

Model-based 3-D Pose Estimation and Feature Tracking for Robot Assisted Surgery with Medical Imaging

C. Doignon, F. Nageotte, B. Maurin and A. Krupa

Abstract—In this paper we address the problem of the pose estimation based on multiple geometrical features for monocular endoscopic vision with laparoscopes and for stereotaxy with CT scanners. Partial and full pose estimation (6 dofs) are considered with applications to minimally invasive surgery. At the University of Strasbourg, we have been developing a set of techniques for assisting surgeons in navigating and manipulating the three-dimensional space within the human body. In order to develop such systems, a variety of challenging visual tracking and registration problems with pre-operative and/or intra-operative images must be solved. This paper integrates several issues where computational vision can play a role. Depth recovery (from the tip of a surgical instrument w.r.t. living tissue), the Plücker coordinates (4 dofs) of a markerless cylindrical instrument, the 6 dofs of a needle-holder with an heterogeneous set of features and stereotaxy are the examples we describe. Projective invariants with perspective projection, quadrics of revolution and stereotactic markers are features which are useful to achieve the registration with uncalibrated or calibrated devices.

Visual servoing-based tracking methods have been developed for image-guided robotic systems, for assisting surgeons in laparoscopic surgery and in interventional radiology. Real-time endoscopic vision and single-slice stereotactic registration has been proposed to retrieve the out-of-field of view instruments, to position a needle and to compensate small displacements like those due to patient breathing or any small disturbances which may occur during an image-guided surgical procedure.

I. INTRODUCTION

The field of vision-based robotics has been widely growing for more than three decades, and more and more complex 3-D scenes are within robot reach thanks to better understanding of the scenes, improvement of computer capabilities and control theory. The achievement of applications like medical robotics, mobile robotics, micro-robotic manipulation, agricultural automation or the observation by aerial or underwater robots needs the integration of several research areas in computer vision and automatic control ([19], [32]).

For the past two decades, medical robot and computer-assisted surgery have gained increasing popularity. They have expanded the capabilities and comfort for both patients and surgeons in many kinds of interventions such as local therapy, biopsies, tumors detection and removal with techniques like multi-modal registration,

C. Doignon and F. Nageotte are with the LSIIT (UMR ULP-CNRS 7005), Louis Pasteur University of Strasbourg, ENSPS, Boulevard Brant, 67412 Illkirch, France. E-mail: name@lsiit.u-strasbg.fr

B. Maurin is now with the Cerebellum Automation Company, 178, route de Cran Gevrier, 74650 Chavanod, France. E-mail: maurin@eavr.u-strasbg.fr

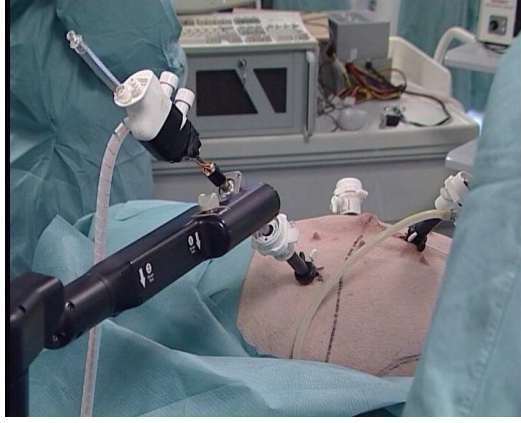
A. Krupa is with the IRISA - INRIA Rennes, Campus de Beaulieu, 35042 Rennes, France. E-mail: Alexandre.krupa@irisa.fr

on-line visualization, simulators for specific interventions or tracking. Medical robots provide a significant help in surgery, mainly for the improvement of positioning accuracy and particularly for intra-operative image guidance [36]. The main challenge in visual 3-D tracking for medical robotic purposes is to acquire the necessary video information from images acquired with endoscopes [5], ultra-sound probe ([17], [21]), scanners ([26], [35])...so as to evaluate the position and the velocity of the object of interest which usually are anatomical or artificial landmarks attached to a surgical instrument.

This paper presents several 3-D pose estimation algorithms and visual servoing-based tracking with monocular vision systems such as endoscopes and CT scanners developed in an attempt to improve the guidance accuracy. These are intended for the 3-D positioning and guidance of surgical instruments in the human body. The efficiency of most of model-based visual servoing approaches relies on correspondences between the position of tracked visual features in the current image and their 3-D attitude in the world space. If these correspondences contain errors then the servoing usually fails or converges towards a wrong position. Overcoming these errors is often achieved by improving the quality of tracking algorithms and features selection methods ([20], [37]). Following this purpose, the paper integrates several issues where computational vision can play a role:

- 1) estimating the distance between the tip of a laparoscopic instrument and the pointed organ with collinear feature points,
- 2) estimating the 3-D pose of an instrument using multiple geometrical feature tracking and Virtual Visual Servoing,
- 3) positioning a degenerate quadric-based instrument,
- 4) registering the position of a robot using stereotaxy and a single CT slice.

The paper is organized as follows. In the next section, the problem of the pose estimation of surgical instruments with markers is stated and solved for full or partial dofs. In section 3, we focus on the positioning of the symmetry axis of a cylindrical-shaped instrument. Applications of both sections use endoscopic vision. The stereotactic registration with a single view is studied as a pose estimation problem in section 4. Much of this material is taken from our recent publications in this area.



(a)



(b)

Fig. 1. (a) The laparoscopic experimental setup. The instrument is mounted on the end effector of a surgical robot and inserted through the abdominal wall while the laparoscope is inserted through another insertion point. (b) An image-guidance with CT scanners feedback control during the percutaneous insertion of a radio-frequency needle.

II. POSE ESTIMATION OF A LAPAROSCOPIC INSTRUMENT WITH LANDMARKS

A. Pose estimation with collinear markers

There exist several difficulties when tackling the problem of estimating the 3-D position of a laparoscopic surgical instruments with a single endoscopic view. One difficulty is the use of monocular vision which gives poor depth information. Another one relies on the highly unstructured nature of the scene with varying lighting conditions and with a background moving due to breathing or heart beating. To solve these problems, we conceived five years ago a special instrument which projects a laser pattern onto the organ surface in order to provide the relative orientation of the instrument with respect to the organ, even if the instrument is not in the camera field of view. Optical markers have been added on the tip of the surgical instrument. These markers (composed of three circular LEDs) were directly projected onto the image and in conjunction with images of the laser pattern, they were used to guide the instrument (see figure 2). We combined image feature points (spots

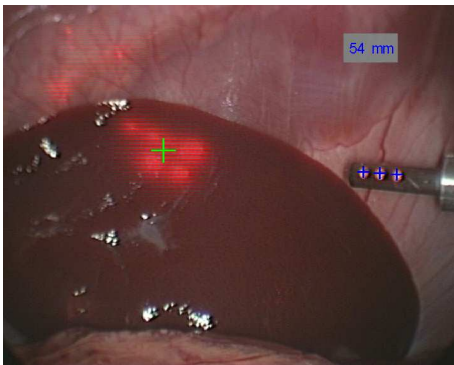


Fig. 2. A surgical instrument with a laser pointing device (laser beam - green cross) and three optical markers (blue crosses). The cross-ratio is computed and controlled in successive images so as to estimate the distance and its variations between the tip and the pointed organ surface.

center coordinates) and depth information for positioning the instrument with respect to the pointed organ [22].

1) *Pose estimation with a calibrated endoscope:* Recovering the relative orientation (2 dof - the unit vector \mathbf{r}) and position (3 dof - a vector \mathbf{t}) of a set of n collinear points such as the optical markers and laser projections in figure 2 with respect to the camera has been previously investigated by Haralick fifteen years ago [14]. The interpoint distances (structure) and a focal length f of the camera are assumed to be known. Haralick solved this problem with a linear algorithm. Let $P_0 = \mathbf{t}$, $P_1 = \mathbf{t} + \lambda_1 \mathbf{r}, \dots, P_{n-1} = \mathbf{t} + \lambda_{n-1} \mathbf{r}$ be n discriminated points where λ_i represents the distance between the $(i+1)$ th and i th points. The first point P_0 is arbitrarily chosen as the origin ($\lambda_0 = 0$), hence the perspective projection $Q_i = (u_i, v_i, 1)^T$ of the i th point is given by

$$[0 \ 0 \ 1] (\lambda_i \mathbf{r} + \mathbf{t}) \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = K^c (\lambda_i \mathbf{r} + \mathbf{t}) \quad (1)$$

where K^c is a (3×3) upper diagonal matrix containing the internal parameters of the camera. From the above equation, Haralick built an homogeneous linear system with an univariate matrix $K^c = \text{diag}(f, f, 1)$ and vectors \mathbf{t} and \mathbf{r} as unknowns

$$\underbrace{\begin{bmatrix} \mathbf{A}_r & \mathbf{A}_t \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \mathbf{r} \\ \mathbf{t} \end{bmatrix} = \mathbf{0} . \quad (2)$$

\mathbf{A} is a $(2n \times 6)$ real matrix and a closed-form solution can be found with $n \geq 3$ discriminated points. This system may be reformulated as a classical optimization problem with an equality constraint:

$$\min \|\mathbf{A}_r \mathbf{r} + \mathbf{A}_t \mathbf{t}\| \quad \text{subject to} \quad \mathbf{r}^T \mathbf{r} = 1 , \quad (3)$$

where \mathbf{A}_r and \mathbf{A}_t are two $(2n \times 3)$ real matrices defined as:

$$\mathbf{A}_r = \underbrace{\begin{bmatrix} \lambda_0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & \lambda_0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & \lambda_{n-1} & 0 \\ 0 & 0 & \cdots & \cdots & 0 & \lambda_{n-1} \end{bmatrix}}_{\Lambda} \mathbf{A}_t \quad (4)$$

$$\mathbf{A}_t = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \left(\mathbf{K}^c - \begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} [0 \ 0 \ 1] \right) \\ \vdots \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \left(\mathbf{K}^c - \begin{bmatrix} u_{n-1} \\ v_{n-1} \\ 1 \end{bmatrix} [0 \ 0 \ 1] \right) \end{bmatrix} \quad (5)$$

The solution for \mathbf{r} is given by the eigenvector associated with the smallest eigenvalue of the following symmetric matrix

$$\mathbf{E} = \mathbf{A}_r^\top \left(\mathbf{I} - \mathbf{A}_t (\mathbf{A}_t^\top \mathbf{A}_t)^{-1} \mathbf{A}_t^\top \right) \mathbf{A}_r \quad (6)$$

and the position vector \mathbf{t} is straightforwardly given by the expression $\mathbf{t} = -(\mathbf{A}_t^\top \mathbf{A}_t)^{-1} \mathbf{A}_t^\top \mathbf{A}_r \mathbf{r}$. We end up with two different estimates for \mathbf{r} (a twofold ambiguity in the sign of \mathbf{r}). However, for real objects placed in front of the camera, the third component of vector \mathbf{t} must be strictly positive assuming that the camera z -axis (usually, the optical axis) is pointed towards the scene. This leads to the uniqueness of the solution for the pose.

It worth pointing out that collinearity is a projective invariant property which is not fully exploited in this technique for pose recovery. Moreover, in presence of both noisy data and close points in the object pattern, matrices \mathbf{A}_r and \mathbf{A}_t are ill-conditioned, which introduces some significant bias in the results. The use of the least mean squares and the lack of data normalization in the original algorithm tend the solution to be sensitive to the condition number. One has to pay attention to data normalization since the pose estimation may be computed with points not always well scattered. This may also lead to numerical problems. To lower the condition number, it seems advisable to normalize data coordinates with an affine transformation as in [16].

2) *Distance with Collinear landmarks*: To perform a 3-D positioning of an instrument with respect to an organ [22], we need to estimate the distance between the instrument and the pointed organ (depth d_0 in figure 3). Since the three optical markers centers P_1 , P_2 and P_3 are placed along the instrument axis, we assumed they are collinear with the laser spots barycentre P . Under this assumption, a cross-ratio can be computed from these four points [28]. This projective invariant can also be computed in the image using their respective projections p_1 , p_2 , p_3 and p (see figures 2 and 3) and can be used to estimate the depth d_0 . In other words, since a 1-D projective basis can be defined either with

$\{P_1, P_2, P_3\}$ or their respective images $\{p_1, p_2, p_3\}$, the cross ratio is a projective invariant built with the fourth point (P or p). Consequently, a 1-D homography H exists between these two bases, so that the straight line Δ corresponding to the instrument axis is transformed, in the image, into a line $\delta = H(\Delta)$ as shown in figure 3. The cross-ratio τ is given by:

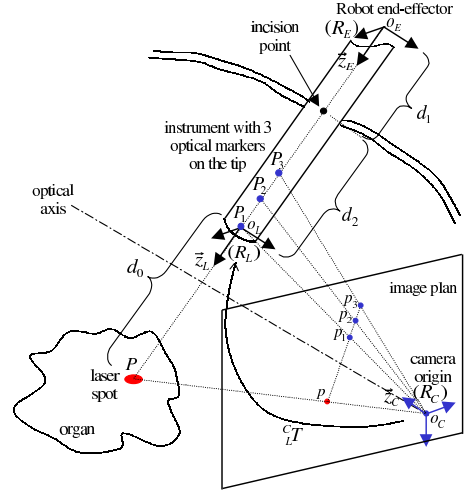


Fig. 3. The basic geometry involved.

$$\tau = \frac{\left(\frac{\overline{pp_2}}{\overline{p_1 p_2}} \right)}{\left(\frac{\overline{pp_3}}{\overline{p_1 p_3}} \right)} = \frac{\left(\frac{\overline{P P_2}}{\overline{P_1 P_2}} \right)}{\left(\frac{\overline{P P_3}}{\overline{P_1 P_3}} \right)} \quad (7)$$

and d_0 is obtained as

$$d_0 = \overline{P P_1} = (1 - \tau) \frac{\overline{P_1 P_3}}{\tau - \frac{\overline{P_1 P_3}}{\overline{P_1 P_2}}} = \alpha \frac{1 - \tau}{\tau - \beta} \quad (8)$$

where α and β depend only on the known relative position of P_1 , P_2 and P_3 . To simplify the computation of the cross-ratio in the image plane, it's necessary to characterize the straight line δ in order to relate the pixels coordinates of an image point $p = (u, v, 1)^\top$ and its projective coordinates $(s\lambda, s)^\top$ on δ . Let $(-b, a)^\top$ be the normalized cosine direction of δ and $p_k = (u_k, v_k, 1)^\top$ a point on δ . This gives:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} -b & u_k \\ a & v_k \\ 0 & 1 \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \quad (9)$$

or

$$\lambda = \begin{bmatrix} -b & a \end{bmatrix} \begin{bmatrix} u - u_k \\ v - v_k \end{bmatrix} \quad (10)$$

with :

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0 \quad (11)$$

where $(-c)$ is the orthogonal distance from δ to the image origin. The computation of the cross-ratio is then:

$$\tau = \frac{\lambda_0 + \overline{p_1 p_2}}{\overline{p_1 p_2}} \frac{\overline{p_1 p_3}}{\lambda_0 + \overline{p_1 p_3}} \quad (12)$$

From equation (8), it is straightforward that \dot{d}_0 is a function of $\dot{\tau}$ which, in turn, is a function of λ_0 , $\overline{p_1 p_2}$ and $\overline{p_1 p_3}$. Similar computations lead to the same relationship between d_2 and another cross-ratio μ defined with the points P_1, P_2, P_3, O_Q and their respective projections provided that o_q , the perspective projection of the incision point O_Q (see figure 3), can be recovered [10]. Since O_Q is generally not in the camera field of view, this can be achieved by considering a displacement of the surgical instrument between two configurations yielding straight lines δ and δ' in the image. Then, o_q is the intersection of these lines given that O_Q is invariant. Finally :

$$\mu = \frac{\left(\frac{\overline{p_1 p_3}}{\overline{p_2 p_3}}\right)}{\left(\frac{\overline{p_1 o_q}}{\overline{p_2 o_q}}\right)} = \frac{\left(\frac{\overline{P_1 P_3}}{\overline{P_2 P_3}}\right)}{\left(\frac{\overline{P_1 O_Q}}{\overline{P_2 O_Q}}\right)}, \quad (13)$$

$$d_2 = \overline{P_1 O_Q} = \frac{\frac{\alpha}{1-\beta}}{\mu + \frac{\beta}{1-\beta}}. \quad (14)$$

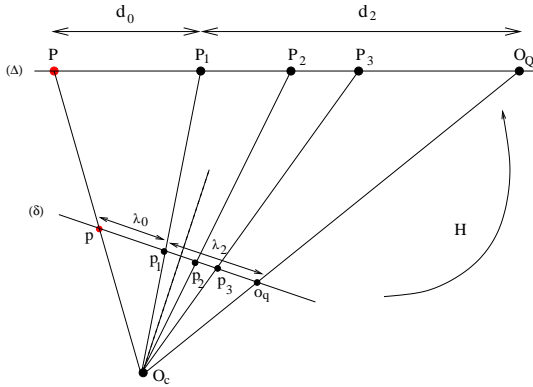


Fig. 4. Markers P_1, P_2, P_3 on the tool axis Δ and their images p_1, p_2, p_3 on line δ . Note that $o_q = H(O_Q)$ is invariant during surgical procedures.

B. Pose estimation with Multiple Features

1) *Objectives and Related work:* In many early works, the images of laparoscopic instruments are segmented, in order to control the position of the endoscopic camera. These methods are based on the structure, mainly the apparent lines of the instrument [1], [6], or on its frequential features [9], [41]. In order to make the detection more robust and accurate, instruments can be marked with structuring markers as described before ([1], [22], [43]) or frequential (colour) markers ([38], [42]). Most of these works use only the 2-D position of the instrument in the image and the accuracy of the features extraction is not so important for aimed applications. On the contrary, the 3-D pose estimation requires a very accurate features extraction step.

None of the known works have focused on the complete 6 dofs detection. To determine the six degrees of freedom, the non symmetric part of the instrument has to be used or the instrument has to be marked. The second solution avoids using a CAD model and can be applied to any kind of instrument. We use the marker presented in figure 5,

which is composed of twelve black spots on a white area, building four "marker lines" which can be discriminated from each other by using the cross-ratio invariant. For more information on the choice of this marker, the interested reader can refer to [30]. The detection of this marker is mainly based on the intensity of the white area and the black spots in the endoscopic images. It can thus be used in grey level images as well as color endoscopic images. The results of the detection technique are shown in figure 5 for laboratory endoscopic images. The image features noted s are n points corresponding to the inertia centers of the visible spots (generally $n \in [0, 6]$) and two lines l^+ and l^- corresponding to the apparent contours of the shaft in the endoscopic image and represented by their distance ρ to the origin of the image and their orientation θ :

$$s = (l^+, l^-, p_1, \dots, p_n)^T. \quad (15)$$

Depending on the size of the white marker area in the image, the complete extraction process can take up to 200 ms. In order to track the instrument at higher rate, we have developed techniques based on the moving edges method due to Bouthemy [3]. The main difficulty is to track the black spots which can appear and disappear, due to the rotation of the instrument around its own axis, and possibly due to occlusions. We have proposed a method based on the prediction of the spots appearance and disappearance which allows to track the markers without the need to register the images of the spots with the real positions [31]. Thus, the tracking of the instrument can be handled at a rate of 20 Hz as long as one spot is visible in the endoscopic images.

2) *Pose Estimation of a Tagged Instrument:* Our model-based pose estimation process requires a calibrated camera. Endoscopic cameras have a large field of view and include large radial distortion. As a consequence, the calibration method must estimate the distortion parameter [2], [39]. Only four degrees of freedom are necessary to estimate the attitude of the instrument axis (see next section). Theoretically, the 4 dofs of the pose can be determined using the contour generator and its image (the apparent contour) of the cylinder [8]. However, the positions of the marking spots not only define the proper rotations and translations, but also give information on the orientation and position of the axis of the shaft. We then chose to estimate the 6 dofs of the instrument. This can be done with analytical methods using both the apparent contours and one known point at the cylinder's surface [29]. Other methods, like those proposed by Horaud [18], Haralick [15], DeMenthon [7] or Quan [33] for instance can also be used.

However, the full pose estimation is interesting for robustness considerations only if all the available information given by the apparent lines and all the spots is used. To this purpose, the Virtual Visual Servoing (VVS) due to Marchand and Sundaeswaran ([24], [34]) may handle the information redundancy. VVS is a numerical iterative method for minimizing the error between the extracted features and the

forward projection of the object in the images and based on the image-based visual servoing (IBVS) schemes. This process needs the computation of an interaction matrix which relates the variations of each image feature and the the camera velocity screw τ . With the image features we use, the interaction matrix L_s has the following form:

$$\begin{pmatrix} \dot{i}^+ \\ \dot{i}^- \\ \dot{p}_1 \\ \vdots \\ \dot{p}_n \end{pmatrix} = \begin{pmatrix} L_{line}(I^+) \\ L_{line}(I^-) \\ L_{pt}(p_1) \\ \vdots \\ L_{pt}(p_n) \end{pmatrix} \underbrace{\begin{pmatrix} {}^cV_{c/i} \\ {}^c\omega_{c/i} \end{pmatrix}}_{\tau}. \quad (16)$$

The interaction matrices associated to a point L_{pt} and to a line L_{line} can be found in the works of Chaumette [11].

In order to guarantee a fast convergence and a good stability of the VVS, it is useful to initialize the algorithm close enough to the real pose. For this purpose, we use either the DeMenthon iterative method when at least four points are visible or the analytical 3-D pose estimation mentioned earlier. From the obtained initial estimate, the following control law is applied to the virtual camera

$$\tau = -\lambda \tilde{L}_s^+(s(r) - s_d) \quad (17)$$

until the control vector becomes smaller than a specified value. The process converges quickly towards the real pose of the camera (see figure 5 (c)).

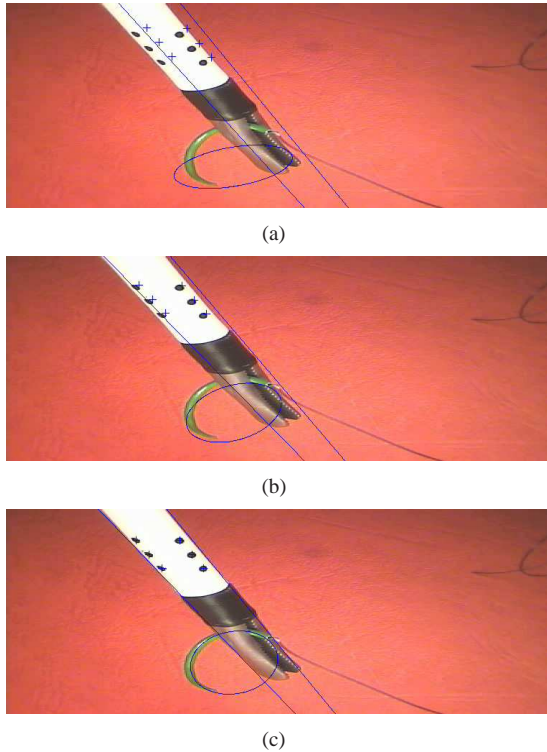


Fig. 5. (a-c) The pose estimation as a virtual visual servoing process with multiple geometric features (apparent lines, marker-lines and circular needle). (a) The blue lines are the projections with the initial virtual camera position. (c) The projections when the error vector $\mathbf{s} - \mathbf{s}_d$ tends to 0.

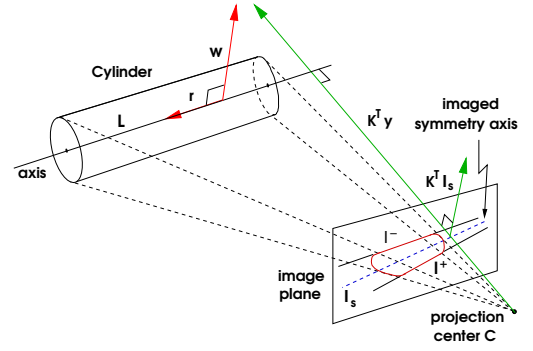


Fig. 7. A cylinder and its image with the perspective projection P . The backprojection of the apparent lines (I^-, I^+) is a pair of planes $P^T I^-$ and $P^T I^+$, passing through the projection center. The image of the cylinder axis Δ is the axis I_s of the harmonic homology H relating the apparent lines.

III. POSE ESTIMATION OF A LAPAROSCOPIC INSTRUMENT WITHOUT LANDMARKS

A. Problem statement and perspective projection

The aim of this section is to briefly present a new algorithm for the determination of the pose of a straight homogeneous circular cylinders (SHCC) without markers, that is to say directly from the apparent contour. More details are provided in [8]. The apparent contour (γ) of a cylinder is a set of points which intersect the viewline and the image plane. It is the projection of a 3-D curve on the cylinder's surface referred to as the contour generator (Γ).

Given the matrix K^c of camera intrinsic parameters, the cylinder radius r_c and the apparent lines $\{I^-, I^+\}$, we look for the estimation of the Plücker coordinates (\mathbf{r}, \mathbf{w}) of the cylinder axis (see figure 7) satisfying the non-linear equation $\mathbf{r}^T \mathbf{w} = 0$. This means that one has to solve a polynomial equation for a unique (double) solution (see [8]) that is, for a null discriminant. This one equals $B^2 - AC = 0$ with

$$\begin{cases} A &= m^T (K^c)^{-T} [\mathbf{r}]_{\times} [\mathbf{r}]_{\times}^T m \\ B &= m^T (K^c)^{-T} [\mathbf{r}]_{\times} \mathbf{w} \\ C &= \|\mathbf{w}\|^2 - r_c^2 \end{cases} \quad (18)$$

and after some computations, it can be expressed as

$$\left(\frac{r_c B}{\sqrt{C}} + \mathbf{w}^T (K^c)^{-1} m \right)^T \left(\frac{r_c B}{\sqrt{C}} - \mathbf{w}^T (K^c)^{-1} m \right) = 0. \quad (19)$$

If the scalar $C \leq 0$, the projection center $C_c = (0, 0, 0, 1)^T$ is located inside the cylinder (or on its surface, if $C = 0$) and yields no real solutions. We do not consider these special cases in the remainder, we rather focus this work on the more practical situation with real solutions. In the case of a circular cylinder with infinite height and a constant radius, equation (19) shows that the apparent contour is a set of two straight lines represented either with the pair of vectors I^- and I^+ satisfying

$$\begin{cases} (I^-)^T m &\equiv \{(K^c)^{-T} (I - \alpha [\mathbf{r}]_{\times}) \mathbf{w}\}^T m = 0 \\ (I^+)^T m &\equiv \{(K^c)^{-T} (I + \alpha [\mathbf{r}]_{\times}) \mathbf{w}\}^T m = 0 \end{cases} \quad (20)$$

with $\alpha = r_c / \sqrt{\|\mathbf{w}\|^2 - r_c^2}$, or alternatively with the real (3×3) matrix $C = I^- I^{+T} + I^+ I^{-T}$ satisfying $m^T C m = 0$.

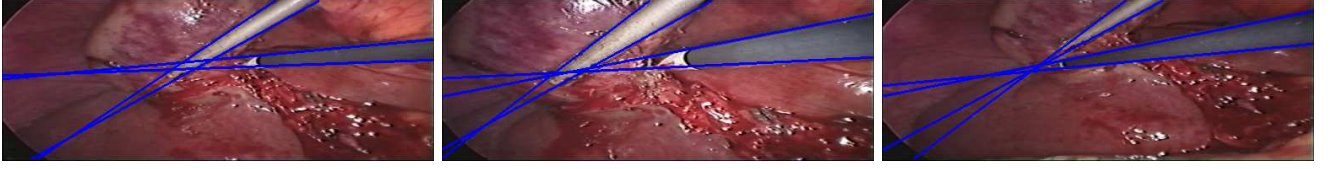


Fig. 6. Segmentation of three endoscopic images of surgical instruments in the abdominal cavity. The degenerate conic fitting (in blue) with the overall apparent contour of the two cylindrical instruments.

The latter is a rank-2 symmetrical matrix defined up to a scale factor and both representations are equivalent to model the apparent contour with 4 parameters.

B. Direct pose computation

In this paragraph we present a linear algorithm for the pose estimation. Starting from (20), the matrix C can be related to the pose parameters since on the one hand, we have

$$\begin{aligned}
K^c T C K^c &\equiv K^c T (I^- I^{+T} + I^+ I^{-T}) K^c \\
&\equiv (\alpha [r]_{\times} - I) \mathbf{w} \mathbf{w}^T (\alpha [r]_{\times}^T + I) \\
&\quad + (\alpha [r]_{\times} + I) \mathbf{w} \mathbf{w}^T (\alpha [r]_{\times}^T - I) \\
&\equiv \alpha^2 [r]_{\times} \mathbf{w} \mathbf{w}^T [r]_{\times} + \mathbf{w} \mathbf{w}^T \\
&\equiv [r]_{\times} \left(\alpha^2 \frac{\mathbf{w} \mathbf{w}^T}{\|\mathbf{w}\|^2} + \frac{[w]_{\times} [w]_{\times}^T}{\|\mathbf{w}\|^2} \right) [r]_{\times}^T \\
&\equiv [r]_{\times} \left(I - (1 - \alpha^2) \frac{\mathbf{w} \mathbf{w}^T}{\|\mathbf{w}\|^2} \right) [r]_{\times}^T \\
&\equiv (I - \mathbf{r} \mathbf{r}^T - \mathbf{z} \mathbf{z}^T) \\
&= \begin{bmatrix} \mathbf{a} & \mathbf{z}_u & \mathbf{r} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - \sigma^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a}^T \\ \mathbf{z}_u^T \\ \mathbf{r}^T \end{bmatrix}
\end{aligned} \tag{21}$$

with $\mathbf{z} = \frac{\sqrt{1-\alpha^2}}{\|\mathbf{w}\|} [r]_{\times} \mathbf{w}$ and the unit vector $\mathbf{z}_u = \mathbf{z}/\sigma$. On the other hand, the SVD has the following expression

$$K^c T C K^c = U D U^T = U \text{diag}(\lambda_1, \lambda_2, 0) U^T. \tag{22}$$

It is easy to see that $U = \begin{bmatrix} \mathbf{a} & \mathbf{z}_u & \mathbf{r} \end{bmatrix}$ and

$$\sigma = \|\mathbf{z}\| = \sqrt{1 - \alpha^2} = \sqrt{1 - \frac{\lambda_2}{\lambda_1}}.$$

Finally, $\mathbf{w} \equiv \mathbf{z}_u \times \mathbf{r} = \mathbf{a}$ and

$$\|\mathbf{z}\| = \sqrt{1 - \frac{r_c^2}{\|\mathbf{w}\|^2 - r_c^2}} \Rightarrow \|\mathbf{w}\| = r_c \sqrt{1 + \frac{\lambda_1}{\lambda_2}}. \tag{23}$$

Many results are provided in [8]. In particular, we have compared the pose computation from the apparent contours of cylinders and the Haralick's method for the pose of a set of collinear points [14] with some artificial markers stucked on the cylinder's surface. Here, we rather focus the discussion on the application of concern which is the image-guidance for intra-operative procedures in minimally invasive surgery (MIS). In laparoscopic surgery, most of surgical instruments have cylindrical parts and are metallic (see figure 6 (top)) leading to grey regions with many specularities in the image. Prior research involving such

endoscopic images have been conducted in the field of color image segmentation [9]. Once regions have been segmented, the region boundaries are ordered and used to perform a degenerate conic-based contours fitting. With the calibrated and distortion-corrected endoscope used in the experiments, the 3-D localization of the two moving surgical instruments in figure 6 (a-c) has been done with success for more than 300 successive images of the abdominal cavity of a pig.

With the proposed method, the location of each insertion point in laparoscopy can be recovered, on-line, with no marker, without any knowledge of robot kinematics and without an external measurement device [10]. Since any laparoscopic instrument is passing through this point, the motion constraint in MIS can be expressed as the intersection of multiple convergent 3-D straight lines. Since any (homogeneous) point \mathbf{X} is on L if $L^* \mathbf{X} = \mathbf{0}$, given n positions corresponding to the set of dual Plücker matrices $\{L_1^*, L_2^*, \dots, L_n^*\}$, the intersection of lines is obtained with a rank-3 ($4n \times 4$) matrix G_n^T such that

$$G_n = [L_1^*, L_2^*, \dots, L_n^*]. \tag{24}$$

That is to say the null-space of G_n^T must be a one-dimensional subspace and the intersection may be computed with n ($n \geq 2$) 3D positions. By computing the SVD of G_n^T , one obtains the common intersection with the singular vector associated to the null singular value (or the smallest one in presence of noisy data). Moreover, the perspective projection of the 3D line L_j is the image line l_j defined by

$$[l_j]_{\times} = K^c P^c L_j (K^c P^c)^T = [(K^c)^{-T} \mathbf{w}_j]_{\times} \Rightarrow \mathbf{l}_j \equiv (K^c)^{-T} \mathbf{w}_j, \tag{25}$$

where P^c is the projection matrix. Since vector \mathbf{l}_j is defined up to a scale, it does not depend on the magnitude of vector \mathbf{w}_j , hence the n convergent image lines l_1, l_2, \dots, l_n must satisfy

$$(\mathbf{l}_1 \ \dots \ \mathbf{l}_n)^T i = \underbrace{(\mathbf{w}_1 \ \dots \ \mathbf{w}_n)^T}_{W_n} (K^c)^{-1} i = \mathbf{0} \tag{26}$$

where i is the image of the insertion point I . It follows that a set of n 3D straight lines is projected to n convergent image lines if the above ($n \times 3$) matrix W_n is of rank 2. It is only a necessary condition which does not ensure the convergence of the 3D lines, but it makes very important the accurate estimation of the imaged cylinder axis (lines l_j), hence the estimation of its Plücker coordinates.

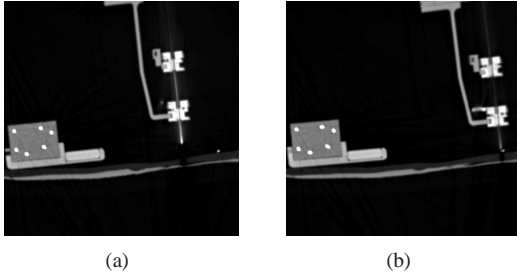


Fig. 8. "Look an move" with CT scanners. (a) The needle is maintained in the needle-holder jaws of a lightweight parallel robotic platform (CT-Bot), which has to be moved to the target point (right). (b) A new acquisition to check for the final positioning.

IV. POSE ESTIMATION OF STEREOTACTIC LANDMARKS

This section deals with the 2-D/3-D registration of a stereotactic frame from a single slice captured with a computed tomography (CT) scanner. A registration with a single image is very well suited for CT-guided robotic systems in interventional radiology, particularly to quickly correct the needle positioning (see figure 8) during percutaneous procedures [27].

In stereotaxy, line fiducials are usually used to produce a set of image points that are further employed in a pose estimation algorithm. To achieve the registration, the matching and pose estimation processes need to be robust and fast enough so as to be convenient in clinical conditions. To this end, a new formulation of the patient-to-modality stereotactic registration with a single image and for any arrangement of the fiducials has been proposed. It is worth pointing out that our solution requires very few fiducials in comparison with previous techniques.

A. The imaging model

Since most CT imaging devices execute some proprietary algorithms to generate image slices and since these algorithms usually are not in the public domain, we consider the imaging device as a black box. In other words, this work is focused on the delivery of a general framework for 2-D/3-D registration rather than a study of the physical properties of each step of image formation. A CT scanner provides slices of objects. It has internal parameters such as the thickness of a slice and scaling parameters that influence the tomographic reconstruction process from the projection measurements. To take care of them, we propose an imaging model composed of an affine transformation accounting for intrinsic scaling parameters, an Euclidean one for the rigid-body transformation that relates the scanner to the stereotactic frame, and an orthographic projection that expresses the projection of a thin slice onto the CT image.

To formulate the registration, we denote with \mathcal{F}_0 the reference frame attached to the fiducials and with \mathcal{F}_{ct} the frame attached to the scanner. A scaled frame \mathcal{F}_I is also attached to the CT image with pixel units instead of millimeters. A point in space like the origin of a reference frame is written in bold as \mathbf{O} . The imaging model relates the coordinates

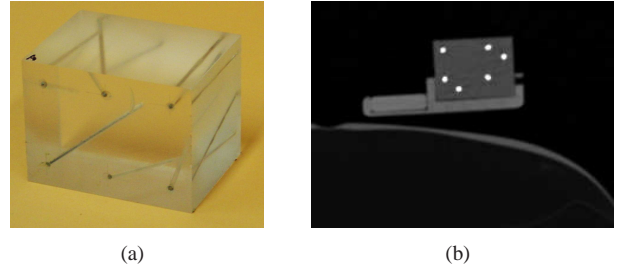


Fig. 9. (a) The plastic cube with the line fiducials used for experiments. - (b) A CT scanner image (magnified) when the cube is placed on a phantom.

of a 3-D point \mathbf{P}_j expressed in \mathcal{F}_{ct} and coordinates of the corresponding point ${}^I\mathbf{Q}_j = [u_j \ v_j]^T$ in the image as

$$\begin{aligned} {}^{ct}\mathbf{O}\mathbf{P}_j &= \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\text{orthographic}} \underbrace{\begin{bmatrix} s_x & g \\ 0 & s_y \end{bmatrix}}_{\text{intrinsic parameters}} \underbrace{\begin{bmatrix} u_j \\ v_j \end{bmatrix}}_{\text{pixel}} \\ &= {}^{ct}\Pi_\pi \pi S_I {}^I\mathbf{Q}_j \end{aligned} \quad (27)$$

where ${}^{ct}\Pi_\pi$ is a (3×2) matrix accounting for the orthographic projection onto the cutting plane (π) and the non-null entries of matrix πS_I are the intrinsic parameters of the scanner. They consist of two scaling factors s_x and s_y and a shearing parameter g accounting for a gantry tilt angle error or table bending during the scan. Usually, this parameter is very small and it is often neglected. However, it may be identified, since in some circumstances, it may decrease the registration accuracy or for MRI [4]). In the rest of the paper, this parameter will be neglected.

Since there exists a rigid-body transformation between \mathcal{F}_0 and \mathcal{F}_{ct} , the expression for the vector $\mathbf{O}\mathbf{P}_j$ in \mathcal{F}_0 is given by ${}^0\mathbf{O}\mathbf{P}_j = R {}^{ct}\mathbf{O}\mathbf{P}_j + \mathbf{t}$ where R is a rotation matrix and \mathbf{t} is a position vector. Then, one may see the following expression

$${}^0\mathbf{O}\mathbf{P}_j = [\mathbf{r}_1 \ \mathbf{r}_2] \pi S_I {}^I\mathbf{Q}_j + \mathbf{t} \quad (28)$$

as a compact representation for the transformation $\mathcal{F}_I \rightarrow \mathcal{F}_0$ including the orthographic projection, where \mathbf{r}_k is the k^{th} column of R . Therefore, the following (3×2) real matrix

$${}^0L_I = [\mathbf{r}_1 \ \mathbf{r}_2] \pi S_I = [\mathbf{l}_1 \ \mathbf{l}_2] \quad (29)$$

must satisfy the quadratic constraints coming from the orthonormality of any rotation matrix ¹:

$$\mathbf{l}_1^T \mathbf{l}_1 = s_x^2, \quad \mathbf{l}_2^T \mathbf{l}_2 = s_y^2 \quad \text{and} \quad \mathbf{l}_1^T \mathbf{l}_2 = 0. \quad (30)$$

Finally, considering the notations for homogenous coordinates of ${}^I\mathbf{Q}_j$ as ${}^I\underline{\mathbf{Q}}_j = (u_j, v_j, 1)^T$, (28) is rewritten as

$${}^0\mathbf{O}\mathbf{P}_j = [{}^0L_I \ \mathbf{t}] {}^I\underline{\mathbf{Q}}_j. \quad (31)$$

¹In accordance with (27), if the shearing parameter g is significant, equation (30) should be replaced by $\mathbf{l}_1^T \mathbf{l}_1 = s_x^2$, $\mathbf{l}_1^T \mathbf{l}_2 = s_x g$ and $\mathbf{l}_2^T \mathbf{l}_2 = g^2 + s_y^2$.

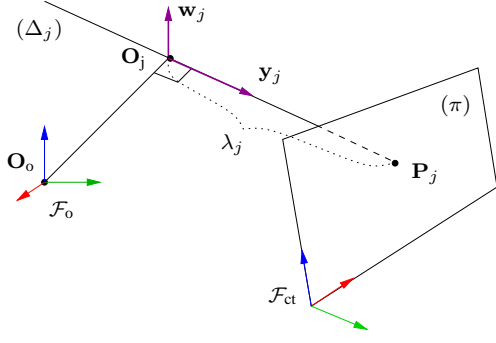


Fig. 10. A 3-D line Δ_j crossing the cutting plane (π) . The pair of vectors $(\mathbf{y}_j, \mathbf{w}_j)$ is the Plückerian representation of the line.

B. Modeling the fiducials

Fiducials used in stereotaxy are usually composed of rods (see figure 9.a) and are represented with straight lines ([13], [23], [35]). Let Δ_j be the j^{th} line. This line may be represented with the origin \mathbf{O}_j (3 dof) and a unit vector \mathbf{y}_j (2 dof). Its intersection with the scanner plane (π) is (generally) a point $\mathbf{P}_j = \Delta_j \cap \pi$ (see figure 9.b), and substituting the expression of ${}^0\mathbf{OP}_j$ in (31), it can be expressed with

$${}^0\mathbf{OP}_j = {}^0\mathbf{OO}_j + \lambda_j {}^0\mathbf{y}_j = [{}^0L_I \quad \mathbf{t}] {}^I\mathbf{Q}_j, \quad \lambda_j \in \mathbb{R} \quad (32)$$

where \mathbf{O}_j is the orthogonal projection of the origin of the frame \mathcal{F}_0 onto Δ_j , thus satisfying $\mathbf{O}_j\mathbf{P}_j \times \mathbf{y}_j = \mathbf{0}$. Therefore, to achieve the registration, one must solve (32) for 0L_I , \mathbf{t} and the $\{\lambda_j\}$'s, that is for $(9+n)$ unknowns with n lines. Consequently, the size of the system to solve increases with the number of rods, leading to large matrices which must be handled with many numerical operations [26]. To reduce the number of unknowns, we introduce the Plückerian representation [16]. The Plückerian coordinates of a 3-D line Δ_j are the pair of orthogonal vectors $(\mathbf{y}_j, \mathbf{w}_j)$ (see figure 10) where \mathbf{w}_j is defined by $\mathbf{w}_j = \mathbf{y}_j \times \mathbf{OP}_j$. With this representation, the origin of the line as well as the $\{\lambda_j\}$'s are removed from the system.

The above definition and the latter expression for ${}^0\mathbf{OP}_j$ can be gathered in the following equation, expressed in \mathcal{F}_0 :

$$[{}^0\mathbf{y}_j]_{\times} [{}^0L_I \quad \mathbf{t}] {}^I\mathbf{Q}_j = {}^0\mathbf{w}_j, \quad (33)$$

where $[y_j]_{\times}$ is the (3×3) skew-symmetric (singular) matrix associated to \mathbf{y}_j . Equation (33) is the basis for our registration approach.

Generally, intersections of straight lines with the cutting plane should provide as many spots as there are lines (see figure 9.b). In practice, several spots may be missing in the image or in contrary some artifacts may appear [23]. In practice, line fiducials are bounded ($\lambda_j^{\min} \leq \lambda_j \leq \lambda_j^{\max}$). It is easy to compute these extremal values for any displacement (R, \mathbf{t}) and to check the relevance of the corresponding spot. To do so, a pre-multiplication with a unit vector ${}^0\mathbf{y}_j^T$ in (32) gives the following expression

$$\lambda_j = {}^0\mathbf{y}_j^T [{}^0L_I \quad \mathbf{t}] {}^I\mathbf{Q}_j. \quad (34)$$

Given n lines/points correspondences, (33) can be expressed as a minimization problem with equality constraints:

$$\min_{\mathbf{x}} \|\mathbf{A} \mathbf{x} - \mathbf{b}\|^2 \quad \text{subject to} \quad \mathbf{x}^T \mathbf{C} \mathbf{x} = 0 \quad (35)$$

where $\mathbf{x} = [\mathbf{1}_1^T \quad \mathbf{1}_2^T \quad \mathbf{t}^T]^T$, \mathbf{C} is a (9×9) symmetrical matrix with null entries except for $C_{14} = C_{25} = C_{36} = C_{41} = C_{52} = C_{63}$, \mathbf{A} is a $(3n \times 9)$ and \mathbf{b} is a $(3n \times 1)$ matrix, respectively defined as:

$$\mathbf{A} = \begin{bmatrix} {}^I\mathbf{Q}_1^T \otimes [{}^0\mathbf{y}_1]_{\times} \\ \vdots \\ {}^I\mathbf{Q}_n^T \otimes [{}^0\mathbf{y}_n]_{\times} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} {}^0\mathbf{w}_1^T \\ \vdots \\ {}^0\mathbf{w}_n^T \end{bmatrix}. \quad (36)$$

C. Registration as a pose estimation problem

This section aims at designing fast algorithms for estimating the parameters of the rigid registration, assuming a calibrated scanner is available (see [26] for uncalibrated scanners). We tackle this rigid registration problem (recovery of R and \mathbf{t}) with the minimum number of fiducials needed and by means of a linear algorithm. Given a single image, and considering the unknown vector $\boldsymbol{\xi} = [\mathbf{r}_1^T \quad \mathbf{r}_2^T \quad \mathbf{t}^T]^T$, (33) becomes

$$[{}^I\mathbf{Q}_j^T \otimes [{}^0\mathbf{y}_j]_{\times}] S_9 \boldsymbol{\xi} = {}^0\mathbf{w}_j, \quad (37)$$

where $S_9 = \begin{bmatrix} {}^{\pi}S_I^T \otimes I_3 & 0 \\ 0 & I_3 \end{bmatrix}$. With exactly 4 lines/points correspondences, (37) is a deficient-rank system which can be solved thanks to rotations properties. Except for some arrangements of the fiducials enumerated in [25], the matrix \mathbf{A} has rank 8 when components contain uncorrupted data. However, with noisy data, the rank may be greater than 8. Hence, we wish to enforce the rank value because of the matrix structure (it is built with singular matrices). Therefore, there is a one-parameter family of solutions and (37) may be solved with the the Singular Value Decomposition (SVD). We summarize it as follows:

- 1) Find the SVD of \mathbf{A} : $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T$, where the diagonal entries d_i of \mathbf{D} are in descending numerical order,
- 2) Set $\mathbf{b}' = [b'_1 \quad b'_2 \quad \dots \quad b'_9]^T = \mathbf{U}^T \mathbf{b}$ (see (36)),
- 3) Build the vector \mathbf{z} defined by $z_i = b'_i/d_i$, for $i = 1, \dots, 8$ and $z_9 = 0$,
- 4) The general solution is $\boldsymbol{\xi} = S_9^{-1} (\mathbf{V}\mathbf{z} + \gamma \mathbf{v}_9)$, where \mathbf{v}_9 is the last column of \mathbf{V} .
- 5) Compute γ with the quadratic relations between $\mathbf{r}_1 = [\xi_1 \quad \xi_2 \quad \xi_3]^T$ and $\mathbf{r}_2 = [\xi_4 \quad \xi_5 \quad \xi_6]^T$.

Equation (37) can be solved provided that all combinations of triplets verify the conditions mentioned in previous section.

Because of the presence of noise, R is not exactly a rotation matrix. One may enforce R to be a rotation by computing the SVD, $R = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$ and by setting eigenvalues to 1. If R' is the corrected rotation matrix, it is given by $R' = \mathbf{U} \text{diag}(1, 1, \det(\mathbf{UV})) \mathbf{V}^T$ ([12], [40]).

A Newton-Raphson (N-R) numerical approach has also been carried out. It uses the initial guesses provided by the

above least-squares method (LS) but we do not describe it here (see [26] for details).

D. Experimental validations

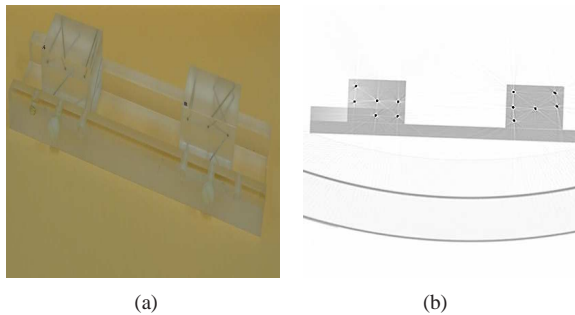


Fig. 11. (a) The two cubes used for the experiments. - (b) image picked up from a CT helicoidal sequence with image size of (512×512) pixel.

Experiments were conducted with a Siemens Somatom Plus CT scanner and with fiducials composed of two cubes with six rods each (see figure 11). The relative position of the two cubes is constrained to by a guide rail on which the cubes are screwed. Each cube of line fiducials has been calibrated as well as their relative position with a Mitutoyo measuring machine which can achieve a precision of $10 \mu\text{m}$.

We have assessed the accuracy of the relative pose recovery by registering only one fiducial cube at a time, and by computing the relative position and rotation. By doing so, it is possible to verify the constancy of the pose estimation between two coordinate frames with a single image. To this end, a helicoidal sequence has been captured while a constant translation of the table is performed. In figure 12, we present the estimated position vector between the two cubes, as the orientation is approximately the identity matrix (it differs to the identity matrix by less than 10^{-5} on each component). The registration has been executed for each cube independently and once it has been done for both (with the LS method and the N-R minimization), the relative position and orientation have been computed. The position $\mathbf{T} = [0 \quad -118.29 \quad 0] \text{ mm}$ was measured during a calibration procedure (orientations are equal for the two cubes' reference frames). Thus, this value (dotted line in figure 12) can be compared to the estimations.

As shown in figure 13, the 3-D pose algorithm works very well on experimental data, since the registration of each cube can serve to predict the position of the rods of the other cube. The estimated error bounding-boxes with an assumed spot location error of 0.25 pixel are also represented. As illustrated, all the detected spots are inside the box with boundaries corresponding to 3-D position errors always less than 2 mm.

V. CONCLUSION

In this paper, we have described some pose estimation problems by means of intra-operative images. We focused the works on endoscopic views for assisted laparoscopy

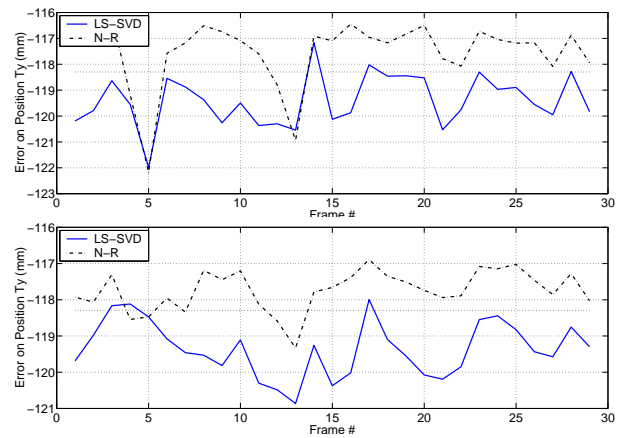


Fig. 12. Relative positions between the cubes during the acquisition (slice thickness is 0.5 mm) while translating the table. The first plot (up) is with 5 fiducials for the estimation while the second (down) is with 6 fiducials.

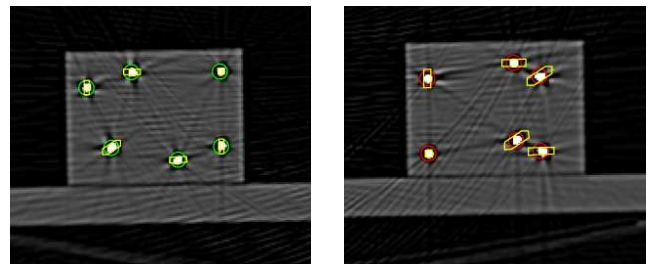


Fig. 13. CT images of the cubes. Each yellow box in one cube is corresponding to 2 mm of error bounds computed with the registration of the other cube.

and CT images slices with X-ray scanners for the image-guidance in interventional radiology.

For vision-based integrated systems used in minimally invasive surgery, we have developed a set of techniques for assisting surgeons in navigating and manipulating the three-dimensional space within the human body. To that purpose, simple geometrical features have been attached to surgical instruments. Alternatively, when the task is sufficiently constrained by the shape of object of interest, we directly solve the pose estimation problem without markers : it is the case for the 4 dofs of a cylindrical needle-holder within the human abdomen.

One path toward safety and reliability is to incorporate all the available video information. Following this road, the virtual visual servoing has been used to combine both the apparent contour of the instrument and artificial markers in a numerical iterative process.

The recovery of out-of-field of view instrument in laparoscopy, the automatic suturing intervention demonstrated in vitro [31], the positioning of a radio-frequency needle with CT scanners [25] are some applications we contribute with the provided solutions for the aforementioned pose problems. Finally, we believe that significant advances are possible when the geometric information is fused across time and across modality. Furthermore, pre-operative informations

like the insertion points localization, the CAD model of instruments, the eye-to-hand calibration or the availability of several statistical atlases of organs can provide some strong constraints on the vision problem. These are crucial factors to achieve reliable dedicated vision systems while compensating small displacements due to patient breathing or any small disturbances which may occur during an image-guided surgical procedure.

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