A Recursive Filter for Despeckling SAR Images

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Abstract—This correspondence proposes a recursive algorithm for noise reduction in synthetic aperture radar imagery. Excellent despeckling in conjunction with feature preservation is achieved by incorporating a discontinuity-adaptive Markov random field prior within the unscented Kalman filter framework through importance sampling. The performance of this method is demonstrated on both synthetic and real examples.

Index Terms—Importance sampling, Markov random field (MRF), speckle, synthetic aperture radar (SAR), unscented Kalman filter (UKF).

I. INTRODUCTION

The synthetic aperture radar (SAR) imaging technique has become popular because of its usability under varied weather conditions, its ability to penetrate through clouds and soil, and due to the independence of SAR image resolution with regard to sensor height. Since SAR systems rely upon coherence properties of the scattered signals, they are highly susceptible to interference effects. A SAR image is a mean intensity estimate of the radar reflectivity of the region being imaged. The difference between a particular measurement and the true mean value is referred to as speckle noise. Fully-developed speckle can be modeled as random multiplicative noise. If $s$ represents the original image and $v$ is speckle noise, then the degraded observation $y$ is given by the relation

$$y(m,n) = s(m,n) \cdot v(m,n)$$

(1)

where $(m,n)$ indicates the pixel location. Noise $v$ is assumed to be independent of $s$ with unit mean and variance $\sigma_v^2$. Speckle severely impedes automatic scene segmentation and interpretation, and limits the resolution of the SAR image as well as its utility. The multiplicative nature of speckle complicates the noise reduction process. A speckle suppression filter should effectively filter homogeneous areas, retain image texture and edges (both straight and curved), and preserve features (linear as well as point-type).

There exist several methods for SAR speckle reduction. In the Lee filter [1], the multiplicative model is first approximated by a linear combination of the local mean and the observed pixel. The minimum mean-square error criterion (MMSE) is then applied to determine the weighting constant. The method by Kuan et al. [2] uses a nonstationary image model and is an extension of Lee’s local statistics algorithm. The Frost filter [3] is an adaptive and exponentially weighted averaging filter. The weights are based on the coefficient of variation which is the ratio of the local standard deviation to the local mean of the degraded image. The enhanced Lee and enhanced Frost filters proposed by Lopes et al. [4] divide an image into homogeneous areas, heterogeneous areas and isolated point targets based on the value of the coefficient of variation (low, intermediate, and high, respectively). These adaptive filters approach the local mean at homogeneous regions; at points of high activity they tend to retain the original observation pixel. The disadvantages are over-smoothing of image texture or ineffective denoising around edges. An adaptive block-Kalman filter (ABKF) has been proposed in [5] which uses a block-wise varying AR model. However, the autocorrelation coefficients of the noise-free image must be identified accurately. Wavelet despeckling approaches also exist and are based on modifying the (log-transformed) speckle noisy wavelet coefficients according to some rule (shrinkage) and reconstructing the filtered image from them. A wavelet method based on soft-thresholding is described in [6]. Xie et al. [7] have proposed a despeckling algorithm that fuses Bayesian wavelet denoising with a regularizing prior. A MAP estimator with an alpha-stable prior within the wavelet framework is proposed in [8]. Bayesian shrinkage which relies on edge information is discussed in [9]. Argenti et al. [10] propose despeckling in the undecimated wavelet domain using a space-variation generalized Gaussian distribution for the wavelet coefficients. An adaptive MAP estimator with a heavy-tailed Rayleigh signal model has been suggested in [11].

In this paper, we propose a novel recursive scheme based on the unscented Kalman filter (UKF) [12], [13] to suppress speckle noise in SAR images while effectively preserving the features. We show that the UKF can be formulated to incorporate a discontinuity-adaptive Markov conditional PDF for the prior and to account for multiplicative noise in the estimation procedure. The first two moments of the prior are estimated using importance sampling. A small set of sigma points capture the prior and the speckle noise statistics, and these are propagated through the multiplicative measurement equation of the UKF to arrive at the final image estimates. Our approach is spatially adaptive (as in [10]) but is computationally less intensive. As compared to [9], the discontinuity preserving prior incorporated in our filter accounts for edges without the need for explicit edge detection. Unlike the ABKF [5], our method does not require inference of the AR parameters of the original image. In contrast to the filters in [9]–[11], the proposed filter does not require parameter estimation or optimization to incorporate non-Gaussian prior.

In Section II, we review the principle of unscented transformation which forms the basis for the UKF. In Section III, we formulate a discontinuity-adaptive prior and discuss a Monte Carlo procedure for its moment estimation. In Section IV, we propose a speckle suppression filter which is a judicious combination of the UKF and the edge-preserving prior through importance sampling. Experimental results and comparisons are given in Section V. We conclude with Section VI.

II. UNSCENTED TRANSFORMATION

The unscented transformation (UT) calculates the moments of a random variable that has undergone a nonlinear transformation. It is founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation [12]. The principle in UT is to 1) capture the Gaussian approximation of the prior distribution with a very small set of carefully chosen deterministic samples known as sigma points, 2) propagate these samples through the nonlinearity, and 3) determine the moments of the posterior from the transformed samples.

Consider propagating an $n_x$-dimensional random variable $x$ through an arbitrary nonlinear function $g: \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_y}$ to generate $y = g(x)$. Assume $x$ has mean $\bar{x}$ and covariance $\Sigma_x$. To estimate the first two moments of $y$ using UT, we proceed as follows: A set of $2n_x + 1$ sigma points $\{X_i\}$ with weights $\{w_i\}$, $i = 0, 1, \ldots, 2n_x$ are deterministically chosen so that they completely capture the true mean and covariance of the (prior) random variable $x$. A selection scheme that
satisfies this requirement [13], [14] is

\[
X_0 = \mathbf{x}; \quad w_0 = \frac{\lambda}{(n_x + \lambda)}
\]

\[
w_i = w_i + \left( 1 - \alpha^2_U + \beta_U \right)
\]

\[
X_i = \mathbf{x} + \left( \sqrt{n_x + \lambda} \mathbf{P} \right)_i, \quad i = 1, \ldots, n_x
\]

\[
X_i = \mathbf{x} - \left( \sqrt{n_x + \lambda} \mathbf{P} \right)_i, \quad i = n_x + 1, \ldots, 2n_x
\]

\[
w_i = \frac{1}{2(n_x + \lambda)}, \quad i = 1, \ldots, 2n_x
\]

\[
\lambda = \alpha^2_U (n_x + \lambda) - n_x.
\]

(2)

Here, \( \sqrt{(n_x + \lambda) \mathbf{P}} \) represents the \( i \)th row or column of \( \mathbf{P} \) of the matrix square root of \( (n_x + \lambda) \mathbf{P} \). Superscripts \( \mu \) and \( \epsilon \) on the respective weights \( w_i \) refer to its use in the calculation of mean and covariance, respectively. The weights \( \left\{ w_i^{(\mu)} \right\} \) satisfy \( \sum_{i=0}^{2n_x} w_i^{(\mu)} = 1 \). Parameter \( \alpha_U \) controls the spread of the sigma point distribution around \( \mathbf{x} \) and is usually set to a value between 0 and 1. The term \( \beta_U \) can be used to incorporate prior knowledge about \( \mathbf{x} \). Each sigma point is instantiated through the nonlinear function as \( Y_i = g(X_i) \), \( i = 0, 1, \ldots, 2n_x \). The estimated mean and covariance of \( \mathbf{y} \) computed as \( \mathbf{E}[Y] = \sum_{i=0}^{2n_x} w_i^{(\mu)} Y_i \), and \( \mathbf{P}_y = \sum_{i=0}^{2n_x} w_i^{(\epsilon)} (Y_i - \mathbf{E}[Y]) (Y_i - \mathbf{E}[Y])^T \) are accurate to the second-order (third-order for symmetric priors) [12] of the Taylor series expansion of \( g(\mathbf{x}) \) for any nonlinear function. When UT is used in the Kalman filter formulation, it leads to the unscented Kalman filter (UKF) [12], [13].

III. DISCONTINUITY-ADAPTIVE PRIOR AND MOMENT ESTIMATION

In recursive filtering, the (current) state is usually predicted from its past through an auto-regressive (AR) model. The drawback lies in the identification of AR parameters which ideally requires the original image. In addition, strong local linear dependence as dictated by the AR model renders it difficult to achieve both effective noise suppression and feature preservation simultaneously. In the past, attempts have been made to circumvent this limitation by introducing space-varying AR models [15] or by using analytically tractable non-Gaussian residuals [16]. However, these methods assume an additive noise model. We propose to handle this problem by incorporating an edge-preserving prior [17], [18].

Bayesian methods enable effective incorporation of prior knowledge. Statistical dependence incorporated using a Markov random field (MRF) model provides great flexibility in enforcing contextual constraints. An MRF possesses Markovian property, i.e., the value of a pixel depends only on the values of its neighboring pixels and on no other pixel [17]. The prior is usually modeled as a homogeneous Gaussian MRF (for mathematical convenience and simplicity). A GMRF prior with a MAP estimate has been used for SAR despeckling in [19]. However, the issue with GMRF is that it imposes smoothness constraint ubiquitously, which inevitably leads to over-smoothing of edges. A better way to handle the situation is to use an appropriate non-Gaussian conditional density function to model the original image.

We model the original image \( s \) by a non-Gaussian MRF with conditional probability density function

\[
P(s(m, n) \mid s(m - i, n - j)) = \frac{1}{Z} \exp \left( -\beta \log \left( 1 + \frac{\eta^2(s(m, n), \bar{s})}{\gamma} \right) \right).
\]

(3)

Here, \( Z \) is a normalizing constant, the neighborhood \( \{(i, j) \mid 1 \leq i \leq M_1, -M_1 \leq j \leq M_1 \} \) \& \{0, 1 \leq i \leq M_1 \}, where \( M_1 \) is the order of the nonsymmetric half plane (NSHP) support. The term \( \eta^2(s(m, n), \bar{s}) = \frac{1}{\rho^2} \sum_{(i, j) \in \Omega} (s(m, n) - \bar{s}(m - i, n - j))^2 \). The quantity \( \rho^2 \) is a measure of the local dependencies. For this MRF model [17], the smoothing strength of the regularizer increases monotonically as \( \eta \) increases within a band \( B_\eta = (\sqrt{\eta}, \sqrt{\eta}) \). Outside, the smoothing decreases and becomes zero as \( \eta \to \infty \). Since this feature enables preservation of image discontinuities, it is called a discontinuity adaptive MRF (DAMRF) model [17], [20].

In a recursive formulation, the predicted mean and variance of the prior are required for the update stage of the filter. Since it is analytically not possible to estimate the variance of the non-Gaussian prior in (3), we resort to a Monte Carlo approach known as importance sampling (IS) [21]. This technique can be used to determine the estimates of the moments of any (non-Gaussian) target PDF (say \( p(z) \)) from the samples of another roughly approximate PDF \( q(z) \) (term as sampler PDF) that includes the (nonzero) support of target PDF in it and is easy to sample. In importance sampling, we draw \( L \) samples, \( \{z^{(l)}\} \) from the sampler PDF \( q \) which concentrates on those points where the function \( p \) is large. If these were under \( p \), we can determine the moments of \( p \) directly from them. When we use samples from \( q \) to determine any estimates under \( p \), in the regions where \( q \) is greater than \( p \), the estimates are over-represented. In the regions where \( q \) is less than \( p \), they are under-represented. To account for this, we use correction weights \( w_i = \frac{p(z^{(i)})}{q(z^{(i)})} \) in determining the estimates under \( p \). For example, to find the mean of the distribution \( p \) we use \( \hat{\mathbf{u}}_p = \left( \sum_{i=1}^{L} w_i z^{(i)} / \sum_{i=1}^{L} w_i \right) \). As \( L \to \infty \) the estimate \( \hat{\mathbf{u}}_p \) tends to the actual mean of \( p \). In our problem, the DAMRF distribution corresponds to \( p \). We choose the heavy-tailed Cauchy distribution as the importance function \( q \).

IV. SPECKLE SUPPRESSION USING UKF

When the observation is linearly related to the state, and the modeling errors are additive white Gaussian, then the Kalman filter (KF) provides an optimal estimate of the state by propagating the first two moments recursively. In speckle reduction, the multiplicative model of speckle noise complicates recursive propagation in the KF formulation.

The unscented Kalman filter (UKF) provides a mathematically tractable way to propagate the first two moments even in the presence of multiplicative noise. We present an algorithm that uses importance sampling to incorporate the DAMRF prior within the recursive UKF framework for speckle noise reduction. We refer to this method as ISUKF. The conditional PDF based on the DAMRF model effectively uses the already estimated pixels in the NSHP support. We employ importance sampling to estimate the first two moments of the conditional PDF. The UKF uses these predicted mean and variances to determine the sigma points and propagates them through the multiplicative model to yield the final estimate of the original pixel intensities. The steps in the proposed algorithm are as follows.

1) Using the DAMRF model (Section III) at each pixel, we construct the state conditional PDF using the past pixels in the NSHP support and the values of \( \rho^2 \) and \( \gamma \) as

\[
P(s(m, n) \mid s(m - i, n - j)) = \exp \left( -\beta \log \left( 1 + \frac{\eta^2(s(m, n), \bar{s})}{\gamma} \right) \right).
\]

(4)

2) Importance sampling is employed to estimate the mean and covariance of the above PDF. We draw samples \( \{z^{(l)}\} \),
Next, we apply the measurement model independently on each of the augmented statistics to determine the corresponding sigma points. We adopt the sigma point selection scheme [(2) in Section II] for UKF to form equivalent amounts of smoothing. To achieve good overall performance, we vary \( \rho^2 \) monotonically as a function of the mean value \( \mu_j \) (of the first-order NSHP pixels) as \( \rho^2 = k \mu_j \), where \( k \) controls the amount of smoothing depending on image texture. It typically takes a value between 0.01 and 0.03. In the following experiments, the number of samples \( L = 100 \) in the importance sampling step. We set \( \beta_{UT} = 1 \), \( \beta_{UT} = 0 \), and \( \kappa = 0 \) for UKF [14].

Quantitative comparisons are made using the following.

1) Signal-to-mean square error ratio (S/MSE) is defined as

\[
S/MSE = 10 \log \left( \frac{\sum_{m,n} s(m,n)^2}{\sum_{m,n} (s(m,n) - \hat{s}(m,n))^2} \right)
\]

Higher the S/MSE value the closer will be the filtered image to the original.

2) Edge correlation factor (ECF) [22] should be close to unity for optimal edge preservation.

3) Equivalent number of looks (ENL) measures the speckle content in a homogeneous region and is given by

\[
ENL = \left( \frac{\mu_f}{\sigma_f} \right)^2
\]

where \( \mu_f \) and \( \sigma_f \) are the mean and standard deviation, respectively, over a chosen uniform region in the image. Theoretically, the square of the reciprocal of the speckle contrast ratio (computed as above) is a measure of the number of independent images that must be averaged to obtain equivalent despeckling in per-

\[
\mathbf{X}^n_{(m,n)/(m,n-1)} = \left[ \hat{x}^n_{(m,n)/(m,n-1)} \quad \hat{x}^n_{(m,n)/(m,n-1)} \right] \pm \sqrt{(n_u + \lambda)\mathbf{P}^n_{(m,n)/(m,n-1)}}
\]
Fig. 2. (a) Original aerial image. (b) Degraded with 3-look Nakagami-distributed speckle [11]. Image estimated using (c) Rayleigh prior MAP estimator [11], and (d) the proposed ISUKF filter.

| TABLE I |
| QUANTITATIVE COMPARISON WITH OTHER FILTERS |

<table>
<thead>
<tr>
<th>(a) Rayleigh MAP-based (Fig. 2)</th>
<th>(b) Real speckle (Fig. 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise (at (165, 100))</td>
<td>0</td>
</tr>
<tr>
<td>Rayleigh MAP technique [11]</td>
<td>18.57</td>
</tr>
<tr>
<td>ISUKF (k = 0.03)</td>
<td>21.14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Wavelet edge-based (Fig. 4)</th>
<th>(d) UDWL-MAP (Fig. 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise (at (90, 160))</td>
<td>89.73</td>
</tr>
<tr>
<td>Wavelet edge-based [9]</td>
<td>90.85</td>
</tr>
<tr>
<td>ISUKF (k = 0.015)</td>
<td>89.25</td>
</tr>
</tbody>
</table>

...fectly uniform regions [7], [10], [23]. A large ENL corresponds to better speckle suppression.

4) Figure of merit (FOM) [6] gives a quantitative evaluation of detection of true edges and suppression of false edges and ranges between 0 and 1. A high value indicates superior edge rendition.

The edge image shown in Fig. 1(a) is degraded with simulated fully-developed 1-look exponential speckle [Fig. 1(b)]. The output of the Enhanced Lee [4], Frost [3], and ISUKF method are shown in Fig. 1(c)–(e), respectively. Fig. 1(c) and (d) has a grainy appearance in the white uniform region due to residual speckle. Even though the noise level is quite high, the proposed method is not only effective in despeckling but also preserves the curved edges between the dark and bright regions. This is also reflected in the FOM and S/MSE values which are higher than those of the standard filters.

We also compared our method with the adaptive MAP technique in [11] which is based on a heavy-tailed Rayleigh model. In Fig. 2(a)–(c), we reproduce the original, the 3-look degraded image, and the output image from [11], respectively. Even though the method proposed in [11] reduces speckle, the output is blurred. The image estimated using the proposed approach [Fig. 2(d)] is sharp and even the fine details are recovered well. The output of our method has higher S/MSE and ENL values as given in Table I(a). Because the output of [11] is blurred, its value is marginally higher. The ISUKF filter is able to preserve even curved edges (as can be inferred from its high ECF value) and its output is much closer to the original image (texture-wise also).

Next, we considered the case of a 1-look “Horse track” image that is affected by real speckle noise [Fig. 3(a)]. The output of the Frost filter, shown in Fig. 3(b), contains noticeable residual speckle in uniform regions. Also, the edge boundaries are noisy. The image estimated by the ISUKF method is shown in Fig. 3(c). Our method is significantly more effective in smoothing speckle over uniform regions (such as the top-left and bottom-right gray regions); yet the edges are sharp and...
clear. Even the small white blobs on the top-right corner are recovered well. Table I(b) gives a quantitative comparison with other filters. The cropped $60 \times 60$ region that was used to calculate the $E N L$ is centered at $(115, 55)$ as given in Table I. The performance of our method is evidently superior even in the real case.

Fig. 4(a) shows a 2-look “Bedfordshire” image in Southeast England with real speckle. The image estimated using a Bayesian wavelet filter which relies on edge information is reproduced from [9] and is given in Fig. 4(b). The output obtained by using the ISUKF filter on the degraded image of Fig. 4(a) is shown in Fig. 4(c). We note that the edges are recovered without any blurring effects while the homogeneous regions are almost free from noise. The isolated point targets which are two white spots in the dark homogeneous region on the left become strikingly visible in Fig. 4(c). A quantitative comparison of the two methods is given in Table I(c).

Finally, we compared our method with a very recent despeckling technique [10] developed in the undecimated wavelet (UDWL) domain. Fig. 5(a) shows a NASA/JPL AIRSAR 4-look image of an airport [10]. This is a very difficult example as it contains lots of weak edges. The outputs of the method in [10] and the proposed method are shown in Fig. 5(b) and (c), respectively. It is quite evident that the ability of the ISUKF method in capturing soft edges is superior to that of [10]. Even the $E N L$ for our method is higher [Table I(d)].

These examples clearly demonstrate the effectiveness of the ISUKF filter in suppressing speckle while simultaneously preserving edges and fine features. It leads to better visual quality while comparing favorably in quantitative performance with existing filters. The method in [9] requires only $O(M N)$ operations for wavelet transformation and edge detection. However, the expectation-maximization step used for parameter estimation in [9] is computationally quite involved and depends on the number of iterations. The method in [11] incurs a high computational cost in formulating the Rayleigh prior and takes about 15 min to execute in MATLAB on a Pentium-IV 1.8-GHz PC for a $1100 \times 1100$ image. The UDWL reportedly requires a few minutes [10] to filter a $512 \times 512$ image on a 1.4-GHz PC running MATLAB. The computational requirement for our method is as follows: at each pixel, it requires $2L$ exponential functional evaluations, $O(L)$ additions and multiplications in prediction, $(n_x^2/6) + (n_x^2/2)$ multiplications and additions for matrix square root computation in the sigma point calculation, and $O(2n_x + 1)$ operations for updation. Since $n_x = 2$, the overall complexity is approximately $O(L M N)$. On a Pentium-IV PC with 256-MB RAM and for a $200 \times 200$ image, our method executes in less than 20 s in MATLAB.

VI. CONCLUSION

We proposed a recursive spatial domain unscented Kalman filter-based despeckling technique for SAR imagery. A small set of sigma points were used to capture and propagate the first two moments through the SAR filter equation for estimating the original image. The method explored the advantage of incorporating non-Gaussian prior in
recursive estimation for speckle reduction. When tested on synthetic as well as real examples, the proposed method was found to be very effective.

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