Sparse Models and Pursuit Algorithms for PIV Tomography

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1 Abstract

The goal of this article is to study the tomographic problem of particle reconstruction within a sparse representation framework. By adopting this class of models for the tridimensional (3D) image reconstruction, we account for an additional prior on the signal to reconstruct. In numerical tests involving a recovery problem over a moderate-to-high wide range of seeding densities, the pursuit algorithms are shown to have comparable performance to various state-of-the-art algorithms, for a complexity reduced by a factor of 10 to 100.

2 Description

The *Tomographic Particle Image Velocimetry (TomoPIV)* is an experimental technique for the retrieval of Eulerian velocity measurements of turbulent fluids introduced in [1]. The technique aims at reconstructing, with very high update rates, 3D motion fields of lightly seeded particles from the images captured by a finite number of cameras.

A crucial step in solving the velocity fields is estimating the volume distribution of the seeded particles. However, the number of observations available for the reconstruction is very limited, which implies solving an underdetermined linear system.

Methods in the current literature exploit prior knowledge on the 3D signal. The classical methods commonly adopted in the TomoPIV community are the so-called algebraic methods for reconstruction ([2]), amongst which we mention the most popular as being the so-called ART and MART algorithms. The latter looks for a solution satisfying the observation model under an entropy-based optimization criterion and it naturally integrates a **non-negativity constraint** on the sought solution. More recently, *Petra et al.* set-up the theoretical context for the study of TomoPIV as a sparse representation matter ([3]) and have empirically shown that application of reconstruction algorithms for finding **sparse solutions** outperforms state-of-the-art algebraic techniques [4].

While these algorithms lead to acceptable reconstruction with respect to accuracy, they each present specific drawbacks: the algebraic techniques induce too dense positive solutions, as opposed to the reconstructed sparse vectors issued by the convex optimization procedure, which suffers from high-complexity without guaranteeing non-negative solutions. Our current investigation focuses on reconstructing a sparse volumetric signal from few projections with respect to accuracy and complexity. In order to achieve a precise sparse signal with a reasonable computational time, we took an interest in a family of algorithms for sparse representations extensively known as **pursuit algorithms**. By applying them to our reconstruction problem, we point out a faster alternative to the state-of-the-art techniques for comparable performance in terms of probability of correct reconstruction.

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The results shown will confirm the efficiency of this family of procedures and will reveal, for MART, basis pursuit and matching pursuit algorithms, the levels of probability of exact reconstruction and their corresponding complexities, with respect to seeding densities in the flow.

References

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