# M2RI, Parcours Images et Données

## Module Acquisition et Représentation des Données

## Traitements et Transformations

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Première partie

# PDF of the slides

- http://people.irisa.fr/Remi.Gribonval/talks-and-tutorials/

  > COURSES AT THE MASTER LEVEL > M2RI

# References

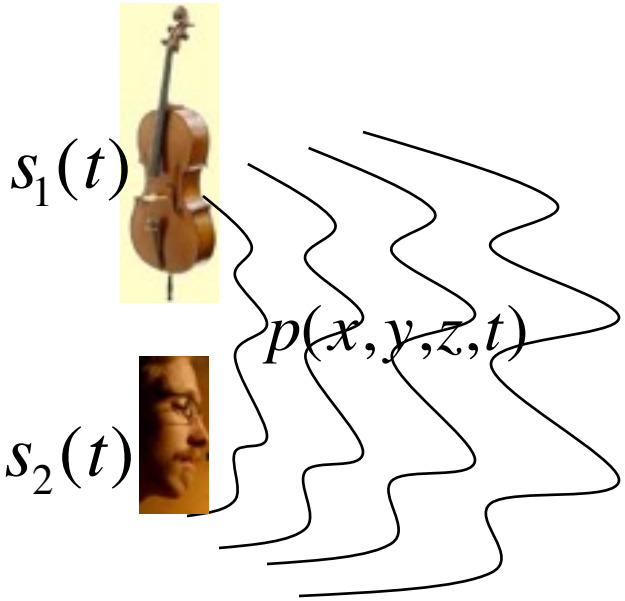
- Oppenheim & Shafer, Discrete-Time Signal Processing, Prentice-Hall
- Rabiner & Shafer, Digital Processing of Speech Signals, Prentice-Hall
- Mallat, a Wavelet Tour of Signal Processing, Academic Press
- Kunt, Traitement numérique des signaux, Dunod

# ARD module: evaluation

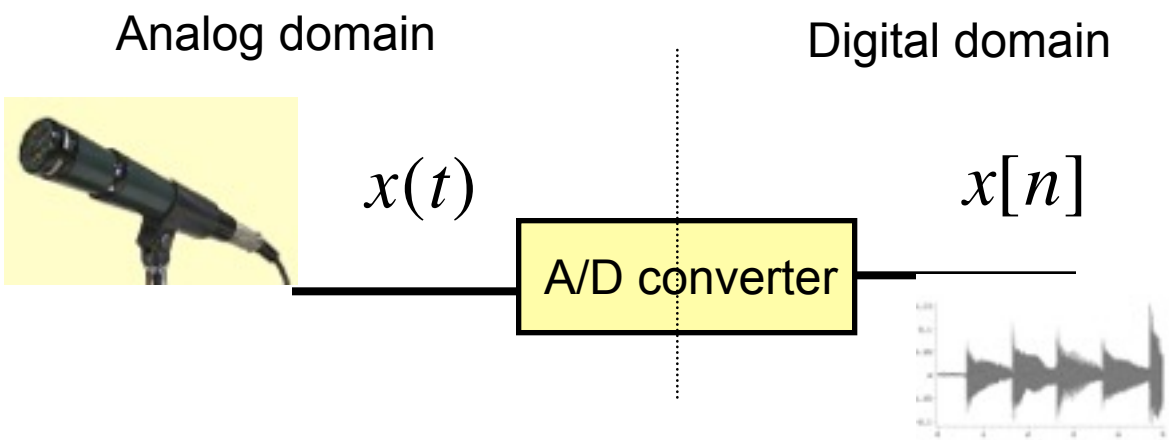
- **Oral exam on the course content**
  - ✓ 2 questions -*November 2, 2015*
  - ✓ 10 minutes preparation + 10 minutes exam

- **Presentation of a research paper**
  - ✓ written report (~8 pages)- *November 4*
  - ✓ oral presentation (10 minutes + 5 minutes of questions) - *November 9*

# Signal acquisition




Reverberation
Background noise
Mixture of sources




Storage

Transmission

Recognition

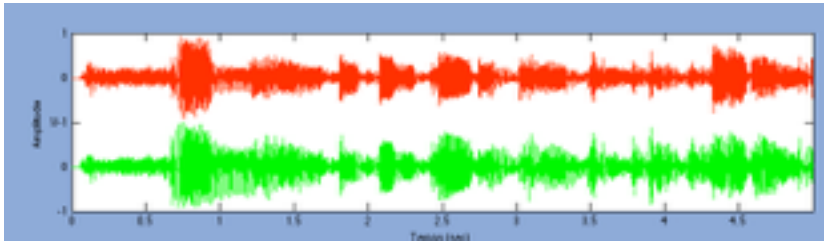
# Goals of signal processing

- **Understand / model** signal acquisition
  - Understand / simulate reverberation (1D)
  - Understand / simulate blur (2D)
  - Understand / design analog-digital conversion
- **Enhance** signal quality
  - De-reverberation / de-blurring
  - Denoising
  - Source separation, inpainting
- **Manipulate**: store, transmit, extract info from digital data
  - Compact / meaningful signal representation

# Example : Audio Source Separation

- « Softly as in a morning sunrise »

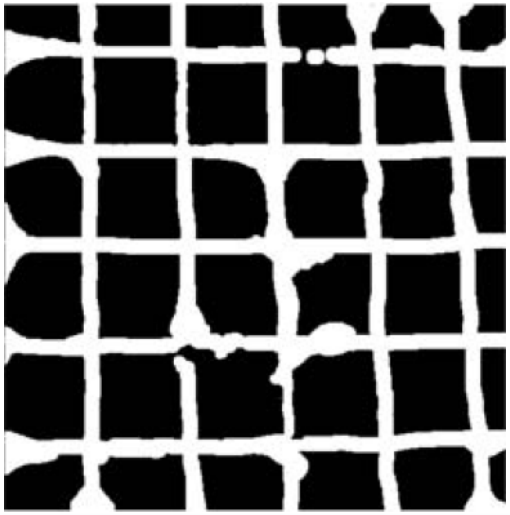
CD



Stereophonic sound = 2 channels

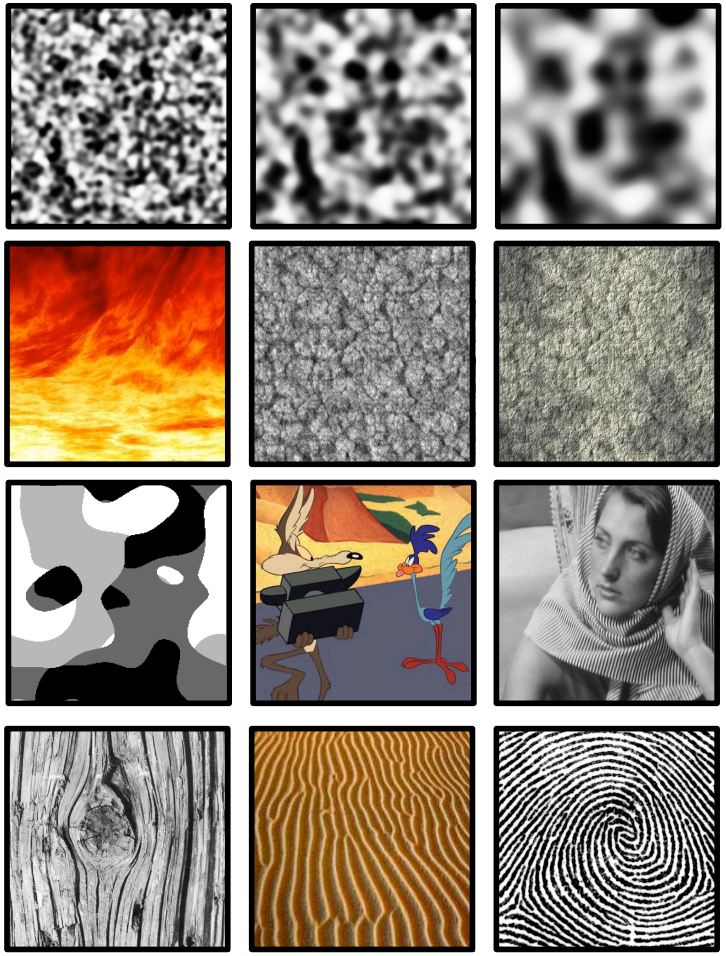
# Image inpainting

Courtesy: G. Peyré, Ceremade, Université Paris 9 Dauphine



Inpainting →

Courtesy : G. Peyré, Ceremade, University Paris 9 Dauphine



# Texture modeling

# Overall program

- Convolution, Filtering
- Fourier transform and (fast)convolution
- Analog - digital conversion
- Time-frequency / time-scale representations
- Sparse representations

# Today's program

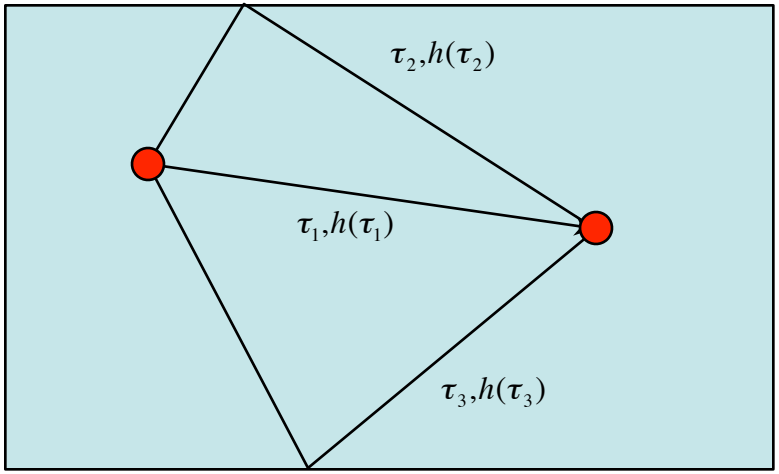
- **Model**: 1D and 2D convolution (reverberation, blur)
- **Problem**: deconvolution for signal / image restoration
- **Tools**: 1D Fourier transforms

# **Model**: convolution process (reverberation / blur)

1D and 2D

# 1D Convolution

- Ideal analog signal measure $x(t) = h.s(t)$
- With propagation delay $x(t) = h.s(t-\tau)$
- Reverberation on room walls

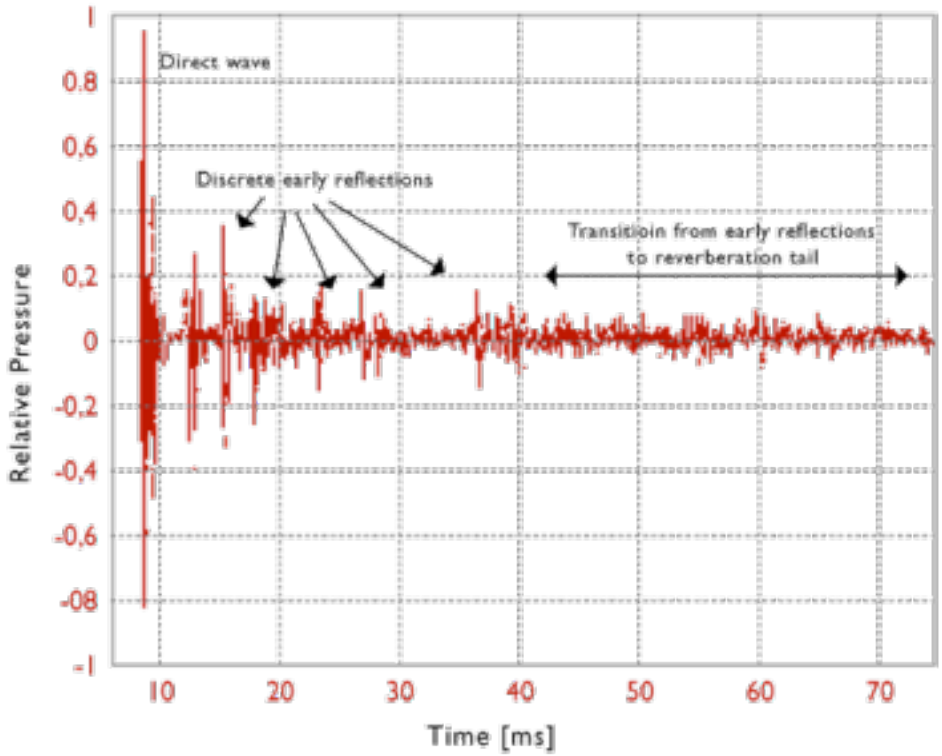



$$x(t) = \int h(\tau).s(t-\tau)d\tau$$

$$x[n] = \sum_k h[k].s[n-k]$$

*h =filter = (room) impulse response*

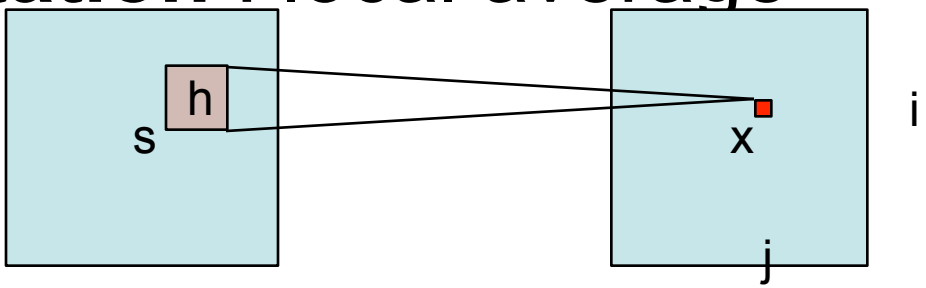
# Example of room impulse response




Reproduced from http://www.am3d.com/technology/acoustical

# 2D Convolution

- **Definition** $$x[i,j] = \sum_{k,l} h[k,l].s[i-k,j-l]$$
- **Interpretation** : local average

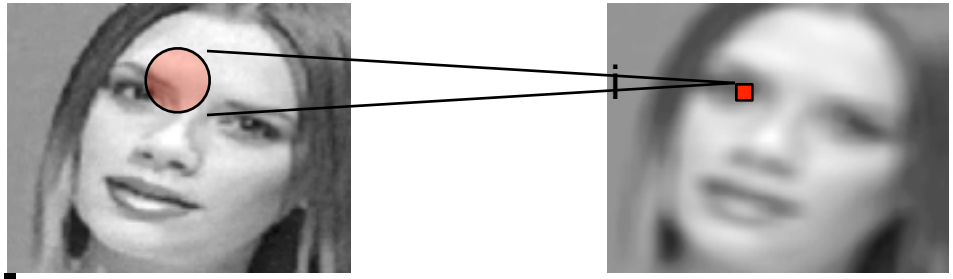



- **Examples** : optical / motion blur

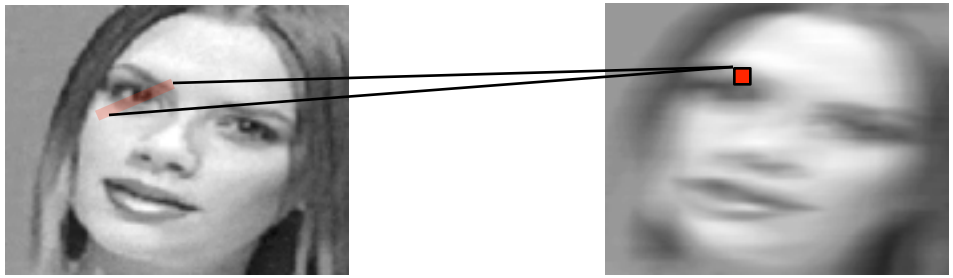
Reproduced from http://www.robots.ox.ac.uk/~improofs/super-resolution/super-res1.html

# 2D convolution

- **Optical blur** *h =point spread function (PSF)*

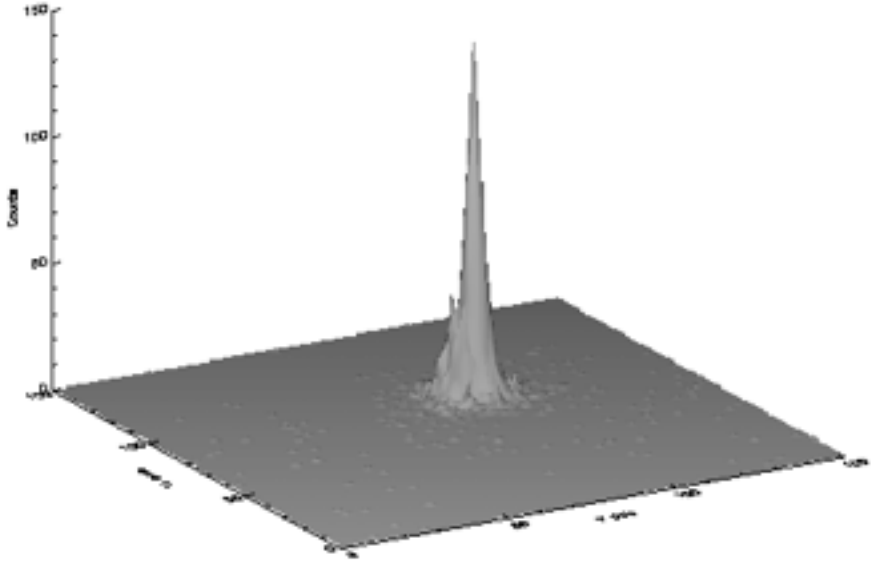


- **Motion blur**



Reproduced from http://www.robots.ox.ac.uk/~improofs/super-resolution/super-res1.html

# Example of point spread function



Reproduced from http://hea-www.harvard.edu/HRC/calib/hrci_qe.html

# **Problem**: deconvolution

- given data *x* and filter *h*, solve

$$x = h * s$$

# Deconvolution ? [Exercise]

- **Sine waves :** $e_f(t) := \exp(2i\pi ft)$

$$(h \star e_f)(t) = \int h(\tau) e_f(t-\tau) d\tau = \boxed{\ldots}$$

- **Combinations of sine waves**

$$s(t) := \int S(f) e_f(t) df$$

$$x(t) = h * \int S(f) e_f(t) df = \int S(f).(h * e_f)(t) df = \boxed{\cdots\cdots}$$

# Sine waves = eigen vectors of convolution operators

- **Convolution operator**: linear & shift-invariant

$$L_h : s \mapsto h * s$$

- **Eigen vectors / eigen values**

$$L_h(e_f) = H(f)\, e_f$$

- **Diagonalization**: Fourier transform *F* & inverse *F**

$$L_h = F^* \circ diag(H) \circ F$$

# Main property of convolution

- **Fourier transform** $F : h \mapsto H := F\{h\}$

$$H(f) := \int h(t).\exp(-2i\pi ft)dt$$

- **Inverse transform** $F^\star : H \mapsto h := F^\star\{H\}$

$$h(t) := \int H(f).\exp(+2i\pi ft)df$$

- Fourier transform of convolution product

$$F\{h * s\}(f) = H(f)S(f)$$

# Deconvolution (continued)

- Deconvolution problem = in Fourier domain

$$x = h * s \xrightarrow{F\{\cdot\}} X(f) = H(f)S(f)$$

$$\Downarrow$$

$$s(t) = \int S(f) e^{2i\pi ft} df \xleftarrow{F^{-1}\{\cdot\}} S(f) = X(f)/H(f)$$

- H(f) := *Transfer function* of filter h(t)

  [i.e. of Point Spread Function or Impulse Response]

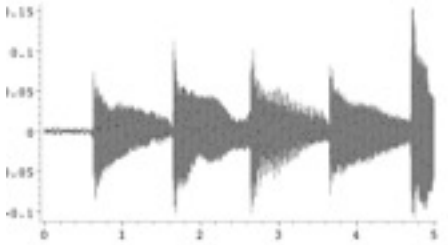
- Issue = zeroes in H(f)

# **Tools**: Fourier transforms

1D

# Fourier representation

- **Time domain representation**
  - N samples = N parameters



- **Sine wave**: N samples = 3 parameters
  - Amplitude
  - Frequency
  - Phase

$$x[n] = a\cos(2\pi fn + \varphi)$$

- **Periodic signal** = sum of few sinusoids

$$x(t) = \int X(f)\, e^{2i\pi ft}\, df$$

# Fourier representation

- **Goal** = express / represent a signal as

$$x(t) = \int X(f)\, e^{2i\pi ft}\, df$$

- **Tool** = Fourier Transform
  - Direct transform (analysis)
  - Inverse transform … (synthesis)

$$X(f) = \int x(t)\, e^{-2i\pi ft}\, dt$$

$$x(t) = \int X(f)\, e^{2i\pi ft}\, df$$

- Similar to *change of orthonormal basis*

# Fourier transforms :properties

- Linear, invertible, *unitary* transform
- Preserves the energy (Parseval equality)

$$\|x\|_2^2 := \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$$

- Converts convolution into multiplication

$$F\{x * y\} = F\{x\}.F\{y\}$$

Up to normalization factor for other equivalent definitions of Fourier transforms (parameter $\omega = 2\pi f$ or other normalization)

# Exercices

# Continuous FT Properties [Exercise]

- Multiplication $x(t).y(t)$
- Time-shift $x(t-u)$
- Frequency-shift $x(t)e^{2i\pi f_0 t}$
- Dilation $x(t/a)$
- Time-reversal $x(-t)$
- Time-derivative $x^{(p)}(t)$

Fourier →

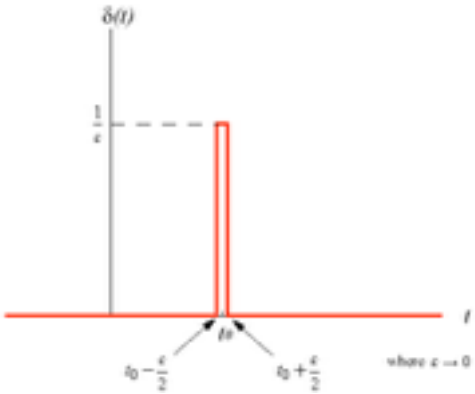
$(2i\pi f)^p X(f)$

# The Dirac impulse

- **Definition**: for any x(t)

$$\int_{\mathbb{R}} \delta(t)x(t+t_0)dt = \int_{\mathbb{R}} x(t)\delta(t-t_0)dt := x(t_0)$$

- **Approximation**




- **Dirac** = idealized instantaneous measure.

# Continuous FT Examples (1) [Exercise]

- Discrete Dirac $\delta(t)$
- Constant $x(t) = 1, \forall t$
- Shifted Dirac $\delta(t-u)$
- Complex exp. $e^{2i\pi f_0 t}$

Fourier →

# Continuous FT Examples (2) [Exercise]

- Real cosine $\cos(2\pi f_0 t + \varphi)$ Fourier →
- Real sine $\sin(2\pi f_0 t + \varphi)$

Hint : $\cos(u) = \dfrac{e^{iu} + e^{-iu}}{2}$

- Rectangle $x(t) = 1_{[-T,T]}(t)$

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