EXPERIMENTAL STUDY OF NEAR WALL TURBULENCE USING PIV

J. Carlier, J. M. Foucaut and M. Stanislas
LML URA 1441, Bv Paul Langevin, Cité Scientifique, 59655 Villeneuve d’Ascq Cedex, France

SHORT ABSTRACT
Experiments have been carried out by means of 2D2C Particle Image Velocimetry in a specific turbulent boundary layer wind tunnel which allows to reach Reynolds numbers based on momentum thickness from 7500 to 19000. Double spatial correlation, Proper Orthogonal Decomposition and Pattern Recognition were applied to the velocity maps in different directions to analyse coherent structures. Some quantitative information about high and low speed streaks, large scale motions and eddy structures have been obtained and are discussed.

INTRODUCTION
What is the contribution of coherent structures and of their interactions in the generation and self-sustaining of wall turbulence? To answer such a question, spatio-temporal data coming from numerical methods such as DNS and LES or from experimental methods such as PIV can be of great help. In this paper, experiments have been carried out by means of PIV in a turbulent boundary layer. The wind tunnel is 1 x 2 m$^2$ in cross section and 20 m in length. The external velocity can be varied between 3 and 10 m/s, giving a Reynolds number based on momentum thickness between 7500 and 19000. The boundary layer thickness is about 0.35 m. PIV measurements were performed with a 2 x 330 mJ ND-YAG laser, a Kodak DCS 460 (2048 x 3072 px$^2$) and a Pulnix TM-9701 (484 x 768 px$^2$). The PIV records were analysed with an in house software based on multigrid process with zero padding. Auto-correlation was used with the first camera and cross-correlation with the second one. Two multigrid iterations were performed, respectively with window sizes of 64 x 64 px$^2$ and 24 x 24 px$^2$. The correlation peaks were interpolated using a gaussian peak fitting algorithm. The raw velocity fields were cleaned using the procedure described by Westerweel (1994). Single holes were filled by interpolation.

DOUBLE SPATIAL CORRELATION
One of the most important tools of the statistical approach to random phenomenon is the correlation operator. Correlation indicates possible deterministic links between “a priori” random variables. In steady flows, one can define the double spatial correlation coefficient of turbulent velocity fluctuations in the following form:

$$ R_{ij}(\ddot{x}, \ddot{x} + d\ddot{x}) = \frac{\langle u'_i(\ddot{x}, t) \cdot u'_j(\ddot{x} + d\ddot{x}, t) \rangle}{\sqrt{\langle (u'_i(\ddot{x}, t))^2 \rangle} \sqrt{\langle (u'_j(\ddot{x} + d\ddot{x}, t))^2 \rangle}} \tag{1} $$

where $\ddot{x}$ is the position vector of a fixed point in space, $d\ddot{x}$ is the distance between a moving point and the fixed point, $u'_k$ is the $k^{th}$ turbulent velocity fluctuation component and $\langle \rangle$ can be a temporal or an ensemble average operator. In PIV, due to the large time separation between each records, ensemble averaging is used.

Fig 1 presents the double spatial correlation coefficient $R_{11}$ in a plane parallel to the wall at $y^+ = y_u/u_\tau = 100$ (where $u_\tau$ is the friction velocity and $v$ is the kinematic viscosity) and $R_0 = 7500$. A homogeneity hypothesis was used to average $R_{11}$ in both directions. The same double spatial correlation coefficient in a plane normal to the wall and parallel to the flow is presented in Fig 2 also at $R_0 = 7500$. The fixed point is at $y^+ = 100$. This coefficient is averaged only in the x direction.
Figure 1 - Double spatial correlation coefficient $R_{11}$ in a plane parallel to the wall at $y^* = 100$.

Figure 2 - Double spatial correlation coefficient $R_{11}$ in a plane normal to the wall and parallel to the flow at $y^* = 100$ for the fixed point.

These figures give a good idea of the three dimensional shape of $R_{11}$ around the fixed point. Fig 2 shows a downstream angle to the wall of about 15°. This angle is comparable to the value often mentioned in the literature for the backs of the large scale motions. However, the shape of $R_{11}$ in the plane parallel to the wall is clearly indicative of a streaky structure of the flow. This streaky structure extends far beyond the size of the PIV window in the x direction, which is also in agreement with the literature.

**PROPER ORTHOGONAL DECOMPOSITION**

Lumley (1967) did propose to define the coherent structures in a turbulent flow as the structures having the largest mean square projection on the velocity field. This problem of maximization leads to a Fredholm integral problem:

$$\int \langle u'_i(\vec{x},t) \cdot u'_i(\vec{x}',t) \rangle \psi_j^n(\vec{x}') d\vec{x}' = \lambda^n \psi_j^n(\vec{x})$$  \hspace{1cm} (2)

where $\Omega$ is the spatial domain under study, $\langle u'_i(\vec{x},t) \cdot u'_i(\vec{x}',t) \rangle$ is the double spatial correlation (see eqn 1) and $\psi_k^n$ and $\lambda^n$ are respectively the $k^{th}$ of the component eigenvector of order $n$ and the corresponding eigenvalue. A POD analysis was performed with the snapshots method introduced by Sirovich (1987a, b, c), which corresponds to solve the following eigenvalue problem:
\[ C_{mn} \alpha_i^n = \lambda^n \alpha_m^n \]  

(3)

where \( C_{mn} = \frac{1}{M} \sum_{\bar{\xi} \in D} \bar{u}(\bar{\xi}, t_m) \cdot \bar{u}(\bar{\xi}, t_k) \) are the M snapshots of the velocity fields available, \( C_{ml} \) is a \( M \times M \) square projection matrix and \( a_k^n \) are the projection coefficients of the \( n^\text{th} \) eigenvector on the \( k^\text{th} \) instantaneous velocity map. The eigenvectors are thus obtained by:

\[ \psi_i^n(\bar{\xi}) = \sum_{k=1}^{M} a_k^n u_i(\bar{\xi}, t_k) \]  

(4)

In the present study, the eigenvalues and eigenvectors were computed in a plane normal to the wall and parallel to the flow, with 200 snapshots for \( R_n = 7500 \) and 13500 and 100 maps for \( R_n = 10500 \) and 19000. Fig 3 presents these eigenvalues divided by the turbulence kinetic energy for the four values of the Reynolds number. For the modes above \( n = 1 \) which correspond to turbulence (mode 1 is the mean flow), the eigenvalues appear fairly independent of the Reynolds number. It seems that 200 velocity maps are enough to reach convergence until the 20\textsuperscript{th} mode. The contribution of the first ten modes after \( n = 1 \) to the turbulence kinetic energy is of the order of 40\%. This convergence poses the problem of the number of significant modes needed to describe turbulence with a dynamical system.

Figure 3 – Eigenvalues distribution in a plane normal to the wall and parallel to the flow.

Fig 4 shows the second eigenvector for \( R_n = 7500 \) (the first one is the mean velocity profile). This eigenvector looks very much like a back of a large scale motion. The angle of this structure is quite comparable to the one observed in the spatial correlation of Fig 2. On the main front going through the map, several eddy structures can be observed.
PATTERN RECOGNITION

It is quite difficult to define what is an eddy structure. Many definitions were reviewed by Jeong and Hussain (1995): maximum value of vorticity magnitude; maximum value of the second invariant of the velocity gradient tensor; minimum negative second eigenvalue of $S^2 + \Omega^2$ with $S$ and $\Omega$ being respectively the symmetric and anti-symmetric parts of the velocity gradient tensor. However, the spatial coherence concept is not considered in these definitions. Ferré and Giralt (1989a, b) propose to use the Pattern Recognition analysis based on convolution to identify part of a HWA velocity signal similar to a model which define a reference coherent structure. In a similar way to Scarano et al. (1999), this method has been applied to detect eddy structures in the velocity maps. The model is, in polar coordinate, a tangential velocity component with gaussian damping:

$$\bar{u}(r, \theta) = \exp\left(\frac{-r}{\sigma \gamma}\right) \cdot \bar{e}_\theta$$  \hspace{1cm} (5)

Measured velocity maps are correlated with the model. The extremum values of this product indicate the presence of eddy structures. Fig 5 shows on the left, the corotating mean eddy structure detected by this procedure in velocity maps normal to the wall and parallel to the flow at $y^+ = 125$ and $R_e = 7500$. On the right, the counterrotating mean eddy structure is presented. These mean values were obtained over 150 samples in the first case and 175 in the second case. The mean eddy structure parameters were determined by fitting an Oseen vortex (see eqn 6) to the spatial velocity distribution.

$$\bar{u}(r, \theta) = \frac{\rho \nu r^2}{r} \left(1 - \exp\left(-\frac{r}{\gamma \eta}\right)\right) \cdot \bar{e}_\theta$$  \hspace{1cm} (6)
Figure 5 – Plane normal to the wall and parallel to the flow at $y^* = 125$ and $R_0 = 7500$ : (a) corotating mean eddy structure ; (b) counterrotating mean eddy structure.

For the corotating mean eddy structure, the radius and the vorticity are in wall units : $r_0 = r_0 \frac{u_*}{v} = 16$ ; $\omega_0 = \frac{\omega_0 u_*^2}{v} = 0.105$. At this location, the local mean velocity gradient is : $\frac{dU^*}{dy^*} = 0.02$ and is included in $\omega_0$. For the counterrotating mean eddy structure one obtains $r_0 = 18$ and $\omega_0 = 0.075$. The mean eddy structure observed in Fig 5a is fairly comparable to those of the mean front of Fig 4 and to the structures observed by several authors (Falco (1977), Head and Bandyopadhyay (1981), Zhou et al. (1999), etc).

CONCLUSION

Three analysis methods based directly or indirectly on the correlation operator were used to study the coherent structure in the turbulent boundary layer from PIV experimental data. The first is the double spatial correlation, the second is the Proper Orthogonal Decomposition and the third is the Pattern Recognition. These methods have allowed to identify eddy structures on slowly raising fronts and to obtain some quantitative information about these coherent structures.

REFERENCES