Tracking closed curves with non-linear stochastic filters

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Abstract. The joint analysis of motions and deformations is crucial in a number of computer vision applications. In this paper, we introduce a non-linear stochastic filtering technique to track the state of a free curve. The approach we propose is implemented through a particle filter which includes color measurements characterizing the target and the background respectively. We design a continuous-time dynamics that allows us to infer inter-frame deformations. The curve is defined by an implicit level-set representation and the stochastic dynamics is expressed on the level-set function. It takes the form of a stochastic differential equation with Brownian motion of low dimension. Specific noise models lead to traditional evolution laws based on mean curvature motions, while other forms lead to new evolution laws with different smoothing behaviors. In these evolution models, we propose to combine local motion information extracted from the images and an incertitude modeling of the dynamics. The associated filter we propose for curve tracking thus belongs to the family of conditional particle filters. Its capabilities are demonstrated on various sequences with highly deformable objects.

1 Introduction

Tracking deformable structures delineated by free curves, with no prior on their possible shapes, is a very challenging problem. As a matter of fact, the shape of a deformable object or even of a rigid body may change drastically when visualized from an image sequence. These deformations are due to object apparent motion, to perspective effects and to 3D shape evolution. This difficulty is amplified when the object becomes partially or totally occluded during even a very short time period. The presence of cluttered background and ambiguities constitutes other difficulties for tracking. For curve tracking numerous approaches based on the level set representation have been proposed [1–7]. These techniques mainly addressed the problem as a succession of instantaneous detection or segmentation problems. At best only discrete snapshots of the location of the object of interest are provided and no dynamical or morphological consistency can be really enforced. Implausible growing/decreasing or merging/splitting cannot be avoided without resorting to shape priors [8–10]. This reduces considerably the generality of the tracker and restrains its use to very specific applications [8, 10].

Such deterministic approaches have also great difficulties to cope with ambiguities and noise. The explicit introduction of a dynamics in the curve evolution law has been considered in [4]. However, the proposed technique, although much more satisfying from the point of view of the forecasting of the curves, is not embedded into a tracking framework. In [11], an approach based on a group action mean shape and a moving average has been proposed. This tracking is restricted to simple motions. Recently an optimal control strategy has been defined for curve tracking [12]. This technique permits to cope with non linear differential evolution laws. It is nevertheless a deterministic technique that only involves Gaussian incertitude on the dynamical system. It is also a batch technique which relies on the entire image sequence. It can hardly be used for on-line tracking.

The extraction of state trajectories relying on past measurements and on a dynamical model, as done with stochastic filtering, permits to handle naturally partial occlusions, cluttered noise and ambiguities. It enables also to rely on an approximate knowledge of the underlying dynamics. However, the state dimension constitutes the Achille's heel of recursive Bayesian filter such as the particle filter. Due to this so called curse of dimensionality, only few works attempted to mix stochastic filtering and level set representation for curve tracking [13, 14]. These works have to face a high dimensional sampling problem and as a consequence rely on a crude discretization of the non linear curve dynamics which may be problematic in some situations.

The approach we proposed for curve tracking is also implemented through a particle filter and a level set representation. This approach includes color measurements characterizing the target and the background respectively [15]. The dynamics involved is formulated as a stochastic differential equation. This allows us to get a continuous-time representation of the curve trajectory and, thus, to infer inter-frame deformations. This gives access to richer dynamics on curves. It would also permit the use of continuous time physical evolution laws in specific contexts. The stochastic dynamics is expressed on the level-set function and takes the form of a stochastic differential equation with Brownian motion of low dimension. Although such an attempt has been done to build stochastic dynamics for image segmentation in [16], our approach is different, as it integrates naturally the contribution of noise in the dynamics derivation. It also allows interpreting additional smoothing terms on the curve as a consequence of the incertitude we have on the curve dynamics. Conceptually, this yields a rigourus derivation of the curve dynamics, enabling to handle topological changes occuring between two frame instants, and also to cope with the propagation of possibly irregular curves driven by noisy motion fields. No adhoc, additional filters are here needed to propagate the curve. Such a smoothing is expicitly handled within the expression of the stochastic expression of the level set dynamics. The evolution models we propose combines local motion information extracted from the image and the modeling of dynamics uncertainty. The associated filter thus belongs to the family of conditional particle filters [17].

2 Stochastic filtering and particle filter

Before introducing in detail the stochastic evolution laws on which we will rely in this work we present in this section the generic problem of continuous time stochastic filtering in presence of discrete-time measurements.

Stochastic filters constitute well known procedures to estimate the posterior pdf $p(\mathbf{x}_k|\mathbf{z}_{1:k})$ (called the filtering distribution) of a state variable of interest at any measurement instant k, given the discrete measurements series $\mathbf{z}_{1:k} = (\mathbf{z}_1 \cdots \mathbf{z}_k)$ until instant k, and an initial distribution $p(\mathbf{x}_0)$. In the following, we consider a continuous time state $\mathbf{x}_t$. We will denote by $\mathbf{x}_{t=k}$ or $\mathbf{x}_k$ its value at the measurement instant k. At each time instant k, the measurement equation relates the observation $\mathbf{z}_k$ to the state $\mathbf{x}_k$. In this work the general system we are dealing with is described by:

$$\begin{cases} d\mathbf{x}_t &= f(\mathbf{x}_t)dt + \sigma(t)d\mathbf{B}_t, \\ \mathbf{z}_k &= g(\mathbf{x}_k) + \mathbf{v}_k, \end{cases} \tag{1}$$

where $\mathbf{B}_t$ is a Brownian motion and $\mathbf{v}_k$ is a noise variable. Functions f and g are non linear in the general case.

Assuming there exists a transition distribution $p(\mathbf{x}_t|\mathbf{x}_{r<t})$ (which should formally be written as $p(\mathbf{x}_t|\mathcal{F}(x_{r<t}))$ where $\mathcal{F}(x_{r<t})$ denotes the filtration generated by Brownian motion up to time r), the inference of the posterior pdf may be obtained in two successive stages: a prediction step and a correction step. The prediction uses the transition distribution $p(\mathbf{x}_k|\mathbf{x}_{r<k})$ to make a first approximation of the next state. Then, the correction step updates the posterior pdf through the likelihood $p(\mathbf{z}_k|\mathbf{x}_k)$ of the new observation $\mathbf{z}_k$ obtained at instant k. Both steps involve integral terms that can be analytically computed only for linear systems with additive Gaussian noise. This case corresponds to the famous Kalman-Bucy filter [18].

2.1 Particle filter

Particle filtering is a sequential Monte Carlo framework that yields an approximate solution of the general stochastic filtering problem (non linear likelihood, non additive and non Gaussian noises). The filtering distribution $p(\mathbf{x}_k|\mathbf{z}_{1:k})$ is recursively approximated by a finite weighted sum of N Diracs centered on hypothesized locations in the state space – called particles – of the initial system $\mathbf{x}_0$. At each particle, $\mathbf{x}_k^{(i)}$ $(i = 1 : N)$, is assigned a weight $\gamma_k^{(i)}$ describing its relevance. This approximation reads:

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) \approx \sum_{i=1:N} \gamma_k^{(i)} \delta_{\mathbf{x}_k^{(i)}}(\mathbf{x}_k). \tag{2}$$

Assuming that the approximation of $p(\mathbf{x}_{k-1}|\mathbf{z}_{1:k-1})$ is known, the recursive computation of the filtering distribution is done by propagating the swarm of weighted particles $\{\mathbf{x}_{k-1}^{(i)}, \gamma_{k-1}^{(i)}\}_{i=1:N}$. At each time instant (or iteration), the set of new particles $\{\mathbf{x}_k^{(i)}\}_{i=1:N}$ is drawn from an approximation of the true distribution $p(\mathbf{x}_{t\geq k-1}|\mathbf{z}_{1:k})$, called the *importance function* and here denoted $\pi(\mathbf{x}_t|\mathbf{x}_{0:k-1}^{(i)}, \mathbf{z}_{1:k})$. The closer the approximation to the true distribution, the more efficient the filter. The importance weights, $w_k^{(i)}$, account for the deviation w.r.t. the unknown true distribution. To maintain a consistent sample, the importance weights are updated according to a recursive evaluation as the new measurement $\mathbf{z}_k$ becomes available:

$$\gamma_k^{(i)} \propto \gamma_{k-1}^{(i)} \frac{p(\mathbf{z}_k|\mathbf{x}_k^{(i)})\ p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{(i)})}{\pi(\mathbf{x}_k^{(i)}|\mathbf{x}_{0:k-1}^{(i)}, \mathbf{z}_{1:k})}, \qquad \sum_{i=1:N} \gamma_k^{(i)} = 1. \tag{3}$$

Different choices are possible for this proposal density [17]. The most common one consists in setting the proposal distribution to the dynamics:

$$\pi(\mathbf{x}_t|\mathbf{x}_{0:k-1}^{(i)}, \mathbf{z}_{1:t}) = p(\mathbf{x}_t|\mathbf{x}_{k-1}^{(i)}). \tag{4}$$

In this case the weight update in (3) is greatly simplified: it amounts to multiplying by the data likelihood $p(\mathbf{z}_t|\mathbf{x}_t^{(i)})$. This version of the particle filter is known as the *bootstrap filter*. This is the kind of filter which we are dealing with. In our case the two steps of the filter reads:

- **Prediction step :** $\mathbf{x}_k^{(i)} \sim p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{(i)})$
- **Correction step :** $w_k^{(i)} \propto w_{k-1}^{(i)}\ p(\mathbf{z}_k|\mathbf{x}_k^{(i)})$**.**

The prediction step consists in sampling trajectories $\{\mathbf{x}_t^{(i)} : k-1 \leqslant t \leqslant k\}_{i=1:N}$ from the stochastic differential equation describing the continuous evolution of the state:

$$d\mathbf{x}_t^{(i)} = f(\mathbf{x}_t^{(i)})dt + \sigma(t, \mathbf{x}_t^{(i)})d\mathbf{B}_t^{(i)}, \tag{5}$$

from the initial condition $\{\mathbf{x}_{k-1}^{(i)}\}_{i=1:N}$ and where $\{B_t^{(i)}\}_{i=1:N}$ are independent Brownian motions. The simulation of the sde (5) can be done through the Euler scheme:

$$\mathbf{x}_{t+\Delta t}^{(i)} = \mathbf{x}_t^{(i)} + f(\mathbf{x}_t^{(i)})\Delta t + \sigma(t)(\mathbf{B}_{t+\Delta t}^{(i)} - \mathbf{B}_t^{(i)}), \tag{6}$$

where the increments $\mathbf{B}_{t+\Delta t}^{(i)} - \mathbf{B}_t^{(i)}$ are independent Gaussian noises with zero mean and variance $\Delta t \mathbb{I}$. Let us note that the discretization step is much smaller than the inter measurement time interval ($\Delta t \ll 1$).

In order to avoid degeneracy of the particle swarm, a resampling step must be applied sufficiently often [19]. This process consists in drawing, with replacements, a new set of particle from the current one according to a probability distribution that depends on importance weights. The particles associated to low weights will tend to disappear whereas the ones with larger weights are likely to be duplicated.

In this work the state variables will consist in closed curves represented by implicit surfaces. Their associate dynamics will be defined in section 3. Before that let us define the likelihood on which we will rely.

2.2 Likelihood definition

In bootstrap filters, the likelihood associated to each particle directly determines its weight. It is therefore crucial for the likelihood to be sufficiently discriminant in order to discard curves which are too distant from the intended result. To this end, we choose to define a likelihood that depends on the similarity between the color distributions inside the curve at times $t = 0$ and $t = k$ respectively. For each particle, it reads:

$$p(\mathbf{z}_t|\mathbf{x}_t^{(i)}) \propto \exp -\lambda d(h_0, h_k^{(i)}), \tag{7}$$

where d is related to the Bhattacharyya distance between h_0 the reference interior color histogram instantiated at time 0 and $h_k^{(i)}$ the interior color histogram associated to the i-th level-set sample at time k, and λ is a positive parameter. For discrete probability distributions p and q defined over the same domain X, the Bhattacharyya distance is defined as:

$$d(p, q) = \left(1 - \sum \sqrt{p(x)q(x)}\right)^{1/2} \tag{8}$$

3 A Stochastic evolution laws for level sets

As mentioned in the introduction, the curve that we want to track is defined by an implicit level-set representation. The stochastic dynamics has thus to be defined on this level-set function which is of infinite dimension (or at least of very high dimension in its discrete form). In order to cope with the curse of dimensionality that makes inefficient any sampling in high dimension, the model we consider relies on a low dimension Brownian motion. To this end we introduce next three different evolution laws and explain how they are related to evolution laws of level sets.

Let $\mathcal{C}_t$ denote a closed Jordan curve $\mathcal{C}_t : [0, 1] \to \mathbb{R}^2$ at time $t \in [t_0, \tau]$ of the image sequence. Let us first assume that this curve evolves in time according to the following evolution law:

$$d\mathcal{C}_t = w_n \boldsymbol{n} dt + \sigma_1 \boldsymbol{n} dB_t^{(1)} + \sigma_2 \boldsymbol{n}^{\perp} dB_t^{(2)}, \tag{9}$$

where $dB_t^{(1)}$ and $dB_t^{(2)}$ are two independent Brownian motions, $\boldsymbol{n}$ is the unit vector normal to the curve and $w_n = \boldsymbol{w} \cdot \boldsymbol{n}$ is the projection of some velocity field $\boldsymbol{w}$ on this normal. In this model, a deterministic drift associated to velocity field $\boldsymbol{w}$ is mitigated with an isotropic Gaussian incertitude that grows linearly in time. As a matter of fact, let us recall that the quadratic variation of the Brownian motion, on the real line for sake of simplicity, is:

$$< \sigma dB_t, \sigma dB_t >_t = \int_0^t (\sigma dB_s)^2 = \lim_{\Delta t \to 0} \sum_0^t |B_{t+\Delta t} - B_t|^2 = \sigma^2 t. \tag{10}$$

Contrary to differentiable deterministic functions, Brownian motion does not have a bounded variation (i.e., its total variation on $[0, t]$ is infinite).

Level set representation As we wish to focus in this work on the tracking of non parametric closed curves that may exhibit topology changes during the time of the analyzed image sequence, we will rely on an implicit level set representation of the curve of interest [5, 7]. Within this framework, the curve $\mathcal{C}_t$ enclosing a region D we wish to track is described at time t through a higher dimensional surface $\Phi : \mathbb{R}^2 \to \mathbb{R}$ and the implicit equation:

$$\mathcal{C}_t = (\boldsymbol{x}_t(p) : \Phi(\boldsymbol{x}_t(p)) = 0)\,, \tag{11}$$

where p stands for a parameter of the curve and $\boldsymbol{x} \in \Omega$ denote image positions. This representation constitutes an Eulerian representation of a curve and enables a natural topology adaptivity. The implicit representation is defined from an initial surface such as a signed distance function to the contours of interest, and evolves according to the curve evolution law. The curve at time t is defined by construction through its implicit representation at time t:

$$\Phi_t = \Phi_0 + \int_0^t d\Phi_s. \tag{12}$$

Assuming the level set representation is uniquely defined from an initial surface and the curve evolution (9), the surface, Φ, constitutes a function of the stochastic process $\mathcal{C}_t$. Its differential must be calculated using the so called Îto formula from stochastic calculus [20, 21].

Stochastic level set evolution law Let us apply Îto formula to the implicit representation of the curve $\Phi(\boldsymbol{X}_t)$ where $\boldsymbol{X}_t = (X_t^x X_t^y)^T \in \Omega$, is driven by an Îto diffusion defined as an extension of the curve velocity:

$$d\boldsymbol{X}_t = w_n^* \boldsymbol{n} dt + \sigma_1 \boldsymbol{n} dB_t^{(1)} + \sigma_2 dB_t^{(2)} \boldsymbol{n}^\perp. \tag{13}$$

In this equation, the drift term w_n^* is an extension to the whole image domain of the curve deterministic drift along the curve normal $\boldsymbol{n} = \boldsymbol{\nabla}\Phi / \|\boldsymbol{\nabla}\Phi\|$. Following Îto formula, the process $\varphi_t = \Phi(\boldsymbol{X}_t)$ is an Îto process defined as

$$d\varphi_t = w_n^* \|\boldsymbol{\nabla}\varphi\| dt + \sigma_1 \|\boldsymbol{\nabla}\varphi\| dB_t^{(1)} + \frac{1}{2} \sum_{i,j=x,y} \frac{\partial^2 \varphi}{\partial x_i, x_j} < dX_t^i, dX_t^j > . \tag{14}$$

The associated quadratic variation reads:

$$\begin{aligned} < dX_t^x, dX_t^x > &= \frac{\sigma_1^2 \varphi_x^2 + \sigma_2^2 \varphi_y^2}{\|\boldsymbol{\nabla}\varphi\|^2} dt, \\ < dX_t^y, dX_t^y > &= \frac{\sigma_1^2 \varphi_y^2 + \sigma_2 \varphi_x^2}{\|\boldsymbol{\nabla}\varphi\|^2} dt, \\ < dX_t^x, dX_t^y > &= \frac{(\sigma_1^2 - \sigma_2^2) \varphi_x \varphi_y}{\|\boldsymbol{\nabla}\varphi\|^2} dt. \end{aligned} \tag{15}$$

Introducing the surface normal expression, the Îto diffusion [21] driving the implicit surface evolution reads finally:

$$\begin{aligned} d\varphi_t = w_n^* \|\boldsymbol{\nabla}\varphi\| + \frac{1}{2\|\boldsymbol{\nabla}\varphi\|^2} &(\varphi_{xx}(\sigma_1^2 \varphi_x^2 + \sigma_2^2 \varphi_y^2) + \varphi_{yy}(\sigma_1^2 \varphi_y^2 + \sigma_2^2 \varphi_x^2) \\ &+ 2(\sigma_1^2 - \sigma_2^2) \varphi_x \varphi_y \varphi_{xy}))dt + \sigma_1 \|\boldsymbol{\nabla}\varphi\| dB_t^{(1)}. \end{aligned} \tag{16}$$

Recalling that the mean curvature can be expressed as:

$$\kappa = \text{curv}(\varphi) = \frac{1}{\|\boldsymbol{\nabla}\varphi\|}(\Delta\varphi - \boldsymbol{\nabla}\varphi^T \boldsymbol{\nabla}^2\varphi\ \boldsymbol{\nabla}\varphi), \tag{17}$$

where $\boldsymbol{\nabla}^2\varphi$ denotes the Hessian matrix and $\Delta\varphi$ the Laplacian, the surface evolution law may be written in a more compact form as:

$$d\varphi = (w_n^*\|\boldsymbol{\nabla}\varphi\| + \frac{\sigma_2^2}{2}\kappa\|\boldsymbol{\nabla}\varphi\| + \frac{\sigma_1^2}{2\|\boldsymbol{\nabla}\varphi\|^2}\boldsymbol{\nabla}\varphi^T\boldsymbol{\nabla}^2\varphi\ \boldsymbol{\nabla}\varphi)dt + \sigma_1\|\boldsymbol{\nabla}\varphi\|dB_t^{(1)}. \tag{18}$$

It can be observed from (18) that if both incertitudes have the same strength (i.e. $\sigma_1 = \sigma_2$) this model takes a particular simple form:

$$d\varphi_t = (w_n^*\|\boldsymbol{\nabla}\varphi\| + \frac{1}{2}\sigma_1^2\Delta\varphi)dt + \sigma_1\|\boldsymbol{\nabla}\varphi\|dB_t^{(1)}. \tag{19}$$

The dynamical model (2) constitutes a general stochastic process allowing to guide a curve through an implicit surface. This stochastic process will enable us to draw samples of curves in our tracking process. Before turning to the experiments, it is interesting to see to what corresponds the expectation of these stochastic processes. It can be shown, through Kolmogorov backward equation (the adjoint of the Fokker-Planck equation) that the expectation $u(\boldsymbol{x}, t) = \mathbb{E}^{\boldsymbol{x}}(\Phi(\boldsymbol{X}_t))$ evolves as:

$$\frac{\partial u}{\partial t} = (w_n^* + \frac{\sigma_2^2}{2}\kappa)\|\boldsymbol{\nabla}u\| + \frac{\sigma_1^2}{2\|\boldsymbol{\nabla}u\|^2}\boldsymbol{\nabla}u^T\boldsymbol{\nabla}^2u\ \boldsymbol{\nabla}u, \ \texttt{and}\ u(\boldsymbol{x}, 0) = \Phi_0(\boldsymbol{x}), \tag{20}$$

where Φ_0 denotes the initial surface, built from an initial value of the contour. This equation gives us the evolution law of the expectation on a fixed grid of an implicit surface driven by a stochastic dynamical model of form (9). This dynamical model includes two independent Brownian uncertainty on the curve motion directed along the curve's tangent and normal respectively. The first term corresponds to the traditional deterministic evolution law of a level set function. The curvature term is here introduced due to the effect of the motion incertitude along the curves tangent. The second term is less usual and corresponds to an uncertainty directed along the surface normal. If both uncertainties are set to the same amplitude then the previous equation simplifies as:

$$\begin{aligned} &\tfrac{\partial u}{\partial t} = w_n^*\|\boldsymbol{\nabla}u\| + \tfrac{\sigma^2}{2\|\boldsymbol{\nabla}\Phi\|^2}\Delta u, \\ &u(\boldsymbol{x}, 0) = \Phi_0(\boldsymbol{x}). \end{aligned} \tag{21}$$

4 Experiments and results

Motion information extracted from the images The evolution laws introduced in the previous section are based on a stochastic force w calculated from the image. We now introduce the force we use in our experiments. It is a linear combination of two main components:

$$\boldsymbol{w}_n^{*(i)} = \beta(t)\boldsymbol{v}^T\boldsymbol{n} + (1-\beta(t))\partial_\varphi F(\varphi^{(i)}) \tag{22}$$

with proportions $\beta(t) \in [0,1]$ and $1-\beta(t)$ respectively. The first component is a motion component obtained from an optical flow computation, while the second corresponds to a photometric edge component obtained from a generalized Chan-Vese operator [12].

Optical-flow component The motion component $\mathbf{v} = (v^x, v^y)^T$ is provided by a robust and fast optical-flow estimator. It is defined as the minimizer of the objective function:

$$\int_\Omega f(\|\boldsymbol{\nabla} I^T\boldsymbol{v} + I(t+dt) - I(t)\mathbf{1}_{p(\mathbf{z}_t|\nu(\boldsymbol{x}))<1-\epsilon}\|)d\boldsymbol{x} + \lambda\int_\Omega(\|\boldsymbol{\nabla} v^x\|^2 + \|\boldsymbol{\nabla} v^y\|^2)d\boldsymbol{x}. \tag{23}$$

Function f is a robust function whose role is to discard data that significantly deviates from the brightness constancy assumption. This function together with the characteristic function defined from a local likelihood computed over a neighborhood $\nu(\boldsymbol{x})$ of $\boldsymbol{x} \in \Omega$ (eq. 7) provides a smooth motion field on the whole image plane that represents only the motion of data points that likely correspond to the object of interest. This motion component is a rough description of the curve's motion. It is reasonable to combine it with a photometric edge force.

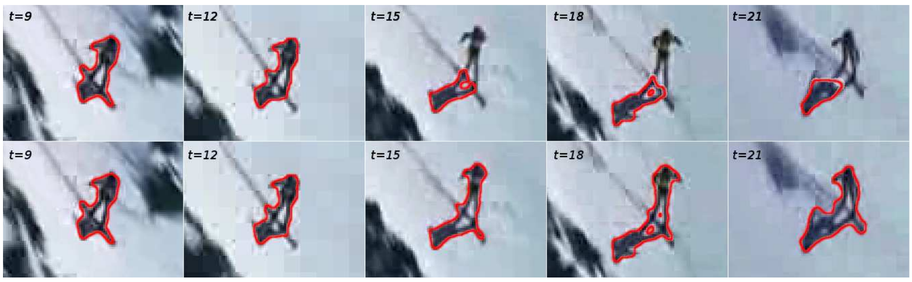

Fig. 1. Tracking of a skier; first row: drift term with only the photometric edge component; second row: drift term defined as a combination of a photometric edge components and a motion component

Photometric edge component The second component is derived from an operator [12] that corresponds to Chan and Vese operator [22] applied to histograms. It is thus defined from the derivative w.r.t. the unknown level set of the following objective function:

$$F(\varphi, I)(\boldsymbol{x}, t) = d(h(\nu(\boldsymbol{x})), h_0)^2\mathbf{1}_{\varphi(\boldsymbol{x})<0} + d(h(\nu(\boldsymbol{x})), h_b)^2\mathbf{1}_{\varphi(\boldsymbol{x})\geq 0}, \tag{24}$$

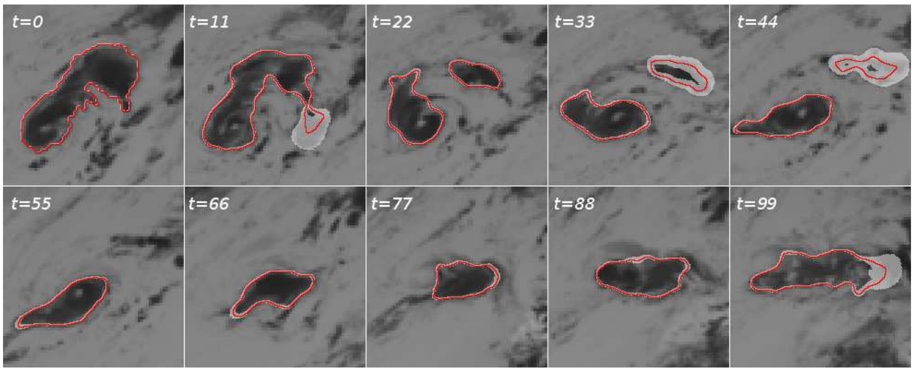

Fig. 2. Tracking cyclone Vince in infrared channel of Meteosat satellite.

where d is the Bhattacharyya distance, h_o and h_b denote respectively the reference interior and exterior color histograms instantiated at time 0, $h(\nu(\boldsymbol{x}))$ represents the local color histogram at point $\boldsymbol{x}$. The gradient of this objective function reads:

$$\partial_\varphi F = (d(h(\nu(\boldsymbol{x})), h_o)^2 - d(h(\nu(\boldsymbol{x})), h_b)^2)\delta(\varphi), \tag{25}$$

where $\delta(.)$ is the Dirac function.

Both components have their own advantages in the time interval between measurement instants k and $k+1$. For our tracking purpose, the photometric component is especially helpful in the temporal vicinity of the second images, whereas the optical-flow component is more likely to be meaningful as a rough component of the motion only in the temporal vicinity of the first image. As a consequence we choose to change gradually the proportion of each according to:

$$\beta(t) = \frac{2t}{\Delta k} - 1, t \in [0, \Delta k]. \tag{26}$$

In order to illustrate the role of each component we show first results on a sequence of 21 frames depicting a skier in action. On Fig. 1, the first row exhibits the results obtained when considering only the photometric component with a constant weight. The second row shows the results obtained from the combination of the optical-flow and the photometric components. Between $t = 13$ and $t = 15$, the skier moves rapidly to the right of the image. It can be observed that in the first case, the tracker quickly focuses on the skier's shadow only. In the second case, the optical flow term allows us to cope with this large displacement and to improve the result.

Let us outline that for visualisation purposes, we have centered all the images on the skier.

The second sequence on which we present results is composed of 100 meteorological images (Meteosat infra-red image) showing the evolution of cyclone

Vince over North Atlantic. In Fig. 2 we show in red the level set associated to the mean of all implicit function particles (after resampling) and the standard deviation of the estimate. As can be observed from these pictures or from the companion video the results are of good quality. The method allows a robust tracking of the regions of interest. When the cyclone collapses at the end of the sequence, the tracking becomes less certain and the variance of the estimation grows. Such an assessment of estimate confidence is another great advantage of probabilistic techniques.

We finally present results on 30 frames of a video showing a lion running in the savanna. The results obtained are shown in Fig. 3.

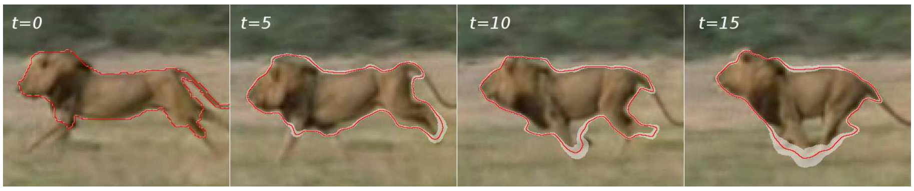

Fig. 3. Tracking of lion running in the savanna with our particle filter on the space of implicit functions.

We can observe on this sequence that for regions where background color is a source of high ambiguities (i.e., around such as the legs), the uncertainty is important. The top of the lion is clearly distinct from the background, it is therefore segmented with better accuracy and confidence. Beside the quality of the results local confidence assessment via variance vizualisation (or analysis) is an interesting feature of our approach. This could probably be of practical interest in medical image applications.

In order to show the advantage of our method, we present in Fig. 4 the same sequence with successive segmentations obtained using the Chan-Vese operator only. We can observe the lack of continuity in the tracking, and the selection of several portions of the background due to color ambiguities. Our method avoids these problems by favoring a continuous evolution of the implicit surface.

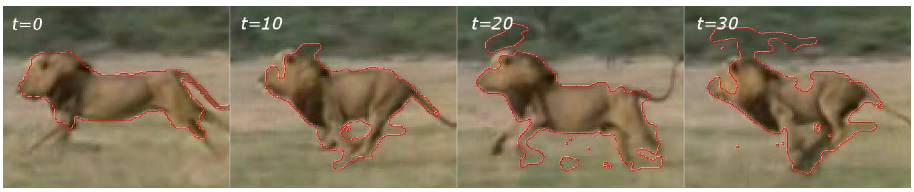

Fig. 4. Successive segmentations of lion running in the savanna.

Our method involves two main parameters, which are related to the incertainty we have on the curve dynamics. The estimation of these parameters is not addressed in this paper but will be investigated in future researches. We have observed that better results were obtained for a noise along the curve tangent that is slightly larger than for the one directed along the normal. For the sequences shown in this paper we chose $\sigma_1 = 3$ and $\sigma_2 = 4$.

5 Conclusions and future work

In this paper we have described a probabilistic filtering method for the tracking of level sets. The technique we propose is implemented through a particle filter and combines discrete-time image measurements with a continuous-time stochastic dynamics. This continuous dynamics relies on two different incertainties on the curve motion, directed respectively along the curve normal and along the curve tangent. The considered curve dynamics has been built from the image data by considering a drift term that combines in varying proportions a motion component and a photometric component. The measurement considered in this filter are built from color histograms of the object delineated by the user at the initial time.

The first perspective concerns the automatic estimation of the two noise variances. The first one is related to the incertainty on the motion whereas the second one corresponds to the level of noise in the image. Another perspective concerns the management of occlusions. To that end, an idea would be to modify the coefficient of the normal noise according to the average of all likelihoods of particles. Thus, in case of loss of the object, the uncertainty would grow, resulting in a spread and expansion of the level sets and, as a consequence, in a more likely recovery of the tracker when the object reappears. Finally, it could be interesting to investigate the use a Brownian motion of higher dimension to capture a larger set of deformations between two consecutive frames.

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