij}$ is chosen such that the rate loss introduced by the suboptimal compander is minimized. Using some elementary calculus, it can be shown that

$$\frac{\partial \tilde{F}_{ij}(x)}{\partial x_{ij}} = \|x_i\|^{-1}, \qquad (8)$$

and, for $l \neq j$,

$$\begin{aligned}\frac{\partial \tilde{F}_{ij}(x)}{\partial x_{il}} &= \frac{x_{il}}{\|x_i\|(x_{ij} + \|x_i\|)} \\ &\quad - \frac{x_{il}}{(c_{ij}^2 + z_{ij}^2)^{\frac{1}{2}}(c_{ij} + (c_{ij}^2 + z_{ij}^2)^{\frac{1}{2}})}, \qquad (9)\end{aligned}$$

which is nonzero, from which we conclude that the compander is suboptimal. However, by inspection of (8) and (9), we have that

$$\lim_{k\to\infty} \|x_i\| \frac{\partial \tilde{F}_{ij}(x)}{\partial x_{il}} = \delta_{lj},$$

where $\delta_{lj}$ is the Kronecker delta which is 1 if $l = j$ and 0 otherwise. Hence, we conclude that the matrix $\tilde{F}'(x)$ becomes diagonal for large vector dimensions $k$ and as a consequence

$$\tilde{F}'(x)^t \tilde{F}'(x) \to \operatorname{diag}\left(\|x_1\|^{-2} I_k, \ldots, \|x_n\|^{-2} I_k\right) = M(x),$$

which means that the compander becomes optimal.

At the decoder, the quantized data has to be inversely mapped by the expander function $\tilde{F}(x)^{-1}$. Since $\det(\tilde{F}'(x)) \neq 0$ ($\tilde{M}(x)$ is positive-definite), $\tilde{F}(x)$ is invertible by the inverse function theorem [17]. Hence, the problem of finding the appropriate quantization levels amounts of finding a fast numerical algorithm for solving $x_{ij}$ in (7). Since both $\tilde{F}(x)$ and $\tilde{F}'(x)$ are known, this can be done efficiently using Newton's method [18]. That is, letting $\tilde{F}_{ij}(x) = \zeta$, the reconstruction points $x_{ij}$ can be approximated iteratively by

$$x_{ij}^{(m+1)} = x_{ij}^{(m)} - \tilde{F}'(x_{ij}^{(m)}) \left(\tilde{F}(x_{ij}^{(m)}) - \zeta\right),$$

where the superscript $^{(m)}$ indicates the iteration number. Note that no side information is needed to reconstruct the data.

## 4. RATE-DISTORTION EVALUATION

In this section we discuss results obtained by computer simulation where we will numerically evaluate the rate-distortion function. Recall that $D = Ed(X, Y)$ and that Shannon's rate-distortion function is given by [19]

$$R(D) = \inf\{I(X;Y) : Ed(X,Y) \leq D\},$$

where $I(X;Y)$ denotes the mutual information per dimension between $X$ and $Y$. As mentioned before, this function is only known is some special cases. However, for locally quadratic distortion measures we have the following result [3]:

**Theorem 4.1.** Suppose $d(x, y)$ and $X$ satisfy some technical conditions, *cf. [3]*. Then

$$\begin{aligned}&\lim_{D\to 0}\left(R(D) + \frac{1}{2}\log_2(2\pi e D)\right) \\ &\qquad = \bar{h}(X) + \frac{1}{2nk} E \log_2\left(\det M(X)\right).\end{aligned}$$

In order to evaluate the rate-distortion performance of the multidimensional companding scheme, we generated $10^6$ i.i.d. Rayleigh distributed random variables with parameter $\sigma = 5$. Hence, the expected value of the source is $\sigma\sqrt{\frac{\pi}{2}}$ and the differential entropy is given by

$$h(X) = \frac{1}{2}\log_2\left(\frac{1}{2}\sigma^2 e^{(2+\gamma)}\right),$$

where $\gamma$ is Euler's constant. The Rayleigh distribution reflects, for example, the distribution of amplitudes in sinusoidal coding applications [10, 11]. For the experiments presented here, we used $k = 2$ (and thus $n = 0.5 \cdot 10^6$) since this is the smallest vector dimension for which the optimal compander does not exist and, according to the discussion above, would yield the largest rate loss (the rate loss decreases as $k$ increases). The rate loss introduced by this suboptimal compander can be computed using (5). By inspection of (8) and (9), we conclude that $\tilde{F}'(x)$ is block-diagonal with $k \times k$ block diagonal elements, and so is $\tilde{M}(x)$. As a consequence, we have that

$$\begin{aligned}\frac{1}{n} E \log_2\left(\det \tilde{M}(X)\right) &= \frac{1}{n} E \log_2\left(\prod_{i=1}^{n} \det \tilde{M}_i(X)\right) \\ &= E\left(\frac{1}{n}\sum_{i=1}^{n} \log_2(\det \tilde{M}_i(X))\right) \\ &\approx \frac{1}{n}\sum_{i=1}^{n} \log_2(\det \tilde{M}_i(x)),\end{aligned}$$

for sufficiently large $n$ by the central limit theorem, assuming that the random variables $\log_2(\det \tilde{M}_i(X))$ are statistically independent and that the Lindeberg conditions on the individual variances are satisfied [20]. Similar results hold for $E \log_2(\det M(X))$, where $\det(M_i(X)) = \|X_i\|^{-k}$. In addition, we have

$$\begin{aligned}E \operatorname{tr}\left(\frac{\tilde{M}(X)^{-1} M(X)}{nk}\right) &= E\left(\sum_{i=1}^{n} \operatorname{tr}\left(\frac{\tilde{M}_i(X)^{-1} M_i(X)}{nk}\right)\right) \\ &\approx \sum_{i=1}^{n} \operatorname{tr}\left(\frac{\tilde{M}_i(x)^{-1} M_i(x)}{nk}\right).\end{aligned}$$

Hence we conclude that the rate loss given by (5) can be accurately approximated using realizations of the process; the source distributions do not have to be known.

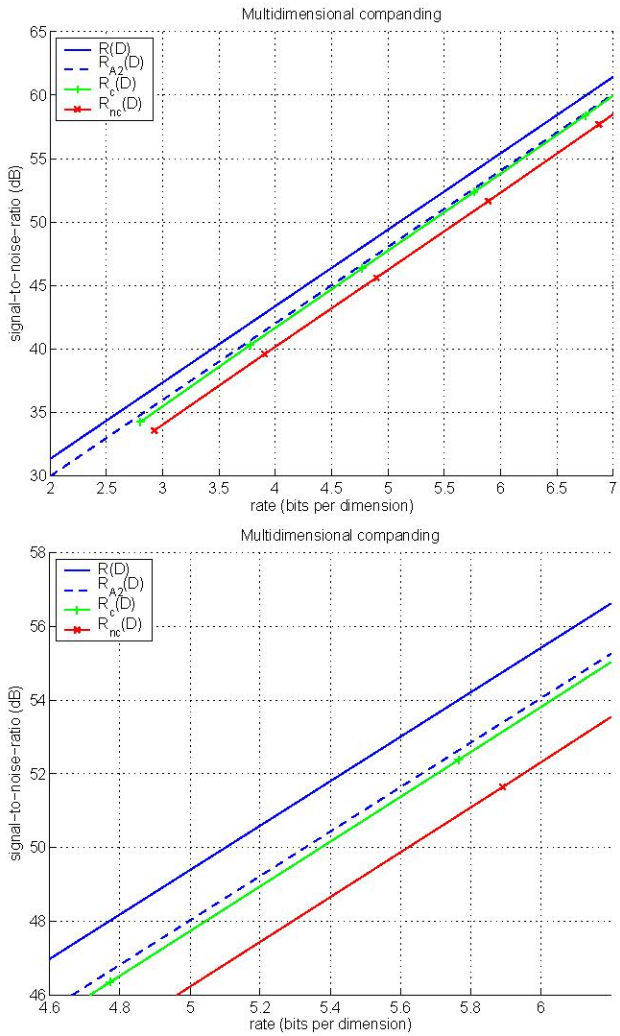


**Fig. 1**. Rate-distortion evaluation of the perceptual distortion measure (1) for a Rayleigh distributed source using multidimensional companding. The bottom plot is an enlarged version of the top plot.

Figure 1 shows the results for the multidimensional compander given by (7). The solid line denotes the rate-distortion function of the source. The dashed line is the optimal performance when using a two-dimensional vector quantizer, which introduces a rate loss of $\frac{1}{2}\log_2(G(A_2)) - \frac{1}{2}\log_2(G(\Lambda_\infty)) = 0.23$ bit per dimension due to the sphere-packing loss of the two-dimensional lattice. Here, $G(A_2) = \frac{5\sqrt{3}}{108}$ denotes the normalized second moment of inertia of the $A_2$-lattice [14] and $G(\Lambda_\infty) = 1/2\pi e$ is the normalized second moment of an infinite-dimensional sphere. The solid line $R_{\mathrm{c}}(D)$ denotes the rate-distortion performance of the proposed multidimensional companding scheme using the $A_2$-lattice. The rate loss, as computed by (5), is about 0.05 bit per dimension which is in line with the experimental results (see bottom plot for an enlarged version). The line $R_{\mathrm{nc}}(D)$ shows the rate-distortion performance without any companding; the source data is quantized directly using the $A_2$-lattice. In that case we have $\tilde{M}_i(x) = I_k$ and the corresponding rate loss becomes, again using

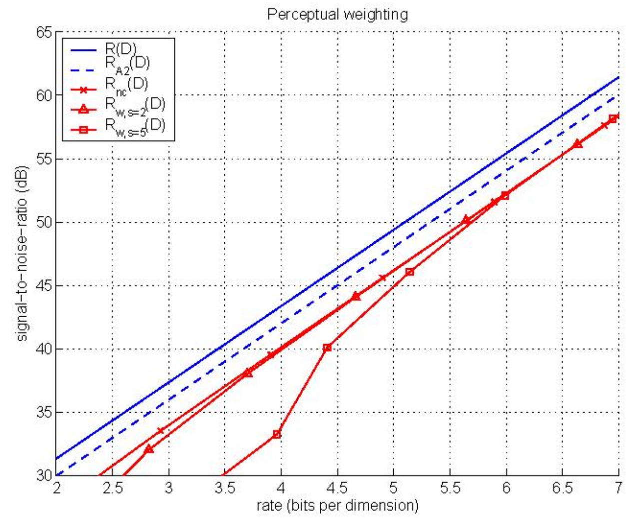


**Fig. 2**. Rate-distortion evaluation of the perceptual distortion measure (1) for a Rayleigh distributed source using perceptual weighting.

(5), 0.30 bit per dimension. Note that the rate loss is independent of the lattice vector quantizer used in the companding scheme and only depends on the companding function.

From Figure 1, we can conclude that the rate loss introduced by direct quantization of the source data is relatively small, namely 0.30 bit per dimension. For $k = 3$, this rate loss reduces to 0.17 bit per dimension. This indicates that there is not much to gain by transforming the distortion measure (1) into a MSE measure as (2) and transmitting the weights separately to the decoder. Figure 2 shows the rate-distortion performance of the perceptual weighting scheme. Again, the two top lines correspond to the rate-distortion function and the optimal performance using the $A_2$-lattice, respectively. The other curves show the results for encoding the weights (to be transmitted as side information) using 0, 2, and 5 bits. The quantizers for quantizing the weights are uniform and centered around the expected value of the weights and the quantizer output is entropy encoded. Note that the zero-bit situation corresponds to the case where we don't use a compander at all (line labeled $R_{\mathrm{nc}}(D)$ in Figures 1 and 2). From Figure 2 we conclude that transmitting the weights to the decoder does not improve the performance of the scheme. In fact, after spending about 2-3 bits for the weights, the additional rate for transmitting the side information will decrease the performance of the scheme at practical bit rates. The only way to reduce the rate loss introduced by the perceptual weighting is to jointly encode the source data and the perceptual weights, thereby increasing the encoder complexity, something which is not necessary in the multidimensional companding scheme.

## 5. CONCLUSIONS

In this paper we studied high-resolution entropy-constrained coding using multidimensional companding. We introduced a perceptual relevant distortion measure to account for auditory perception and introduced a compander function that becomes optimal for large vector dimensions. Using two-dimensional vector quantization, we showed that the rate loss introduced by the proposed compander is less than 0.05 bit per dimension. We compared the companding scheme to a scheme based on perceptual weighting and showed that, for the case $k = 2$, transmitting the weights separately to the decoder does not improve the performance of the scheme resulting in a rate loss of 0.30 bit per dimension.

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