A GAMMA MIXTURE MODEL FOR THE ENERGY OF BLOCKS DEDICATED TO VECTOR QUANTIZATION OF SPARSE SIGNALS

L. Guillemot, S. Moussaoui, J.M. Moureaux

CRAN – CNRS UMR 7039
Université Henri Poincaré, Nancy 1, BP 239,
F-54506 Vandœuvre-lès-Nancy Cedex
Tel.: +33 3 83 68 44 76, Fax: +33 3 83 68 44 61
e-mail: *firstname.lastname@cran.uhp-nancy.fr*

ABSTRACT

Most of works related to vector quantization with structured codebooks deal with i.i.d. signal assumptions. We have shown in previous works that this hypothesis limited the efficiency of such coders applied on real signals. Indeed, we have proposed the use of a vector dead zone (VDZ) which outperforms previous schemes. The VDZ takes advantage of sparsity and clustering properties of sources like discrete wavelet transform of images. Here we give an interpretation of these properties under the angle of the norm distribution. We propose a probabilistic model of the norm of source sample blocks based on Gamma mixtures. Theoretical arguments, as well as experimental results show that our model fits well the source block distribution.

1. INTRODUCTION

In this paper we propose a probabilistic model dedicated to sparse and clustered signals like digital images in the multiresolution domain. It is based on the following issue: the L^α norm of blocks of samples can be a good measure of local activity, yielding thus to cluster significant coefficients within significant blocks[1]. This has been used with benefit in two schemes previously proposed by the authors: dead zone lattice vector quantization (DZLVQ) for image compression [8], and modulated lattice vector quantization (MLVQ) for joint compression and watermarking [3]. They have in common to use a block thresholding technique with respect to the norm of the blocks. This new technique, called vector dead zone, enables to better encode high energy[2] vectors (by putting more bits on them), while removing low energy ones. Thus, at constant bit rate, distortion is decreased, or equivalently, at fixed distortion, bit rate is decreased with respect to a classical LVQ scheme without dead zone.

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[1]In the rest of the paper we will call vectors the blocks of sample.
[2]In the whole paper, $\|X\|_\alpha^\alpha$ of a vector X stands for its energy.

Besides the nice performances offered by this approach in the context of image coding using discrete wavelet transform (DWT), the need of an appropriate probabilistic model of the L^α norm is crucial for MLVQ and DZLVQ. Indeed, in transform coding, variable-length coding of the norm of vectors is a key point of LVQ-based schemes. Thus, a model of the L^α norm will allow to design a cheap rate allocation procedure (computationally speaking). Moreover, such a model could also be of interest in other applications like denoising using block thresholding (see for example [2][4]).

The paper is organized as follows. First we present the key points of our two coders (DZLVQ and MLVQ) dedicated to sparse and clustered signals. Then we give details about our model based on a mixture of Gamma distributions for the norm of the blocks and we show how accurate it is. Finally, we conclude on the performances of the proposed block based schemes.

2. BLOCK BASED METHODS

The choice of vector quantization (VQ) for the two previously cited coders has been motivated by a well-known result of Information Theory: VQ offers better performances (in the rate-distortion sense) than scalar quantization (SQ), especially as it is able to take into account correlations existing between neighbor samples. Moreover, VQ with structured codebooks, like lattice vector quantization has low complexity and can benefit from the statistical properties of data when its design includes an adapted codebook shape (with respect to the source distribution) and entropy coding of the norm of the codebook vectors [8]. However, the majority of the works proposed in this domain is based on the hypothesis of i.i.d. data, whereas a lot of real data have spatial dependencies, like for example cluster properties of DWT coefficients from digital images.

Our main contribution in LVQ is related to this issue: from the point of view of LVQ, sparsity and clustering of

wavelet coefficients lead to a large amount of vectors with low norm (see figure 4) which cannot be modelled from the classical i.i.d Laplacian hypothesis for coefficient distribution. Thus, we have proposed a new scheme, called DZLVQ, based on the thresholding of vectors according to their L^1 norm [8]. Figure 1 illustrates the efficiency of DZLVQ compared to classical LVQ. We have shown that DZLVQ could outperform at low rates both standard JPEG2000 and SPHIT algorithms, which are references in image coding [8]. Furthermore, the gain involved by the proposed vector dead zone is emphasized when applied to joint compression and watermarking. Indeed, when the well-known quantization index modulation [1] (QIM) algorithm is straightforwardly integrated to a transform coding scheme, experimental results show that such a method is not adapted to compression (see figure 2). This is due to the fact that the quantized values depend on the message to embed. Consequently, QIM is unable to take advantage of the sparsity of the original data. In this context, MLVQ which is based on the vector dead zone is an appropriate answer to maintain sparsity of the quantized data (as it is shown in figure 2).
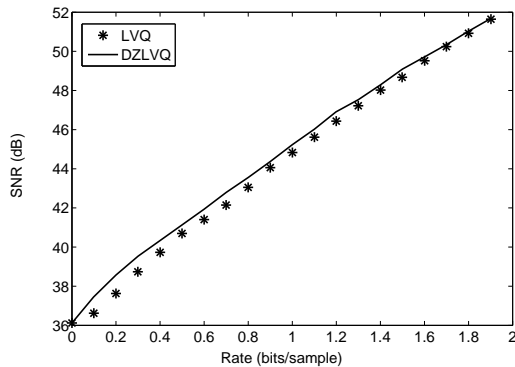

Fig. 1: Rate – SNR functions for DZLVQ and LVQ applied on the vertical sub image of Lena (level 3)

Whatever the coder we consider, the efficiency of the vector dead zone is based on the statistical properties of the energy of vectors. However, these properties cannot be represented from a classical i.i.d. probabilistic model. In the following, we will show that a mixture of Gamma distributions permits to solve this problem and thus to improve efficiency of block based methods.

3. THE GAMMA MIXTURE MODEL

The main properties of the norm distribution can be observed on the histogram represented in figure 4. It is non negative, it has a mode close to zero (which is due to sparsity of blocks) and a heavy tail corresponding to clusters
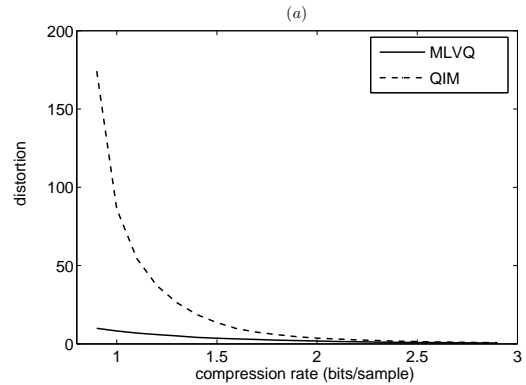

Fig. 2: R-D distortion curve of MLVQ and QIM schemes applied to the vertical dyadic wavelet subimage (level 3) of Lena image.

of high magnitude samples. All these properties naturally lead to a density mixture [2]. The use of mixture of densities allows to give a flexible model to describe the distribution of the energy of vectors. In order to account for both non-negativity and sparsity of the energy of vectors, we use a Gamma mixture model. The Gamma distribution is expressed by

$$r \sim \mathcal{G}(r; a, b) \Leftrightarrow$$
$$p(r; a, b) = \frac{b^a}{\Gamma(a)} r^{a-1} \exp[-b\, r] \mathbb{I}_{r>0}, \quad (1)$$

where $\Gamma(a)$ is the Gamma function, r the random variable corresponding to the energy of the block and $\mathbb{I}_{r>0}$ is the indicator function. The parameters (a, b) allow to adjust the shape of this distribution. As shown in figure 3, for $0 < a < 1$ the distribution has a peaky shape, which is well suited to sparse signals, and for $a > 1$, the distribution has a mode in $(a-1)/b$ and is heavy tailed.
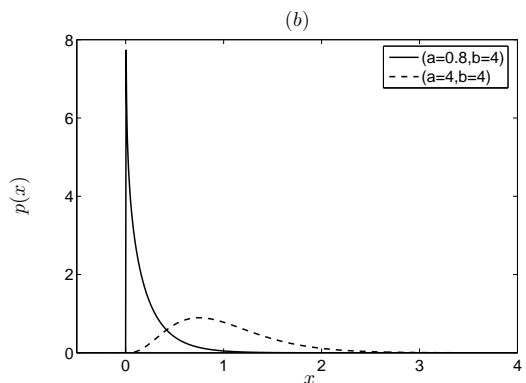

Fig. 3: Illustration of the Gamma distribution typical shapes.

In addition to its flexibility, the use of the Gamma model presents a well stated theoretical justification. Consider random variables distributed as a Generalized Gaussian distribution of power parameter α and shape parameter β:

$$z \sim \mathcal{GG}(z; \alpha, \beta) \Leftrightarrow$$
$$p(z; \alpha, \beta) = \frac{\alpha \, \beta^{1/\alpha}}{2\Gamma(1/\alpha)} \exp[-\beta z^\alpha], \quad (2)$$

and take vectors $\mathbf{z} = [z_1, ..., z_n]$. The energy $(\|\mathbf{z}\|_\alpha^\alpha = \sum_{k=1}^{n} |z_k|^\alpha)$ of these vectors is Gamma distributed with parameters $a = n/\alpha$ and $b = 1/\beta^\alpha$. This property can be proven by making a substitution of variables $r = |z_1|^\alpha + \cdots + |z_n|^\alpha$ and performing a probability transformation using the pdfs $p(z_i; \alpha, \beta)$ to get the density $p(r)$

In the framework of our application, the energy of vectors is assumed to belong to two classes, low and high energy states, noted E_1 and E_2. This assumption leads to the following mixture model:

$$p(r; \mathbf{a}, \mathbf{b}) = \sum_{k=1}^{2} c_k \, \mathcal{G}(r; a_k, b_k), \quad (3)$$

with $c_k = P(r \in E_k)$, $\mathbf{a} = [a_1, a_2]$ and $\mathbf{b} = [b_1, b_2]$. To estimate the parameters (c_1, c_2) we use a Monte Carlo Markov chain algorithm [5], where the Gamma parameters $\{\boldsymbol{a}, \boldsymbol{b}\}$ are simulated using the algorithm given in [6]. Figure 4 shows that the two Gamma mixture permits to fit successfully the main properties of the energy distribution: the peak and the heavy tail. A second point is that the modes of the two distributions are disjoint which facilitates the estimation of the parameters.
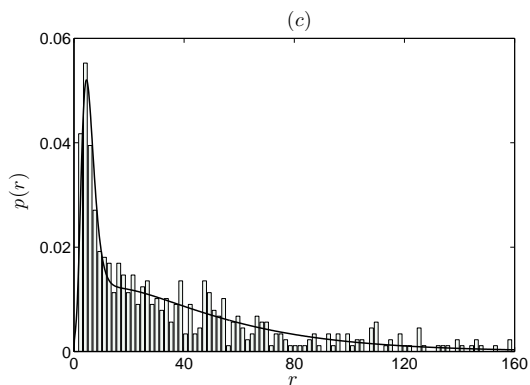

Fig. 4: Histogram of the energy (vectors of size 8) of the vertical sub-image of level 3 of Lena and the estimated two Gamma mixture density with parameters $\mathbf{a} = [5.12, 1.49]$, $\mathbf{b} = [0.92, 0.03]$ and $\mathbf{c} = [0.24, 0.76]$.

4. CONCLUSION

In this paper we have proposed a probabilistic model related to a block approach permitting to take into account clustering and sparsity properties of signals like images in a multiresolution domain. It is based on the estimation of the energy of the blocks using a mixture of two gamma distributions. On one hand, this model is well suited to the estimation of the parameters and on the other hand, it corresponds to the distribution of the energy in the case of a generalized gaussian *a priori*.

The perspectives of this work seem to be rather large. Particularly, we aim at deducing an analytical rate-distortion model in order to design an efficient rate allocation procedure for DZLVQ and MLVQ. Finally, the accuracy of the mixture and its advantages concerning the estimation stage suggest that our approach could be applied in an other domains like, for exemple, denoising.

5. REFERENCES

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