The goal of geometric numerical integration is the simulation of evolution equations preserving their geometric properties over long times. Of particular importance are Hamiltonian partial differential equations typically arising in application fields such as quantum mechanics or wave propagation phenomena. They exhibit many important dynamical features such as energy preservation and conservation of adiabatic invariants over long time. In this setting, a natural question is how and to which extent the reproduction of such long time qualitative behavior can be ensured by numerical schemes.

Starting from numerical examples, these notes provide a detailed analysis of the Schrödinger equation in a simple setting (periodic boundary conditions, polynomial nonlinearities) approximated by symplectic splitting methods. Analysis of stability and instability phenomena induced by space and time discretization are given, and rigorous mathematical explanations for these.

The book grew out of a graduate level course and is of interest to researchers and students seeking an introduction to the subject matter.