A bottom-up efficient algorithm learning substitutable languages from positive examples

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Motivation

Distributional Hypothesis (words that occur in the same contexts tend to have similar meanings [Harris, 1954]. "a word is characterized by the company it keeps" [Firth, 1957]) has been for long an influential idea in Linguistics :

- Part of the language acquisition discussion...
- Base of Statistical Semantics
- Unsupervised POS parsing (Constituent-Context Models [Klein & Manning, 2001]...)
- Learning expressive grammars from positive examples only
  - Heuristics : EMILE [Adriaans, 1992 ; Adriaans and Vervoort, 2002)] , ABL [van Zaanen, 2002], ADIOS [Solan et al., 2005]...
  - Characterizable inference of *substitutable languages* : [Clark & Eyraud 2007, Yoshinaka 2008, ...]

and [CGN2012] for proteins !

Substitutable Languages

L is *substitutable* [CLARK & EYRAUD, 2007] if :

$x_1, y_1, z_1, x_2, y_2, z_2 \in \Sigma^*, y_1, y_2 \neq \lambda$:

$$x_1 y_1 z_1 \in L \wedge x_1 y_2 z_1 \in L \wedge x_2 y_1 z_2 \in L \Rightarrow x_2 y_2 z_2 \in L$$

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“Weak-implies-Strong” Generalization

Let K be the following set of strings :

Major General was here yesterday morning.
Major General went here yesterday morning.
Major General will be there tomorrow morning.
He will be gone tomorrow evening.

Strings to add to get a $1,1$-local substitutable language :

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⋮

Learning k, l-local substitutable languages

Adaptation of SGL algorithm [Clark & Eyraud, 2007] :

SGL_{LS} (Substitution Graph Learner for Local Substitutable languages)

Input: Set of sequences K on alphabet Σ, int k, int l
Output: Grammar $G = \langle \Sigma, N_K, S_K, P_K \rangle$

```
/* Partition Sub(K) in Local Substitutability classes */
C_K ← LS_classes(K, k, l) /* ∀y ∈ Sub(K), C_K(y) = C ∈ C_K : y ∈ C */
/* Build grammar */
N_K ← ∅, P_K ← ∅
for C ∈ C_K do
    /* A non-terminal for each substitutability class */
    N_K ← N_K ∪ {⟦C⟧}
    /* Productions rules for each substring in the class */
    for y ∈ C do
        if |y| > 1 then
            /* Branching rules: a 'CNF' rule for each split */
            for y_1 ∈ Σ^+, y_2 ∈ Σ^+ : y_1 y_2 = y do
                P_K ← P_K ∪ {⟦C⟧ → ⟦C_K(y_1)⟧⟦C_K(y_2)⟧}
        else
            /* Terminal rule */
            P_K ← P_K ∪ {⟦C⟧ → y}
S_K ← ⟦C_K(k)⟧ : k ∈ K
return ⟨Σ, N_K, S_K, P_K⟩
```

Local substitutability classes

LS_classes()

Input: Set of sequences K on alphabet Σ, int k, int l
Output: k, l local substitutability classes on K

```
/* Build substitutability graph on substrings */
```

$V \leftarrow \{y \in \Sigma^+ : y \in Sub(K)\}$

$E \leftarrow \{\{y_1, y_2\} \in V \times V :$
$\quad uy_1v \in Sub(K), uy_2v \in Sub(K), y_1 \neq y_2, u \in \Sigma^k, v \in \Sigma^l\}$

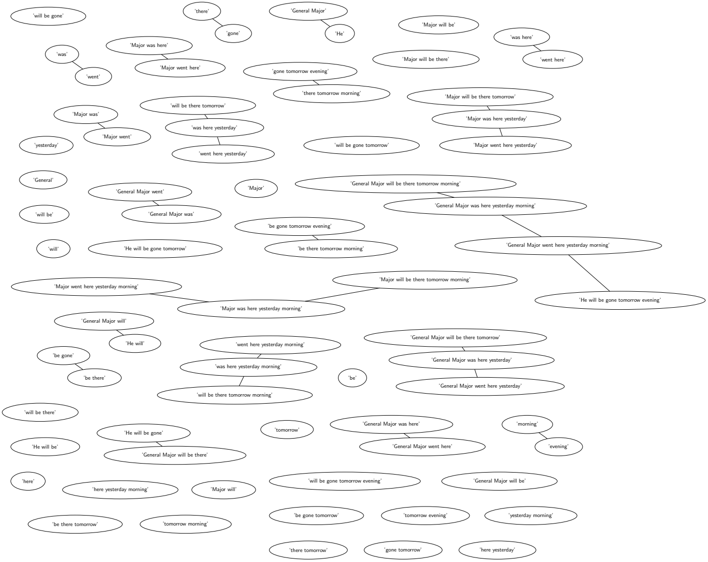
```
/* Return connected components of graph */
```

return `Connected_components`$(\langle V, E \rangle)$

where $Sub(K)$ denotes the set of substrings of K

- This is the unique difference with SGL!
- To learn other substitutable classes, change E and V initialization.

Substitutability graph



source: Gaelle Garet

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Resulting grammar :

$X_{47} \rightarrow '\ yesterday'$
$X_{46} \rightarrow X_3X_{43}|X_{11}X_{19}|X_{23}X_{13}$
$X_{45} \rightarrow X_{20}X_2$
$X_{44} \rightarrow X_{20}X_1|X_{45}X_{29}|X_{34}X_{16}|X_9X_{15}$
$X_{43} \rightarrow X_{20}X_{19}|X_{45}X_{13}$
$X_{42} \rightarrow '\ tomorrow'$
$X_{41} \rightarrow X_{42}X_{15}$
$X_{40} \rightarrow X_{30}X_{46}|X_{21}X_{43}|X_{39}X_{19}|X_{25}X_{13}|X_{21}X_{34}|X_8X_{13}$
$N_0 \rightarrow X_{30}X_{24}|X_{21}X_{31}|X_{10}X_{32}|X_{36}X_{17}|X_{26}X_{15}|X_{39}X_1$
$|X_{25}X_{29}|X_{40}X_{41}|X_{21}X_{44}|X_8X_{29}|X_{40}X_{16}|X_{33}X_{15}$
$X_{29} \rightarrow X_{13}X_{16}|X_4X_{15}|X_{13}X_{41}|X_{38}X_{15}$
$X_{28} \rightarrow X_{27}X_{47}$
$X_{25} \rightarrow X_{30}X_{23}|X_{21}X_{45}|X_{39}X_2$
$X_{24} \rightarrow X_3X_{31}|X_{18}X_{32}|X_{37}X_{17}|X_{14}X_{15}|X_{11}X_1|X_{23}X_{29}|X_{46}X_{41}$
$X_{27} \rightarrow '\ here'$
$X_{26} \rightarrow X_{30}X_{14}|X_{21}X_{12}|X_{10}X_{28}|X_{36}X_{47}|X_{39}X_{35}|X_{25}X_{38}|X_{40}X_{42}$
$X_{21} \rightarrow '\ He'|X_{30}X_3$
$X_{20} \rightarrow '\ will'$
$X_{23} \rightarrow X_3X_{45}|X_{11}X_2$
$X_{22} \rightarrow X_6X_{27}$
$X_8 \rightarrow X_{21}X_{45}|X_{39}X_2$
$X_9 \rightarrow X_{20}X_5|X_{45}X_4|X_{34}X_{42}$
$X_2 \rightarrow '\ be'$
$X_3 \rightarrow '\ Major'$

$X_1 \rightarrow X_2X_{29}|X_{19}X_{16}|X_5X_{15}|X_{19}X_{41}|X_{35}X_{15}$
$X_6 \rightarrow '\ was'|'\ went'$
$X_4 \rightarrow X_{13}X_{42}$
$X_5 \rightarrow X_2X_4|X_{19}X_{42}$
$X_{32} \rightarrow X_{27}X_{17}|X_{28}X_{15}$
$X_{33} \rightarrow X_{21}X_9|X_{39}X_5|X_8X_4|X_{40}X_{42}$
$X_{30} \rightarrow General$
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$X_{38} \rightarrow X_{13}X_{42}$
$X_{39} \rightarrow X_{30}X_{11}|X_{21}X_{20}$
$X_{18} \rightarrow X_3X_6$
$X_{19} \rightarrow X_2X_{13}$
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$X_{11} \rightarrow X_3X_{20}$
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$X_{13} \rightarrow '\ there'|'\ gone'$
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$X_{15} \rightarrow '\ morning'|'\ evening'$
$X_{16} \rightarrow X_{42}X_{15}$
$X_{17} \rightarrow X_{47}X_{15}$

source : GaelleGaret

Limitations

- A lot of non-terminals and rules
- A lot of redundancy and ambiguity

$\Rightarrow$ Parsing time and learning time problems[1] (+ Illegible)

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A solution

Reduce the grammar before parsing

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$\Rightarrow$ Parsing time and learning time problems [1] (+ Illegible)

A solution

Reduce the grammar during the inference

1. About a day for one experiment on a simple set of proteins

Ideas

1 Avoid unnecessary derivations by removing non-terminals with a deterministic derivation :
If A is the left-hand-side of only one rule $A \rightarrow \alpha$ then delete them and replace all $B \rightarrow \ldots A \ldots$ by $B \rightarrow \ldots \alpha \ldots$.

2 Reduce the right-hand-sides of the production rules :
For each $N \rightarrow \ldots \beta \ldots$, if there exists $\alpha : |\alpha| < |\beta| \wedge \alpha \Rightarrow \beta$, then replace $N \rightarrow \ldots \beta \ldots$ by $N \rightarrow \ldots \alpha \ldots$

(Teaser : it ensures also maximal generalization !)

1. Avoid unnecessary derivations

Keep prime classes

Recall : Each non-terminal corresponds to a substitutability congruence class.
Slight abuse of notation : Let $[x]$ denote the class of a non-terminal or terminal x.
N is deterministically derived by $N \to \alpha = \alpha_1 \dots \alpha_{|\alpha|}$ implies

$$[N] = [\alpha_1]...[\alpha_{|\alpha|}].$$

We name such useless class a *composite* class (They are those giving rise to *vacuous local derivation trees* [Clark, 2011]).
We say that a class is *prime* if it is not composite. We have :

Primality test

Let a language L whose set of non-zero and non-unit congruence classes is C^+.
A class $[y]$ in C^+ is *prime* for L iff $\forall y_1 y_2 \in [y], [y] \not\subset [y_1][y_2]$.

Sufficient test since for syntactic congruence, we have $[y_1 y_2] \supseteq [y_1][y_2]$

K-Primes

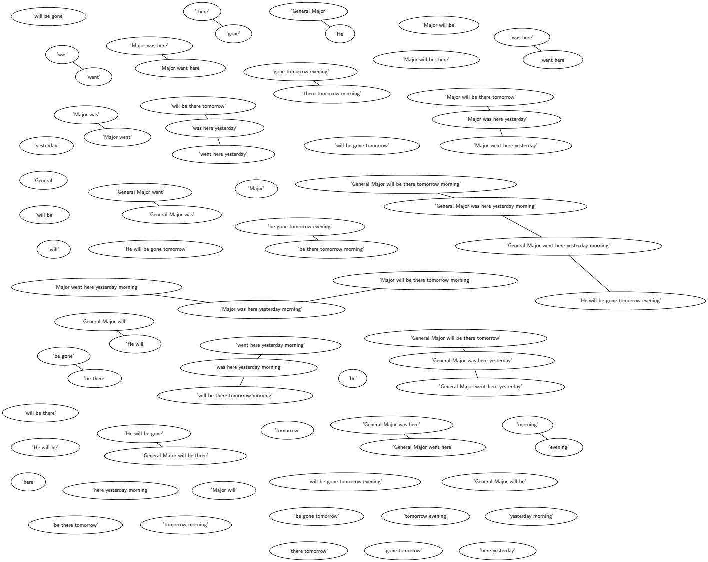
Due to monotonicity of generalization, it is safe to filter out composite classes on the basis of the sample K.
We introduce the function Prime() for that purpose :

Primes()

Input: Set of substitutability classes : $\mathcal{C}_K$
Output: Set of substitutability classes satisfying primality test in $\mathcal{C}_K$: $\mathcal{P}$
$\mathcal{P} \leftarrow \emptyset$
for $C \in \mathcal{C}_K$ **do**
  If $(\nexists C' \in \mathcal{C}_K : \forall y \in C, \exists y' \in C', \exists v \in \Sigma^+, y = y'v)$
  and $(\nexists C' \in \mathcal{C}_K : \forall y \in C, \exists y' \in C', \exists u \in \Sigma^+, y = uy')$
  $\mathcal{P} \leftarrow \mathcal{P} \cup C$

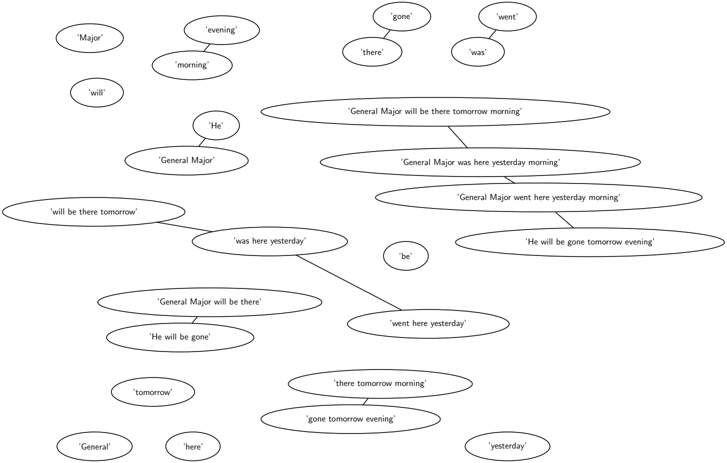
Primality test in K not in L !
but works well in practice and if the sample is informative enough.

On the example

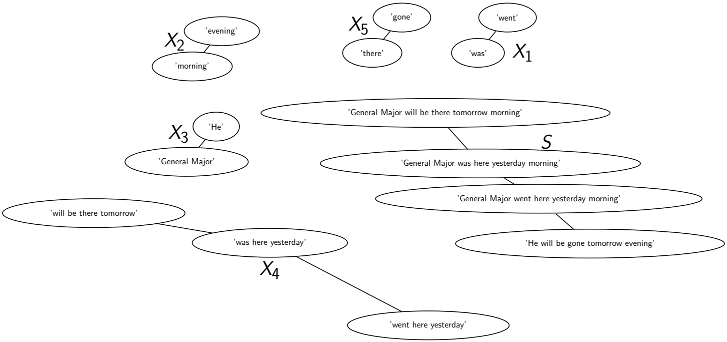


source: Gaelle Garet

On the example



On the example



ReGLiS Part1 (Learning **R**educed **G**rammar by k, l-**L**ocal **S**ubstitutability)
simplified !

Input: Set of sequences K on alphabet Σ, int k, int l
Output: Grammar $G = \langle \Sigma, N_K, S_K, P_K \rangle$

```
/* Prime substitutability classes on K */
```
$\mathcal{C}_K \leftarrow \texttt{Primes}(\texttt{LS\_classes}(K, k, l))$

```
/* Build initial grammar */
```
$N_K \leftarrow \emptyset, P_K \leftarrow \emptyset$

for $C \in \mathcal{C}_K$ **do**

```
    /* A non-terminal for each K-prime */
```
$\quad N_K \leftarrow N_K \cup \{[\![C]\!]\}$

```
    /* A direct production for each substring in the class */
```
$\quad$ **for** $y \in C$ **do**

$\quad\quad P_K \leftarrow P_K \cup \{[\![C]\!] \rightarrow y\}$

```
/* So far, we have built the initial grammar */
```

ReGLiS Part1 (Learning **R**educed **G**rammar by k, l-**L**ocal **S**ubstitutability) simplified !

Input: Set of sequences K on alphabet Σ, int k, int l
Output: Grammar $G = \langle \Sigma, N_K, S_K, P_K \rangle$

```
/* Prime substitutability classes on K */
C_K ← Primes(LS_classes(K, k, l))
/* Build initial grammar */
N_K ← ∅, P_K ← ∅
for C ∈ C_K do
    /* A non-terminal for each K-prime */
    N_K ← N_K ∪ {⟦C⟧}
    /* A direct production for each substring in the class */
    for y ∈ C do
        P_K ← P_K ∪ {⟦C⟧ → y}
/* So far, we have built the initial grammar */
```

Initial 'bottom' grammar

$G =< \Sigma, V_k, P, S >$
$\Sigma = \{$ General, Major, will, be, gone, there, was, went, He, here, tomorrow, yesterday, morning, evening $\}$
$V_k = \{ S, X_1, X_2, X_3, X_4, X_5 \}$
$P = \{$
S $\rightarrow$ General Major will be there tomorrow morning | General Major was here yesterday morning | General Major went here yesterday morning | He will be gone tomorrow morning
$X_1 \rightarrow$ was | went
$X_2 \rightarrow$ morning | evening
$X_3 \rightarrow$ He | General Major
$X_4 \rightarrow$ will be there tomorrow | was here yesterday | went here yesterday
$X_5 \rightarrow$ there | gone
$\}$

- A non-terminal for each K-Prime
- $L(G) = K$ (NO language generalization)
- for each non-terminal N, $L(G, N) = [N]$

2. Reduce right-hand-sides

And generalize at once

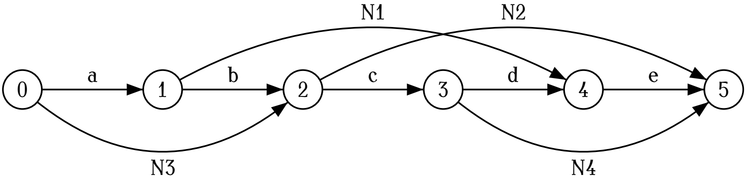
- We 'know' the interesting substitutability classes (but only part of their content)
- If a substring from a substitutability class is present in one right-hand side, we have to replace this occurrence by the non-terminal of the class (Weak-implies-Strong generalization)

$$\frac{N_1 \rightarrow \ldots \beta \ldots,\ N_2 \Rightarrow^* \beta}{N_1 \rightarrow \ldots N_2 \ldots,\ N_2 \Rightarrow^* \beta}$$

- Take care of overlapping occurrences
- Don't keep/introduce redundant rules, introduce most general only
- Some kinship with *Minimal Grammar Parsing* for smallest grammar problem [Carrascosa et al 2011, Gallé, 2011], but with more than one substring per non-terminal

Example

Let us consider that so far the grammar is such that
$P = \{ \ldots ; N \rightarrow abcde \mid \ldots ; N_1 \rightarrow bcd \mid \ldots ; N_2 \rightarrow cde \mid \ldots ; N_3 \rightarrow ab \mid \ldots ; N_4 \rightarrow de \mid \ldots ; \ldots \}$ (where a, b, c, d, e is a terminal or non-terminal symbols)
The parsing graph for $N \rightarrow abcde$ is then :



Example

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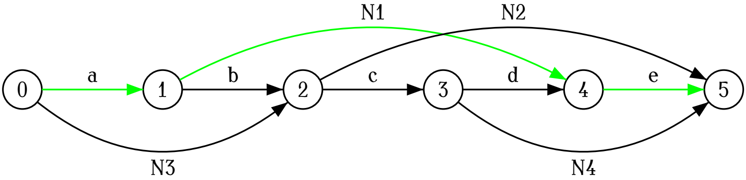

$N \to aN_1e$

Example

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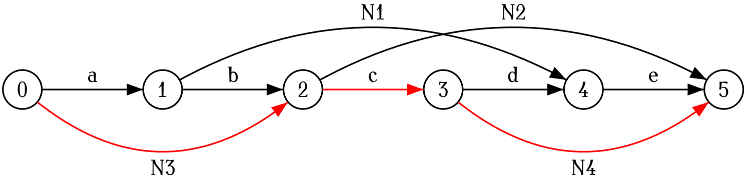

Can be reduced ! (025 strict subsequence[2] of 0235)

$N \rightarrow aN_1e$

2. typo in final version of paper ! replace p7 'strict substring' by 'strict subsequence'

Example

Let us consider that so far the grammar is such that
$P = \{ \ldots ; N \to abcde \mid \ldots ; N_1 \to bcd \mid \ldots ; N_2 \to cde \mid \ldots ; N_3 \to ab \mid \ldots ; N_4 \to de \mid \ldots ; \ldots \}$ (where a, b, c, d, e is a terminal or non-terminal symbols)
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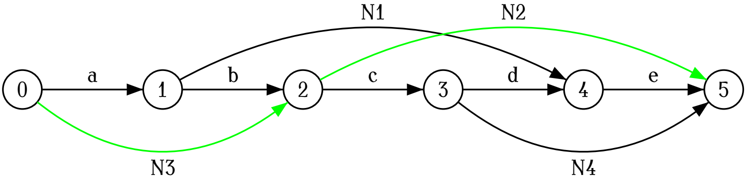

$N \to aN_1e | N_3N_2$

Example

Let us consider that so far the grammar is such that
$P = \{ \ldots ; N \rightarrow abcde \mid \ldots ; N_1 \rightarrow bcd \mid \ldots ; N_2 \rightarrow cde \mid \ldots ; N_3 \rightarrow ab \mid \ldots ; N_4 \rightarrow de \mid \ldots ; \ldots \}$ (where a, b, c, d, e is a terminal or non-terminal symbols)
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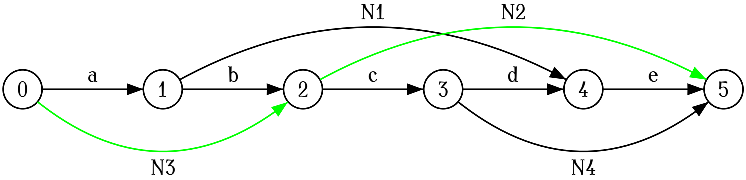


$N \rightarrow aN_1e|N_3N_2$

Implemented by dynamic programming on vertices in function *Non_redundant_rhs*()

ReGLiS Part2

simplified !

```
/* Generalization */
repeat
    P ← P_K ; P_K ← ∅
    /* Branching rules */
    for ([[C]] → α) ∈ P ordered by increasing |α| do
        PG ← Build_parsing_graph(α, P)
        for β ∈ Non_redundant_rhs(PG) do
            P_K ← P_K ∪ ([[C]] → β)
until P_K = P
S_K ← [[C_K(k)]]: k ∈ K
return ⟨Σ, N_K, S_K, P_K⟩
```

Build_parsing_graph

Input: Sequence α, Set of rules P
Output: Parsing graph $\langle V, E \rangle$

```
V ← {i ∈ [0, |α|]} /* vertices */ ; E ← ∅ /* labeled directed edges */
for i ∈ V do
    for j ∈ V : i < j and (i, j) ≠ (0, |α|) do
        if ∃([[C]] → α[i + 1, j]) ∈ P then
            E ← E ∪ (i, j, [[C]])
return ⟨V, E⟩
```

ReGLiS Part2 simplified !

```
/* Generalization */
repeat
    P ← P_K ; P_K ← ∅
    /* Branching rules */
    for ([[C]] → α) ∈ P ordered by increasing |α| do
        PG ← Build_parsing_graph(α, P)
        for β ∈ Non_redundant_rhs(PG) do
            P_K ← P_K ∪ ([[C]] → β)
until P_K = P
S_K ← [[C_K(k)]] : k ∈ K
return ⟨Σ, N_K, S_K, P_K⟩
```

Build_parsing_graph

Input: Sequence α, Set of rules P
Output: Parsing graph $\langle V, E\rangle$

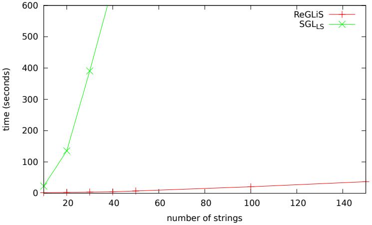
```
V ← {i ∈ [0, |α|]} /* vertices */ ; E ← ∅ /* labeled directed edges */
for i ∈ V do
    for j ∈ V : i < j and (i, j) ≠ (0, |α|) do
        if ∃([[C]] → α[i + 1, j]) ∈ P then
            E ← E ∪ (i, j, [[C]])
return ⟨V, E⟩
```

Final Grammar

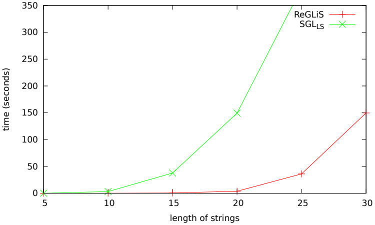
$$\mathbf{S} \rightarrow \mathbf{X_3}\ \mathbf{X_4}\ \mathbf{X_2}$$
$$\mathbf{X_1} \rightarrow \text{was} \mid \text{went}$$
$$\mathbf{X_2} \rightarrow \text{morning} \mid \text{evening}$$
$$\mathbf{X_3} \rightarrow \text{He} \mid \text{General Major}$$
$$\mathbf{X_4} \rightarrow \text{will be } \mathbf{X_5} \text{ tomorrow} \mid \mathbf{X_1} \text{ here yesterday}$$
$$\mathbf{X_5} \rightarrow \text{there} \mid \text{gone}$$

Complexity

- Complexity : $\mathcal{O}(\max(\mathbf{l^3}, \mathbf{l.t}))$
  - l : size of longest sequence
  - t : size of target grammar
- Run time comparison between old and new learning algorithms implementations :



wrt number of strings in training sample



wrt length of strings in training sample

- For our protein experiments : from a day to a few minutes

Protein experiments (10-fold cross-validation)

	Zinc finger			MPI phos.		
	Precision	Recall	F-measure	Precision	Recall	F-measure
Subst.	1	0.1	0.36	1	0.15	0.26
3,3-LS	1	0.2	0.33	1	0.5	0.67
4,4-LS	1	0.25	0.4	1	0.6	0.75
5,5-LS	1	0.33	0.5	**1**	**0.67**	**0.8**
6,6-LS	1	0.5	0.67	1	0.62	0.77
7,7-LS	**1**	**0.55**	**0.7**	1	0.53	0.69
SCFG	1	0.1	0.18	1	0.3	0.46
[Dyrka & Nebel,	0.15	1	0.26	0.5	1	0.67
2009]	0.75	0.87	0.85	0.98	0.89	0.93

	PS00219			PS00063		
	Precision	Recall	F-measure	Precision	Recall	F-measure
Subst.	1	0.2	0.33	1	0.23	0.37
3,3-LS	**1**	**0.72**	**0.84**	1	0.58	0.73
4,4-LS	1	0.7	0.82	1	0.6	0.75
5,5-LS	1	0.68	0.8	1	0.66	0.8
6,6-LS	1	0.6	0.75	**1**	**0.7**	**0.82**
7,7-LS	1	0.5	0.67	1	0.65	0.79
Prosite	1	0.6	0.75	1	0.8	0.89
SCFG	-	-	-	1	0.05	0.1
[Dyrka & Nebel,	-	-	-	0.1	1	0.18
2009]	1	1	1	0.79	0.65	0.71

Conclusion

- ReGLiS : (the ReG.*iS family : ReGiS, ReGCis, ReGLiS, ReGLCiS)
  - Bottom-up generalization from initial grammar
  - Efficient by dynamic programming on parsing graph
  - No parsing required, iterative
- Practical algorithm
  - faster inference
  - faster parsing (non redundant minimal grammar)
- Reduced grammar
  - Easier to read
  - Canonical form ! cf [Clark, 2013]

  → Polynomial dentification in the limit property (cf Remi's talk yesterday)
- Confirmation of good results on proteins (with some preprocessing but no statistical parameters)
- Another step towards practical application of (local-)substitutability

Perspectives

- Practical
  - Choice of initial classes : data-driven heuristics
  - Grammar weighting for biological sequences
  - Better understand why (local-)substitutability seems pertinent for biological sequences. . .
  - Prototype to efficient implementation ?
- Theoretical
  - Better understand/characterize interest of outer loop in generalization wrt *SGL*
  - What happens exactly when sample is not characteristic ?
  - Is it possible to ensure always returning a grammar in the target class ?

And attend Gaelle's thesis (Dec 2014)