Feature points tracking: robustness to specular highlights and lighting changes

Michèle Gouiffès[1], Christophe Collewet[2], Christine Fernandez-Maloigne[1], and Alain Trémeau[3]

[1] SIC, University of Poitiers, 86962 Futuroscope, France
gouiffes@sic.sp2mi.univ-poitiers.fr
[2] IRISA/INRIA Rennes 35042 Rennes cedex, France
[3] LIGIV, University Jean Monnet 42000 Saint Etienne, France

Abstract. Since the precise modeling of reflection is a difficult task, most feature points trackers assume that objects are lambertian and that no lighting change occurs. To some extent, a few approaches answer these issues by computing an affine photometric model or by achieving a photometric normalization. Through a study based on specular reflection models, we explain explicitly the assumptions on which these techniques are based. Then we propose a tracker that compensates for specular highlights and lighting variations more efficiently when small windows of interest are considered. Experimental results on image sequences prove the robustness and the accuracy of this technique in comparison with the existing trackers. Moreover, the computation time of the tracking is not significantly increased.

1 Introduction

Since many algorithms rely on the accurate computation of correspondences between two frames through an image sequence, feature tracking has proved to be an essential component of vision systems. Indeed, many high level tasks can depend highly on it, such as 3D reconstruction, active vision or visual servoing for example. Nevertheless, robust feature tracking is still a problem to be addressed. It becomes far more complicated when no mark (edges or lines for example), can be extracted from the observed object, such as in natural environment [1]. In such a context, only points, among other possible features, are likely to be easily detectable. However, tracking a point into an image sequence is not a trivial task since the only available information is the luminance of the point and of its neighboring pixels. The seminal works in this domain are due to Lucas and Kanade [5, 9] who assume the conservation of the point luminance during the image sequence [3]. The measure of a correlation function between two successive frames determines the translation motion undergone by the point. Thereafter, some more robust tracking approaches have been proposed [8, 10]. However, such methods still assume that the luminance remains constant between two successive frames, which is often wrong. Indeed, most surfaces are not Lambertian and lighting conditions are mostly variable during an image sequence. To solve this problem, Hager and Belhumeur [2] acquire an image data base of the scene under several illuminations and use these data to improve the tracking. This method is efficient but requires a prior learning step, which can be seen as too restrictive. An easier way to cope with illumination changes is to achieve a photometric normalization as in [10] for example. In [4], the tracking task compensates for affine illumination changes by computing the contrast and illumination variations during the whole image sequence. These two methods will be detailed in Section 3.

In this paper, we propose a new feature point tracking algorithm, based on the study of reflection models, which is robust to specular highlights occurrence and lighting changes. In addition, we show clearly on which assumptions the approaches mentioned above are based. Besides, we will see that the proposed algorithm provides a more appropriate model of the illumination changes, particularly for specular surfaces.

This article is structured as follows. Section 2 focuses on the modeling of luminance changes, especially in cases of specular reflections and lighting variations. Section 3 details some of the existing tracking approaches: the Shi-Tomasi-Kanade tracker [5,8,9], the normalized one [10] and the tracker with an affine illumination compensation [4]. Thereafter, section 4 describes the proposed tracking method. To finish, section 5 shows experimental results, in order to compare the different tracking techniques according to their robustness and accuracy. In addition, this section will prove the efficiency of our approach.

2 Modeling of luminance changes

Suppose f and f' to be respectively the images of an object acquired at two different times. A point P of this object projects in image f to p of coordinates (x_p, y_p) and to p' of coordinates (x'_p, y'_p) in the image f' after a relative motion between the camera and the scene. The luminance at p depends on the scene geometry. Fig.1 describes the vectors and the angles used in this paper. $\mathbf{V}$ and $\mathbf{L}$ are respectively the viewing and the lighting directions, which form the angles θ_r and θ_i with the normal $\mathbf{n}$ in P. $\mathbf{B}$ is the bisecting line between $\mathbf{V}$ and $\mathbf{L}$, it forms an angle ρ with the normal $\mathbf{n}$. According to the most widely used reflection models, such as the Torrance-Sparrow [11] and the Phong [7] ones, the luminance at p can be described as follows

$$f(p) = K_d(p)a(p)\cos\theta_i(P) + h_f(p) + K_a \tag{1}$$

where K_a is the intensity of ambient lighting and K_d a diffuse coefficient corresponding to the direct lighting intensity. These values depend also on the gain of the camera. The term $a(p)$ is related to the *albedo*[4] in P. The function h_f expresses the contribution of the specular reflection, which vanishes in case of a

[4] The albedo is the ratio of the amount of light reflected by a small surface in P to the amount of incident light. It depends only on the material and its texture.

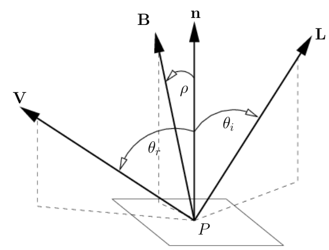


Fig. 1. Vectors and angles involved in the reflection description.

pure diffuse reflection, that is for Lambertian objects. Consequently, with such objects and for a given lighting direction $\mathbf{L}$, the luminance at p remains constant whatever the viewing direction $\mathbf{V}$ is. Phong describes in [7] the specular reflection as a cosine function of ρ

$$h_f(p) = K_s(p)\cos^n(\rho(P)) \tag{2}$$

where n is inversely proportional to the roughness of the surface and K_s is the specular coefficient of the direct lighting. Torrance-Sparrow [11] describes h_f by an exponential function depending on ρ and on the surface roughness ς. For each model, h_f reaches a maximum value for $\rho(P) = 0$, that is when $\mathbf{B}$ coincides with $\mathbf{n}$. Let us also notice that the specular reflection depends on the roughness, the lighting and the viewing directions.

After a relative motion between the camera and the scene, when no lighting change occurs, θ_i, K_d and K_a are constant at P during the time. In the same way, the albedo is constant at P leading to $a(p') = a(p)$. However, the specular component h_f, which depends on the viewing direction, may vary strongly during the motion of the camera. In those conditions, the luminance f' is given by

$$f'(p') = K_d(p)a(p)\cos\theta_i(P) + h_{f'}(p') + K_a \tag{3}$$

where $h_{f'}$ is the specular function.

Now, let us consider that some lighting shifts ΔK_a, ΔK_d and $\Delta\theta_i$ are respectively provoked on K_a, K_d and θ_i. Thus, the luminance can be expressed as

$$f'(p') = K_d^{'}(p)a(p)\cos\theta_i^{'}(P) + K_a^{'} + h_{f'}(p') \tag{4}$$

with $K_d^{'}(p) = K_d(p) + \Delta K_d(p)$, $\theta_i^{'}(P) = \theta_i(P) + \Delta\theta_i(P)$ and $K_a^{'} = K_a + \Delta K_a$. The specular term $h_{f'}(p')$ includes the intensity change of the specular coefficient K_s if necessary. From (4), the next section will clearly show the assumptions on which the most widely used tracking methods are based.

3 Analysis of existing tracking methods according to their robustness to illumination changes

Let be m a point located in a window of interest $\mathcal{W}$ of size $\mathcal{N} \times \mathcal{N}$ centered around p. Point m is the projection of a physical point M in f and m' is its projection in f'. The tracking process consists in computing a motion model δ parameterized by a vector $\mathbf{A}$ between f and f'. According to the tracking method, the assumptions about the photometric model are different. By eliminating $a(m)$ between (1) and (4), it yields to the following relationship between two images of the same sequence $f'(m') = \lambda(m)f(m) + \eta(m)$. This relationship is found in [6] in an optical flow context, but λ and η are supposed to be constant locally. According to our analysis based on the reflection models, $\lambda(m)$ and $\eta(m)$ are expressed by:

$$\lambda(m) = \frac{(K_d(m) + \Delta K_d(m))\cos(\theta_i(M) + \Delta\theta_i(M))}{K_d(m)\cos\theta_i(M)} \tag{5}$$

$$\eta(m) = -(h_f(m) + K_a)\lambda(m) + h_{f'}(m') + K_a + \Delta K_a \tag{6}$$

From these relations we deduce the assumptions on which the classical trackers are based.

The classical approach. The classical point feature tracker [5, 8, 9] assumes a perfect conservation of luminance at point M during the sequence: $f'(m') = f(m)$, $\forall m \in \mathcal{W}$ and $\forall \mathcal{W}$. Owing to owing to (5) and (6), that implies $\lambda(m) = 1$ and $\eta(m) = 0$ $\forall m \in \mathcal{W}$, which is correct when no lighting change occurs ($\Delta\theta_i(M) = 0$, $\Delta K_d(m) = 0$ $\forall m \in \mathcal{W}$) and when objects are strictly lambertian ($h_f(m) = h_{f'}(m') = \Delta K_a = 0$, $\forall m \in \mathcal{W}$ and $\forall \mathcal{W}$). Because of noise and because of the strong assumptions considered on the motion and photometric models, it is more suitable to minimize the following criterion

$$\epsilon_1(\mathbf{A}) = \sum_{m\in\mathcal{W}} \left(f(m) - f'(\delta(m, \mathbf{A})\right)^2 \tag{7}$$

This approach leads to good results in most cases but can suffer from lighting changes and specular highlights occurrence. In order to cope with this problem, a photometric normalization can be performed.

Use of an affine photometric model. In [4] the authors propose a tracking method in order to compensate for contrast and intensity changes by computing the parameters of the following criterion

$$\epsilon_2(\mathbf{A}, \lambda, \eta) = \sum_{m\in\mathcal{W}} \left(\lambda f(m) - f'(\delta(m, \mathbf{A})) - \eta\right)^2 . \tag{8}$$

According to (5), $\lambda(m)$ is supposed to be constant at each point of $\mathcal{W}$. That is correct for any surface curvature (and then $\forall\Delta\theta_i$ and $\forall\theta_i$) and for any lighting ($\forall K_d$ and $\forall\Delta K_d$) only if each function $\Delta\theta_i$, θ_i, ΔK_d, K_d is constant in $\mathcal{W}$. Then, it assumes that $\eta(m)$ is constant at each point of $\mathcal{W}$, which is correct for any surface curvature and for any roughness when both $h_{f'}(m)$ and $h_f(m)$ are constant at each point of $\mathcal{W}$.

As a conclusion, this technique assumes that the illumination changes are the same at each point of $\mathcal{W}$. Practically, that can be a coarse assumption when the surface projected onto $\mathcal{W}$ is non planar.

The photometric normalization approach. This approach is based on the minimization of the criterion $\epsilon_2(\mathbf{A})$ except that λ and η are measured at each step of the minimization process instead of being computed simultaneously with the motion parameter $\mathbf{A}$. Their values are $\lambda = \frac{\sigma_{f'}}{\sigma_f}$ and $\eta = \mu_{f'} - \frac{\sigma_{f'}\mu_f}{\sigma_f}$, where μ_f and $\mu_{f'}$ are the average values respectively of f and f' in $\mathcal{W}$ and σ_f, $\sigma_{f'}$ are the standard deviations. λ and η are supposed to be constant at each point m of $\mathcal{W}$, so that the same assumptions as the previous technique hold. Indeed, in those conditions, the values μ_f and σ_f are given by

$$\begin{cases} \mu_f = K_d \cos(\theta_i)\mu_a + K_a + h_f \\ \sigma_f = K_d \cos(\theta_i)\sigma_a \end{cases} \tag{9}$$

where μ_a is the average value of a and σ_a its standard deviation. Consequently, $f(m) - \mu_f$, $a(m) - \mu_a$ and $f'(m') - \mu_{f'}$ are invariant to highlights occurrence. Finally we show easily that the following ratios are also invariant to ambient and direct lighting changes, and to gain variation

$$(f(m) - \mu_f)/\sigma_f = (a(m) - \mu_a)/\sigma_a = (f'(m') - \mu_{f'})/\sigma_{f'},\ \forall m \in \mathcal{W}. \tag{10}$$

Nevertheless, these properties are true only if the specular reflection and the lighting changes are the same at each point of $\mathcal{W}$, as it as been mentioned above. In some cases, these assumptions are not realistic, particularly when $\mathcal{W}$ is the projection of a non planar surface of the scene. In addition, these values can be ill-defined when $\sigma_a \approx 0$, that is when the intensities almost saturate or more generally when they are almost homogeneous in $\mathcal{W}$.

Our method is described in the next section. It states the assumption that the illumination changes can be approximated by a continuous function on $\mathcal{W}$.

4 The proposed approach

It has been shown in section 2 how each kind of illumination changes can be expressed. By considering (1) and (4), we immediately obtain the relationship between f' and f

$$f'(\delta(m, \mathbf{A})) = f(m) + \psi(m) \tag{11}$$

In the general case the photometric change is written

$$\begin{aligned} \psi(m) = a(m)(K'_d(m)\cos(\theta_i(M) + \Delta\theta_i(M)) - K_d(m)\cos\theta_i(M)) + \\ h_{f'}(\delta(m, \mathbf{A})) - h_f(m) + \Delta K_a \end{aligned} \tag{12}$$

When the lighting (or the gain of the camera) does not vary, the total temporal change ψ in the specular component at a fixed scene point m is equal to

$$\psi(m) = h_{f'}(\delta(m, \mathbf{A})) - h_f(m) \tag{13}$$

According to the most widely used reflection models (see (2) for example), the function ψ is variable on $\mathcal{W}$ since it depends on the viewing and lighting angles and therefore on the normal $\mathbf{n}$ at each point of $\mathcal{W}$. When lighting changes are caused, it depends also on the albedo. We suppose that this function can be approximated by a continuous and derivable function ϕ on $\mathcal{W}$. In that case, a Taylor series expansion can be performed at m in a neighborhood of p leading to the following expression for $\phi(m)$ by neglecting the higher order terms

$$\phi(m) = \phi(p) + \left.\frac{\partial \phi}{\partial x}\right|_p (x - x_p) + \left.\frac{\partial \phi}{\partial y}\right|_p (y - y_p). \tag{14}$$

Finally, by using (14) in (11) with $m = (x, y)^T$ and noting $\alpha = \left.\frac{\partial \phi}{\partial x}\right|_p$, $\beta = \left.\frac{\partial \phi}{\partial y}\right|_p$ et $\gamma = \phi(p)$, the proposed tracker consists in computing the motion parameter $\mathbf{A}$ and the reflection parameters $\mathbf{B} = (\alpha, \beta, \gamma)^T$ by minimizing the following criterion:

$$\epsilon_3(\mathbf{A}, \mathbf{B}) = \sum_{m \in \mathcal{W}} \left(f(m) - f'(\delta(m, \mathbf{A})) - \mathbf{U}^T \mathbf{B}\right)^2 \tag{15}$$

where $\mathbf{U} = (x - x_p, y - y_p, 1)^T$. Contrary to the previous approaches, this method does not make assumptions about the scene. In particular, the incident angle θ_i, the viewing angle θ_r, the parameter K_s and the roughness n (or ς) can vary. Therefore, specular highlights and lighting changes can be different at each point of $\mathcal{W}$. However, when lighting changes occur, the method assumes that the values of the albedo can be approximated by a polynomial of first degree in $\mathcal{W}$. Only in that case, the proposed approach is more adapted for small windows of interest $\mathcal{W}$. In the next section, the different tracking methods are compared through experiments.

5 Experimental results

In the following experiments, the tracking methods are based on the computation of an affine motion model between the first frame and the current one. The tracking algorithm integrates an outlier rejection module, based on the analysis of the residues convergence ϵ_i, $i = 1 \ldots 3$. A point is rejected of the tracking process as soon as its residues become greater than a threshold $\mathcal{S}_{conv} = \mathcal{N}^2 E_{ave}^2$, where E_{ave} is the tolerated intensity variation for each point in $\mathcal{W}$ between f and f'. In these experiments, $E_{ave} = 15$. We consider some sizes from $\mathcal{N} = 9$ to $\mathcal{N} = 13$. In each case, the sequence is played from the first image to the last one and then from the last image to the first one, in order to evaluate the symmetry of the residuals and photometric curves. To compare the trackers we compute several criteria: 1) the *robustness* of the tracking, that is to say the number of points that have been tracked during the whole sequence; 2) the *accuracy* of the tracking: we compute the average of the residuals for all the points that have been tracked during the whole sequence by the considered method. This criterion provides information about the relevance of the photometric model. The lower the residues are, the better the illumination variations are compensated for; 3) the reflection parameters α, β and γ, computed by the parametric method. We will also compare the computation time and the conditioning of the matrices used in the minimization algorithm. Moreover, simulations results compare the accuracy of the tracking in terms of position estimation. In order to simplify the explanations of results, we introduce the following notations: C (classical approach), J (affine model compensation), T (photometric normalization) and P (proposed approach).

First, the tracking algorithms are tested on scenes showing specular highlights. For each image sequence, the scene (objects and lighting) is motionless, only the camera moves. The scenes are lighted by a direct and an ambient lighting.

• **Experiment *Book***. The first sequence (see Fig. 2a) shows a book. The motion of the camera leads to specular highlights which appear and disappear during the sequence. A number of 97 points has been selected in the first frame but 11 points are lost because they are occluded by the book or because they get out of the camera field of view. The number of points correctly tracked is counted in table 1a. These results prove that the proposed approach (P) is far more robust than the existing ones (C, J and T) since the number of tracked points is always much larger. Fig. 2b compares the average residuals obtained by the trackers during the sequence for $\mathcal{N} = 9$. Until the 100^{th} frame, P obtains the lowest convergence residuals: the proposed photometric model fits best to the specular occurrence. After the 100^{th} frame, the residuals of T become lower than the P ones. However, these values are computed by averaging the residuals of 33 points for T and 68 points for P. In order to compare correctly these two approaches, let us consider Fig. 2c, which shows the average residuals computed only on the few points that are correctly tracked by T and P simultaneously. Here, the residuals are lower for the proposed approach. That shows the good adequacy of the local model proposed to compensate for specular highlights. Let us notice that J is less convincing than T even though these two methods are based on the same photometric assumptions. As it will be shown later on, this is due to the ill-conditioning of J. Fig. 2d depicts the behavior of the photometric parameters that have been computed by P. Let us notice that these curves are perfectly symmetric, in agreement with the symmetry of the sequence and therefore with the symmetry of the illumination changes.

• **Experiment *Marylin***. The image sequence *Marylin* (see Fig. 3a) shows different specular objects, planar or not, lighted by the daylight and the spotlights of the room. Different types of material are considered (ceramic, glass, glossy paper, metal). This sequence is particularly noisy since the camera used has an interlaced scan mode. Besides, we can see some specular reflections es-

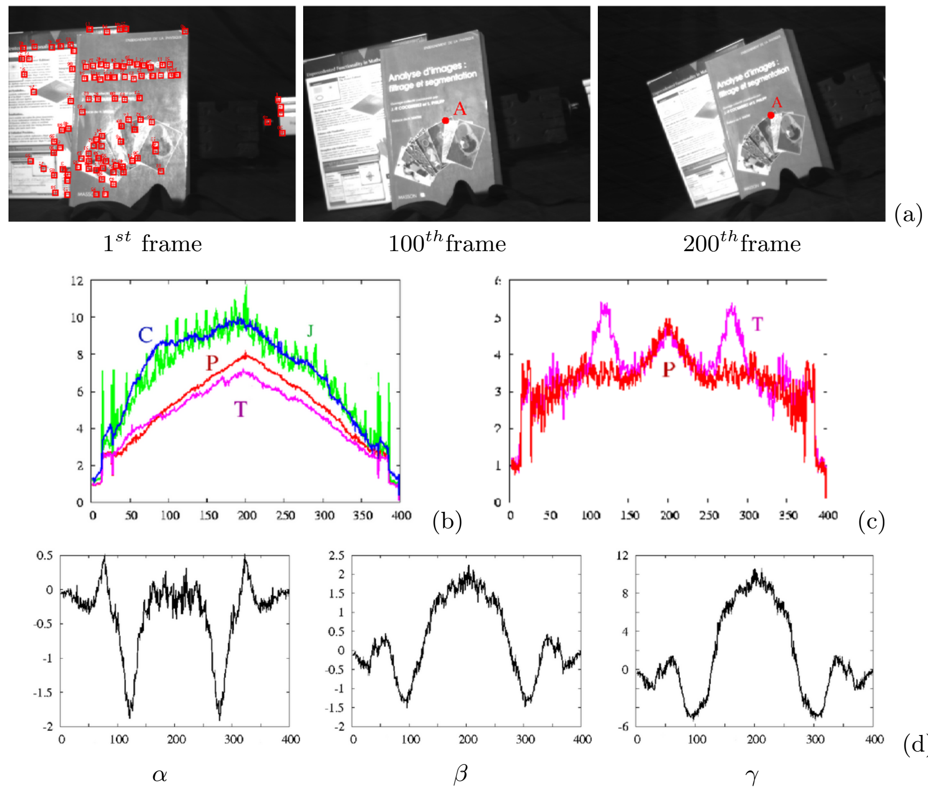
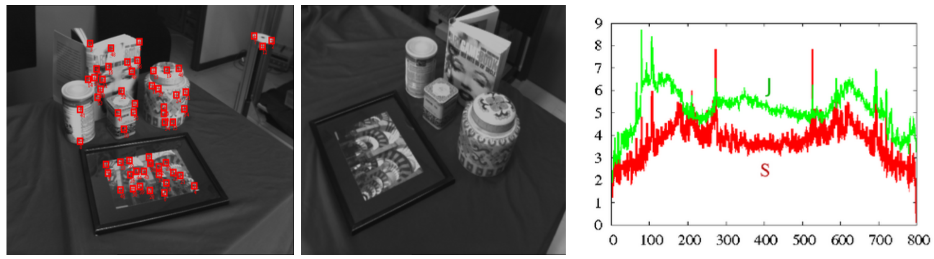


Fig. 2. Experiment *Book* (a) Three images of the sequence. (b) and (c): Residuals for $\mathcal{N}$=9, (b) is the average of the residual computed on all the points tracked and (c) the average of the residuals for the points that are correctly tracked by T et P simultaneously. (d) Illumination parameters computed with the proposed approach.

pecially on the glass of the photograph. A total number of 56 points has been selected in the first frame, but a large number of points (20 points) is lost because of occlusions and noise. Table 1b collects the number of points that have been tracked until the end of the sequence by each technique. Contrary to the previous sequence, T is less powerful than J, which could mean that T is less robust to noise than J. P tracks a larger number of points in each case. As it can be seen from the residuals obtained on the points that have been tracked by the whole of the approaches (see Fig. 3b), this technique is also more accurate.

Now, let us compare the techniques on a sequence showing lighting changes.

• **Experiment *Lighting changes***. The scene consists of a specular planar object lighted by the daylight and by one direct spotlight (see Fig. 4a). Strong variations are produced on the direct lighting intensity since it varies periodically, from a minimum value to a maximum one each 10 frames. Table 1c collects the number of points that have been tracked, on 58 points selected initially. Obviously, the proposed method provides a better robustness of the tracking since



(a) 1^{st} frame (b) 400^{th} frame (c) Residuals

Fig. 3. Experiment *Marilyn* (a) and (b) Two images of the sequence. (c) Average of the residuals for $\mathcal{N}$=9 obtained by J and P.

the whole of the points are tracked. Fig. 4b shows the average residuals obtained. For this kind of illumination changes, P and T are the most accurate techniques and obtain quite similar residuals. However, Fig. 4c, compares the average residuals computed for the points that have been tracked by the two techniques simultaneously. P gets the lowest residuals. The temporal evolution of the illumination parameters is depicted on Fig. 4d. Their periodical variations correspond to the lighting changes that have been provoked.

Accuracy of the tracking. In order to evaluate the accuracy of the points positioning, we use a software that simulates the appearence of an object lighted by an ambient light and a direct spotlight, by using a Phong model (see (2)). In the simulated sequence from Fig.5a and 5b, a motionless cylinder is viewed by a moving camera. The direct light is moved, inducing some specular highlights changes. A point is rejected as soon as its error (the euclidian distance between the real position of the point, computed by the software, and its estimated position by the tracker) is greater than 0.5 pixel. 13 points are initially selected, the method C loses 13 points, T 7 points, J 6 points and P loses no point. The figure 5c shows the evolution of the average of positioning errors obtained by each technique. P obtains the lowest errors all along the sequence and the number of points that are correctly tracked is higher.

Discussions. As expected, the classical tracker described in section 3 is not robust neither to specular highlights occurrence nor to lighting variations. A large number of points are lost, sometimes the whole of the points. Obviously, the tracking with photometric normalization and the approach with an affine compensation roughly improve the results. Besides, their efficiency increases for large windows of interest. Indeed, for small windows, the computation of σ_f, σ_g and λ are sensitive to noise. Because these values are multiplied or divided by the luminance values f, an error caused on these parameters have a huge influence, and can yield to the computation of an incorrect motion parameter $\mathbf{A}$. On the other hand, it could seem surprising that J and T behave differently although they are based on the same photometric assumptions. Actually, J suffers from a bad convergence for small windows. As an example, table 2 contains

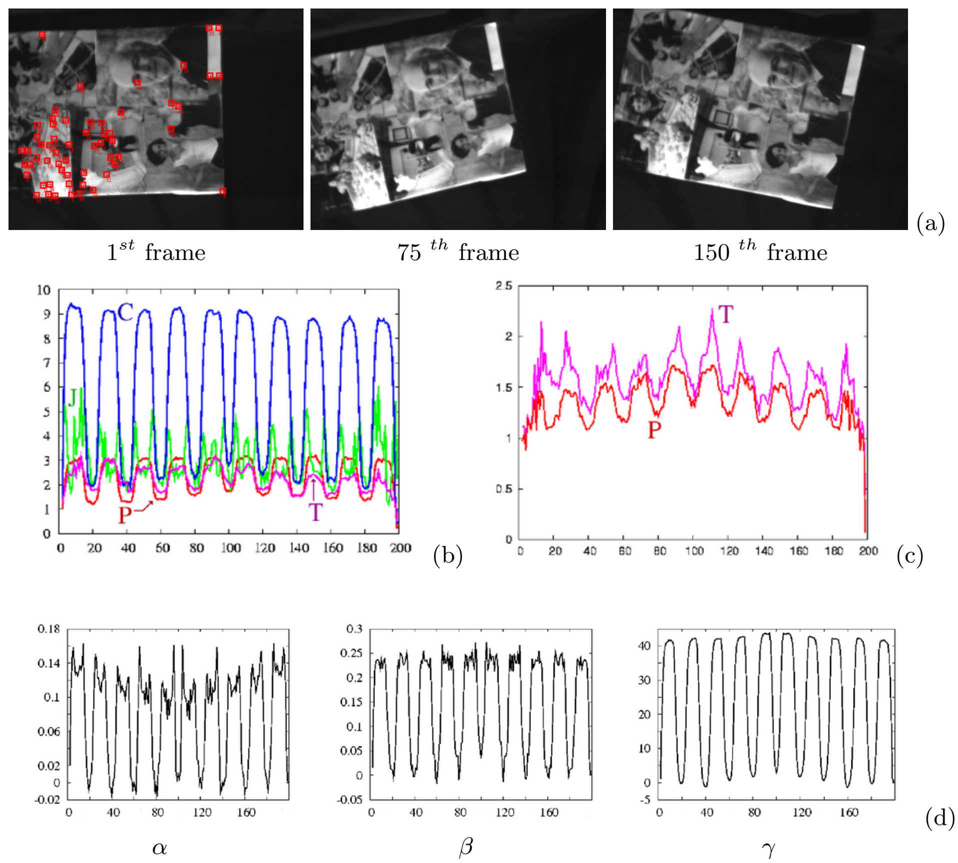


Fig. 4. Experiment *Lighting changes*. (a) Three images of the sequence. (b) and (c) Residuals for $\mathcal{N}$=9, (b) is the average of the residuals computed on all the points tracked and (c) the average of the residuals for the points that are correctly tracked by T et P at the same time. (d) Illumination parameters of point A computed with the proposed method.

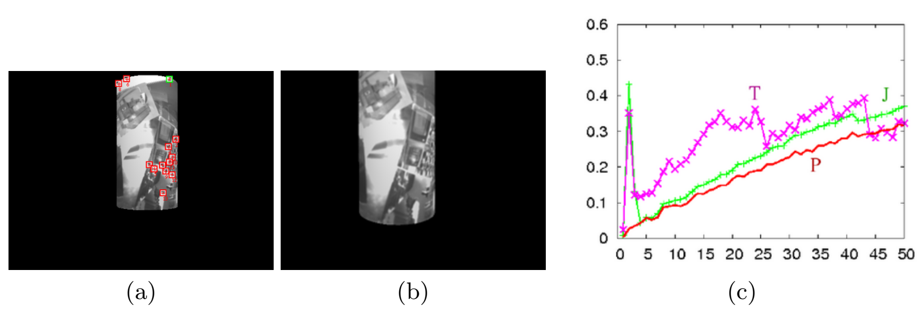


Fig. 5. Simulation. (a) and (b) Images of the sequence. (c) Average of the positioning errors (in pixels).

the ratios CN given by the ratio of the Condition Number of the tracker to the Condition Number of the classical tracker (or the T approach since it has the same condition number), computed on 10 points of the sequence *Book* and for different sizes of window. It collects the maximum (Max), the minimum (Min) and the average values (Ave) obtained. These values show clearly that J is less well-conditioned than the other approaches included ours. To finish, the method P tracks a larger number of points than the other methods. It really compensates for the variability of specular reflection and lighting variations on $\mathcal{W}$, which proves the relevance of the modeling of ψ by (14). However, let us notice that the difference of performance between the three methods (P, J and T) is less significant when lighting changes are caused than when only specular highlights occur, in particular for large windows of interest. J and T are well adapted to contrast changes, and not to specular highlights, which are generally not constant in $\mathcal{W}$. However, let us note once again that J is less efficient because of its ill-conditioning. The proposed method is perfectly adapted to specular highlights since their variations in $\mathcal{W}$ is well modeled. In case of lighting variations, the albedo must be approximated by a polynomial of first degree. However this approximation is satisfying for small windows of interest.

Up to now, we did not compare the computation time of the methods. For example, in sequence *Book*, the average tracking time of one point is 1.3ms for classical method, 1.7ms for the J technique, 4.6ms for the T one (let us note that the computation of the means and standard deviations are costly) and 1.4ms for our approach [5]. As a conclusion, the computation time is not significantly increased in comparison to the classical approach.

6 Conclusion

The existing tracking methods are based on several assumptions that had never been, or partially, specified explicitly before. In this paper, the analysis of these methods is led according to specular reflection models. Contrary to the classical approach, the tracking methods based on an affine photometric compensation are, to some extent, robust to illumination changes. However, the illumination parameters are assumed to be constant around the point to be tracked. This can be incorrect when the surfaces are non planar or when specular highlights occur. Our approach overcomes these issues. We assume that an illumination change can be approximated by a continuous and derivable function around the points to be tracked. This model is well adapted for small windows of interest, since it improves the robustness of the tracking against highlights occurrence and lighting changes. The computation duration of this method is not significantly increased in comparison with the classical technique and the accuracy of tracking is improved. In addition, its convergence properties are more satisfying than the technique involving an affine model since better conditioning numbers have been obtained. Our future work will focus on a more appropriate modeling of illumination changes for larger windows of interest.

[5] With a processor Pentium III, 1.8GHz, 512Mo RAM.

Table 1. Number of points tracked during the whole sequence versus $\mathcal{N}$.

(a) *Book* (86 points to track)

Tracker	$\mathcal{N}$=9	$\mathcal{N}$=11	$\mathcal{N}$=13
C	27	20	16
T	33	49	53
J	15	25	48
P	68	68	69

(b) *Marylin* (36 points)

Tracker	$\mathcal{N}$=9	$\mathcal{N}$=11	$\mathcal{N}$=13
C	0	0	0
T	1	3	7
J	4	8	15
P	23	21	21

(c) *Lighting changes* (58 points)

Tracker	$\mathcal{N}$=9	$\mathcal{N}$=11	$\mathcal{N}$=13
C	37	29	23
T	45	51	53
J	39	48	51
P	58	58	58

Table 2. Conditioning number on 10 points selected on sequence *Book*. Maximum (Max), minimum (Min) and average value (Ave).

Tracker	Max	Min	**Ave**
J	9	1	**3.5**
P	2	0.5	**1.4**

References

1. F.X. Espiau, E. Malis, and P. Rives. Robust features tracking for robotic applications: towards 2d1/2 visual servoing with natural images. In *IEEE Int. Conf. on Robotics and Automation, ICRA'2002*, Washington, USA, May 2002.
2. G. D. Hager and P. N. Belhumeur. Efficient region tracking with parametric models of geometry and illumination. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 20(10):1025–1039, 1998.
3. K.P. Horn and B. G. Schunck. Determinig optical flow. *Artificial Intelligence*, 7:185–203, 1981.
4. H. Jin, S. Soatto, and P. Favaro. Real-time feature tracking and outlier rejection with changes in illumination. In *IEEE Int. Conf. on Computer Vision*, pages 684–689, Vancouver, Canada, July 9-12, 2001.
5. B.D. Lucas and T. Kanade. An iterative image registration technique. In *IJCAI'81*, pages 674–679, Vancouver, British Columbia, August 1981.
6. S. Negahdaripour. Revised definition of optical flow:integration of radiometric and geometric cues for dynamic scene analysis. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20(9):961 – 979, September 1998.
7. B-T Phong. Illumination for computer generated images. *Communications of the ACM*, 18(6):311–317, June 1975.
8. J. Shi and C. Tomasi. Good features to track. In *IEEE Int. Conf. on Computer Vision and Pattern Recognition, CVPR'94*, pages 593–600, Seattle, Washington, USA, June 1994.
9. C. Tomasi and T. Kanade. Detection and tracking of point features. Technical report CMU-CS-91-132, Carnegie Mellon University, April 1991.
10. T. Tommasini, A. Fusiello, E. Trucco, and V. Roberto. Improving feature tracking with robust statistics. *Pattern Analysis & Applications*, 2:312–320, 1999.
11. K.E. Torrance and E.M. Sparrow. Theory for off-specular reflection from roughened surfaces. *Journal of the Optical Society of America*, 57(9), September 1967.