Reasoning by cases without contraposition in default logic

or the Birth of Pre-requisite free Defaults

(Version légèrement remaniée, le 24 avril 1995, de [13])

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Abstract. Default logic, one of the best known formalisms to express common sense reasoning, does not allow to reason by cases in its standard formulations. We propose a natural and easy way of translating rules with exceptions into standard defaults, which allows to reason by cases without giving any unwanted contraposition. This solution is the simplest possible extension to the first coming idea consisting in taking the defaults by sets. Thus we keep the attractive simplicity of default logic basically intact. Our solution is independant of the notion of "commitment" and can as well be applied to the versions of default logic caring about commitment. We provide several examples in order to make precise what is wanted. We study various other proposals in the literature, showing that either they give some kind of unwanted contraposition or that they need a new and non standard form of disjunction and that they greatly modify the notion of default reasoning.

1 Introduction

Default logic as defined in [15] is a very appealing formalism to express common sense reasoning and mainly rules with exceptions, being natural and powerful. One problem is that it does not deal correctly with reasoning by cases: Given that 1) *Tweety is a bird or a bat*, 2) *birds fly (with exceptions)*, and 3) *bats fly (with exceptions)*; we cannot conclude that *Tweety flies*. Several propositions allow to reason by cases [3, 2, 14, 4, 18] but they also give the contraposition of the rules involved, and this may be unexpected. Knowing that *birds fly (with exceptions),* we do not always want to conclude that *non flying animals are not birds (with exceptions).* For instance, imagine we are writing a field guide about *birds and bats*, then we are describing a world in which: *1) birds fly (with exceptions), 2) bats fly (with exceptions), and 3) a non flying animal is a bird (with exceptions);* which cannot be done naturally if every rule comes with its contraposition.

We modify Reiter's definition in order to be able to reason by cases, without giving unwanted contrapositions of the rules. We precise in the next two sections what we want exactly, thanks to basic examples and thanks to the oldest proposals (which use defaults without prerequisite). The most natural way of modifying default logic in order to be able to reason by cases is to allow to take the defaults by sets, instead of individually. Several authors have proposed such a solution, but generally only in a few lines at the end of a paper, stating that this a good idea but that there was not enough space left in the paper to develop it. In section 3 we study this option, showing why the naïve way is highly unsatisfactory. Then, we propose what we think are the most natural improvements of this simple idea which give more satisfactory results. We show that this solution is in fact equivalent to a solution using (individually) a new kind of defaults without pre-requisite. Thus, this solution is simpler yet that the usual way of using defaults. After having motivated our solution, we verify that our formal definitions comply with our requirements, thanks to representative exemples. In section 4, we show what is wrong with the solutions suggested by several authors. Particularly we show why the proposal of [6, 9, 20] is not completely satisfying. Firstly, we must accept to live with two different kinds of disjunctions. These two distinct disjunctions can be bewildering for the user. Secondly this solution restricts default reasoning to its "skeptical" or "cautious" form, which is not a harmless restriction.

2 The motivating examples

Let us first remind the classical definition of Reiter's extensions. W is a set of formulas in a propositional language $\mathcal{L}$. D is a set of *defaults*, that is "rules" written $\frac{a : b_1, \ldots, b_n}{c}$, where a, b_i and c are formulas in $\mathcal{L}$. A default may be understood as: "If a is assumed, and if each b_i is possible, then let us assume c". a is the *prerequisite*, b_i a *justification* and c the *conclusion* of the default. A default such as $\frac{a : c}{c}$ is called *normal* and $\frac{a : b_1 \wedge c, \ldots, b_n \wedge c}{c}$ is *semi-normal* (every justification implies the consequent).

Definition 2.1 [15] A set E is an extension (here called *R-extension*) of a default theory (W, D) iff there exists (E_i) with $E = \cup E_i$, where $E_0 = Th(W)$ and $E_{i+1} = Th(E_i \cup \{c \;/\; \frac{a : b_1, \cdots, b_n}{c} \in D, a \in E_i, \neg b_j \notin E \ (1 \leq j \leq n)\})$.

Here is the fundamental example, introducing the problem.

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ECAI 94. *11th European Conference on Artificial Intelligence* Edited by A. Cohn
Published in 1994 by John Wiley & Sons, Ltd.

Example 2.1 $W = \{a \vee b\}, \quad D = \{d_1 = \frac{a : c}{c},\ d_2 = \frac{b : c}{c}\}$.
This default theory has one extension $E = Th(a \vee b)$: the defaults are of no use. You may think in a as meaning "*Tweety is a bird*", b meaning "*Tweety is a bat*", and c meaning "*Tweety can fly*". This behavior is a serious drawback of default logic: reasoning by cases is not allowed. From W only we cannot conclude a (or b), and default theory allows to consider only one default at a time.

One default is enough to exhibit this problem:

Example 2.2 $W = \{a \vee c\}, D = \{\frac{a : c}{c}\}$. (W, D) has one extension $Th(a \vee c)$. Again, the expected conclusion would be c: knowing "*Tweety is a bird or Tweety can fly*" and "*if Tweety is a bird then he can fly (if Tweety is not an exception)*", it seems natural to conclude that "*Tweety can fly*".

Remark 2.1 A proposition which concludes c in ex. 2.1 and not in ex. 2.2 must be rejected. Indeed, from $\{\frac{a : c}{c}, \frac{\neg a : c}{c}\}$, we would conclude c, even with $W = \emptyset$. Now, when the default $\frac{\neg a : c}{c}$ is replaced by the corresponding "rule" without exception $\neg a \Rightarrow c$ (standard implication), we should *a fortiori* conclude c (a rule without exception should be at least as powerful as the corresponding rule with exceptions).

Some solutions have been given. First came the defaults without prerequisite, introduced in [3] and regularly rediscovered since (see e.g. [14, 4]). A rule such as "a's are b's (with exceptions)" is not translated by $d = \frac{a : b}{b}$, but by $d' = \frac{: a \Rightarrow b}{a \Rightarrow b}$. Examples 2.1 and 2.2 become respectively:

Example 2.3 $W = \{a \vee b\}, \quad D = \{\frac{: a \Rightarrow c}{a \Rightarrow c}, \frac{: b \Rightarrow c}{b \Rightarrow c}\}$
There is one extension $Th(a \vee b, a \Rightarrow c, b \Rightarrow c)$, which means that the defaults can be used here ($c \in E$), as expected.

Example 2.4 $W = \{a \vee c\}, \quad D = \{d = \frac{: a \Rightarrow c}{a \Rightarrow c}\}$.
Again, the only extension $Th(a \vee c, a \Rightarrow c)$ contains c.

So far, so good; however this translation gives the contraposition of every rule with exceptions:

Example 2.5 $W = \{\neg c\}, \quad D = \{d = \frac{: a \Rightarrow c}{a \Rightarrow c}\}$.
The only extension $Th(\neg c, a \Rightarrow c)$ contains $\neg a$, which could be unexpected.

Generally, authors agree in thinking that an appropriate translation of rules with exceptions should **not** give the automatic contraposition of the rules. Thus, this solution is not appropriate in every situation. [7] made an improvement: the rule "a's are b's (with exceptions)" is translated into the semi-normal default without prerequisite: $\frac{: b}{a \Rightarrow b}$. In [14], the same trick is used, except that the justifications are conjuncted, not only juxtaposed as in Reiter's definition. Let us look what happens to our basic examples with [7]'s solution.

Example 2.6 $D = \{d_1 = \frac{: c}{a \Rightarrow c},\ d_2 = \frac{: c}{b \Rightarrow c}\}$.

a) $W = \{a \vee b\}$: the only extension contains c.
b) $W = \{\neg c\}$: the only extension $Th(\neg c)$ does not contain $\neg a$.

Example 2.7 $D = \{d = \frac{: c}{a \Rightarrow c}\}$

a) $W = \{a \vee c\}$: the only extension $Th(a \vee c, a \Rightarrow c)$ contains c.
b) $W = \{\neg c\}$: the only extension $Th(\neg c)$ does not contain $\neg a$.

Thus it seems that this is the ideal solution: we do not get the contraposition (the justification c is stronger now, and it prevents the use of the rule when we know $\neg c$) while reasoning by cases is possible. The problem is that some "shadow contraposition" remains, as the following two examples show:

Example 2.8 $W = \{\neg b \vee \neg c\}, \quad D = \{\frac{: b}{a \Rightarrow b}, \frac{: c}{a \Rightarrow c}\}$.
The only extension $Th(\neg b \vee \neg c, a \Rightarrow b, a \Rightarrow c)$ contains an unwanted answer $\neg a$. Again, one default is enough to provoque this shadow contraposition:

Example 2.9 $W = \{\neg a \vee \neg c\}, \quad D = \{\frac{: c}{a \Rightarrow c}\}$:
$\neg a \in Th(\neg a \vee \neg c, a \Rightarrow c)$.

In example 2.8, Poole's solution does not give the unwanted $\neg a$. This is due to the particular treatment of the justifications in [14]: the two defaults d_1 and d_2 cannot be applied simultaneously without contradicting $\neg b \vee \neg c$. However, Poole's solution also falls pray to example 2.9, and thus it does not avoid the shadow contraposition problem. We have to find a better solution. Also, Pooles treatment of the justification is not classical, even if some "representational" grounds have been given to this conjunction of justifications (see papers about "commitment" in default logic). The primary aim of such a modification was apparently not to improve the expressivity, but to simplify the complexity of the search for extensions (a very desirable goal indeed).

3 The quest for a natural solution

A lot of authors (among them is [21]) have tried to include reasoning by cases in default logic with the natural solution allowing **sets** of defaults instead of isolated defaults only. Definition 2.1 is thus modified: let us say that a default $\frac{a : b}{c} \in D$ is *applicable at stage i* if and only if[2]: $E_i \vdash a$ and $E \not\vdash \neg b$, and that when a default is *applied at stage i*, then we add c to E_i. Here is definition 2.1 when *sets of defaults* are allowed:

Definition 3.1 A finite set of defaults $\{\frac{a_\alpha : b_\alpha}{c_\alpha}\}_{\alpha \in I} \subseteq D$ is *applicable at stage i* if and only if: $E_i \vdash \bigvee_{\alpha \in I} a_\alpha$ and $E \not\vdash \neg b_\alpha$ for every $\alpha \in I$. When a set of defaults is *applied at stage i*, we add $\bigvee_{\alpha \in I} c_\alpha$ to E_i.

In the basic situation for reasoning by cases, **example 2.1**, we get the expected answer c, using the set D. Here is the behavior of this definition with the other typical examples.

Example 3.1 (Contraposition, cf examples 2.5 and 2.7). $D = \{\frac{a : b}{c}\}$, $W = \{\neg c\}$. We do not get the unwanted $\neg a$.

Example 3.2 (Shadow contraposition, cf example 2.8).
$W = \{\neg c' \vee \neg c\}$, $D = \{\frac{a : b}{c}, \frac{a : b'}{c'}\}$. We do not get the unwanted answer $\neg a$: we cannot use the set D, as $W \not\vdash (a \vee a)$.

However, things get not so good with the other examples:

[2] For the sake of simplicity in notations, we use defaults with only one justification b. Remind that for the same reason, we restrict ourselves to propositional formulas.

Example 3.3 (cf ex. 2.2, 2.4 and 2.7.a)

$W = a \vee c \vee e$, $D = \{d = \frac{a:b}{c}\}$. The only extension is nothing more than $Th(W)$, as d is not applicable. This is contestable, if we consider that from $(W, D' = \{d, \frac{c:b}{c}, \frac{e:b}{e}\})$ we would deduce $c \vee e$. A default such as $\frac{c:b}{c}$ provoques some effects, which is a major drawback.

Example 3.4 $W = \neg c$, $D = \{d_1 = \frac{a:b}{c}, d_2 = \frac{\neg a:b}{\neg a}\}$.

From $\neg a \vee a$, using the set $\{d_1, d_2\}$, we get $c \vee \neg a$, which, with $\neg c$, gives $\neg a$. However, if d_2 is removed from D, we do not get anything more than $Th(\neg c)$, as d_1 is not applicable. Here the effect of the default $\frac{\neg a:b}{\neg a}$ is that we get an unwanted "contraposition".

We solve the problem of the last two examples with two modifications:

1) We add the (generally infinite) set of defaults $\{\frac{a:}{a}$ / for any sentence a in the language $\boldsymbol{\mathcal{L}}\}$.

2) We strengthen the justifications. Several arguments justify this. Firstly, as we had to weaken the conditions on the prerequisite, it seems judicious to strengthen the conditions on the justifications. Konolige has solved part of the problem in this way. Secondly, when the prerequisite a must be established, it is useless to add that a must be "possible". But now, it is important to add this condition, if we want to keep the original spirit of defaults with prerequisite, as far as possible.

Definition 3.2 Let $T = (W, D)$ be a default theory, we define the set $D_{\boldsymbol{\mathcal{L}}} = D \cup \{\frac{a:}{a}$ / for any sentence a in $\boldsymbol{\mathcal{L}}\}$.

A finite set of defaults $\{\frac{a_\alpha : b_\alpha}{c_\alpha}\}_{\alpha \in I} \subseteq D_{\boldsymbol{\mathcal{L}}}$ is *applicable at stage i* iff: $E_i \vdash \bigvee_{\alpha \in I} a_\alpha$ and $E \not\vdash \neg(a_\alpha \wedge b_\alpha)$, for any $\alpha \in I$.

When a finite set of defaults is *applied at stage i*, then we add $\bigvee_{\alpha \in I} c_\alpha$ to E_i.

Examples 2.1 and 2.2 behave nicely with this definition, however, with the next example, we get into troubles.

Example 3.5 (cf example 2.9) $W = \{\neg a \vee \neg c\}$, $D = \{\frac{a:b}{c}\}$.

There is no extension here. The set $D \cup \{\frac{\neg a:}{\neg a}\}$, should be applicable, as neither $\neg(a \wedge b)$ nor a is in $Th(W)$. However, when we apply this set, we get $\neg a \vee c$ and thus $\neg a$: this set should not have been applied.

This classical behavior with non normal defaults is unacceptable here, being a consequence of our definitions of applicability of sets of defaults. While trying to solve the problem of reasoning by cases, we fell into the more serious problem of inexistence of extensions. For solving this problem, we strengthen again the justification, adding the condition that the consequent must also be possible. This can be done in various ways, we choose the simplest, and most fruitful one.

Definition 3.3 • A finite set of defaults $\{\frac{a_\alpha : b_\alpha}{c_\alpha}\}_{\alpha \in I} \subseteq D_{\boldsymbol{\mathcal{L}}}$ is *applicable at stage i* if and only if: $E_i \vdash \bigvee_{\alpha \in I} a_\alpha$ and $E \not\vdash \neg(a_\alpha \wedge b_\alpha \wedge c_\alpha)$, for any $\alpha \in I$. When a set of defaults is *applied at stage i*, we add $\bigvee_{\alpha \in I} c_\alpha$ to E_i. Precisely:

$T = (W, D)$, $D_{\boldsymbol{\mathcal{L}}} = \{\frac{a_\alpha : b_\alpha}{c_\alpha}\}_{\alpha \in J}$. E is a set of sentences in $\boldsymbol{\mathcal{L}}$. We define: $E_0 = Th(W)$, $E_{i+1} = Th(E_i \cup \{\bigvee_{\alpha \in I} c_\alpha$ / there is a finite $I \subseteq J$ such that $E_i \vdash \bigvee_{\alpha \in I} a_\alpha$ and $E \not\vdash \neg(a_\alpha \wedge b_\alpha \wedge c_\alpha)$ for any $\alpha \in I\})$. E is a *C-extension* of T iff $E = \cup E_i$.

One argument against this definition is that it "semi-normalizes" every default, thus modifying somehow its intended meaning. This is necessary to avoid unexpected inexistency of extensions. Moreover, the non semi-normal defaults have a rather strange behavior and semi-normal defaults are expressive enough and allow modularity.

We sum up the modifications made from definition 2.1:

1) The "implicit defaults" $\frac{\phi :}{\phi}$ are useless in definition 2.1 but they must be added when sets of defaults are allowed.

2) When a default is taken in combination with other defaults, it becomes necessary to precise that its pre-requisite must be possible. Also the consequent must be possible, otherwise cases of inexistence of extensions would become too numerous and unforeseeable.

This new definition can be given a much simpler form:

Definition 3.4 Let (W, D) be a default theory. D' is the set of defaults $\{\frac{: a \wedge b \wedge c}{a \Rightarrow c}$ / $\frac{a:b}{c} \in D\}$. E is a *C'-extension* of (W, D) iff E is a R-extension of (W, D').

This definition greatly reduces the complexity of the search for the extensions. C-extensions are more complex than ordinary R-extensions: we add an infinite set of defaults and at each stage i we check the applicability of every subset of defaults. With C'-extensions, we get a much simpler notion than ordinary R-extensions. Indeed, defaults without pre-requisite simplify considerably the search for extensions:

Property 3.1 Let $D' = \{\frac{: a_j \wedge b_j \wedge c_j}{a_j \Rightarrow c_j}\}_{j \in I}$, $D = \{\frac{a_j : b_j}{c_j}\}_{j \in I}$, $E'_1 = Th(W \cup \{a_j \Rightarrow c_j$ / $j \in I$ and $E \not\vdash \neg(a_j \wedge b_j \wedge c_j)\})$.
E is a C'-extension of (W, D) iff $E = E'_1$.

This is definition 2.1 applied to defaults without pre-requisite.

Property 3.2 E is a $C-$extension of (W, D) if and only if E is a $C'-$extension of (W, D).

(Proof somehow tedious, but without real difficulties.)

Thus, after [3, 14, 7] we have rediscovered again defaults without pre-requisite (however there is some mild kind of pre-requisite as a takes place in the justification and thus plays a role, enhenced by the presence of c). The filiation between our proposal and those of [3, 7] is obvious. The filiation from [14] is less obvious, due to the different nature of Poole's defaults. We give only the idea, providing a modification to Poole's method: Instead of taking only $b \wedge c$ as justification, we take $a \wedge b \wedge c$. In Poole's term, this amount to take the *constraint* $\neg d \Leftarrow (\neg a \vee \neg c)$ instead of $\neg d \Leftarrow \neg c$ as given in [14]. This modification solves the problem of example 2.9 with Poole's definition exactly as modifying the default $\frac{:c}{a \Rightarrow c}$ of [7] into $\frac{: a \wedge c}{a \Rightarrow c}$ solves this problem with Konolige's defaults.

Now, we examine how our definitions behave with the basic examples (we may use either C-extensions or C'-extensions):

Example 3.6 (cf ex. 3.2) $W = \{\neg c' \vee \neg c\}$, $D = \{\frac{a:b}{c}, \frac{a:b'}{c'}\}$.
We do not get the unwanted $\neg a$. We cannot use the set $D \cup \{\frac{\neg a:}{\neg a}\}$, as $W \cup \{\neg a \vee c, \neg a \vee c'\} \vdash \neg a$. There are two C-extensions: $Th(\neg c' \vee \neg c, \neg a \vee c)$ and $Th(\neg c' \vee \neg c, \neg a \vee c')$.

Example 3.7 (cf ex. 3.5) $W = \{\neg a \vee \neg c\}$, $D = \{d_1 = \frac{a:b}{c}, d_2 = \frac{\neg a:b'}{c'}\}$. We do not get the unwanted result c' (or any unwanted result). Indeed, the only C-extension is $Th(\neg a \vee \neg c, a \vee c')$. No set including d_1 is applicable.

Example 3.8 (cf ex. 3.1 and 3.4) $W = \emptyset$, $D = \{d_1 = \frac{a:b}{c}\}$ (or equivalently now $D = \{d_1, \frac{\neg a:b}{\neg a}\}$). We get one C-extension $Th(\neg a \vee c)$.

Example 3.9 $W = \emptyset$, $D = \{\frac{:b}{c}, \frac{:\neg b}{c'}\}$. We get one C-extension $Th(c, c')$. The two defaults have been used (individually) even if one justification contradicts the other one. This is what happens with the original definitions 2.1. We want to stay as close as possible to these definitions.

Example 3.10 $W = \{a \vee a'\}$, $D = \{\frac{a:b}{c}, \frac{a':\neg b}{c'}\}$.
We get one C-extension $Th(a \vee a', c \vee c')$ (cf above comment).

Example 3.11 $W = \{\neg(b \wedge a \wedge a'), a \vee a'\}$.
- $D_1 = \{\frac{a:b}{c}, \frac{a':b}{c}\}$: one C-extension $Th(\neg(b \wedge a \wedge a'), a \vee a', c)$.
- $D_2 = \{\frac{a \vee a':b}{c}\}$: we get exactly the same C-extension.

However, with e.g. $W = \{\neg a \vee \neg b \vee \neg c\}$, the two sets of defaults would give different results: $(a \vee a') \Rightarrow c$ would be in the extension for D_2, while only $a' \Rightarrow c$ would be in the extension for D_1, and not $a \Rightarrow c$. This feature seems acceptable.

Example 3.12 $W = \{a \vee c, \neg a \vee \neg b\}$, $D = \{\frac{a:b}{c}\}$. We get one C-extension $Th(W)$. If you are not conviced by abstract propositions, think in a as *"Tweety is a bird"*, b as *"Tweety has normal wings"* and c as *"Tweety can fly"*. Then, knowing that *"Tweety is a bird or Tweety can fly"*, *"Tweety is not a bird having normal wings"* and that *"Any bird can fly, except if we know that it does not have normal wings"*, it does not seem judicious to conclude that *"Tweety can fly"*.

4 Comparison with other proposals

Konolidge's solution came from a modal translation of default logic. Another proposal of this kind comes in [18] (or slightly modified in [17], these differences may be ignored here):

Definition 4.1 [18] Every default $\frac{a:b}{c} \in D$ is translated into a formula $a \wedge Hb \Rightarrow Lc$, H and L being two modal operators, L obeying to the rules of the system T: if $\vdash \alpha$, then $\vdash L\alpha$ (*necessitation rule*), $Lp \Rightarrow p$ (*axiom schema of necessity*) and $L(p \Rightarrow q) \Rightarrow (Lp \Rightarrow Lq)$ (*axiom schema of consequence*).

An *S-extension* E is a set, including all the translated defaults, all the formulas $L\phi$ where $\phi \in W$ and all the instances of the axiom schema: $Hp \Rightarrow \neg L \neg p$. Finally E must be a minimal set verifying the following properties:
– closed for consequence in T;
– completed in $H\phi$ (for every $H\phi$ appearing in some translated default, either $H\phi$ is in E or $\neg H\phi$ is in E);
– maximal in $\{H\phi\}$: there exists no E' verifying the above conditions and such that $\{H\phi / H\phi$ appears in some translated default and $H\phi \in E\} \subset \{H\phi / H\phi$ appears in some translated default and $H\phi \in E'\}$.

In S-extensions we are only interested with the non modal formulas ϕ such that $L\phi \in E$. This set corresponds roughly to an extension of (W, D). Also, as there are quite a lot of such extensions, there is an optional test: we may consider only the S-extensions E in which $L\neg p \in E$ whenever $\neg Hp \in E$.

In [18], the standard definition translates $\frac{a:b}{c}$ into $La \wedge Hb \Rightarrow Lc$. The translation given here is also presented (ex. 6 in [17, 18]), as a convenient way of introducing reasoning by cases in default reasoning, without the contraposition. We develop this point, showing that even if this formalism solves some of the problems listed above, it is not really satisfactory.

Example 4.1 (cf ex. 2.5, 2.6.b, 2.7.b) $D = \{\frac{a:b}{c}, \frac{a':b'}{c}\}$.
a) $W = \{a \vee a'\}$ gives one extension which contains the expected answer c ($Lc \in E$, because E contains $a \wedge Hb \Rightarrow Lc, a' \wedge Hb' \Rightarrow Lc$ and $a \vee a'$).
b) If $W = \{\neg c\}$, then we cannot derive the unwanted result $\neg a$. More precisely, we may derive $\neg a$ (and $\neg a'$ as well), from $L\neg c$ which gives $\neg Lc$ and from $a \wedge Hb \Rightarrow Lc$, but we cannot derive $L\neg a$ (or $L\neg a'$) so, by definition 4.1 we conclude that $\neg a$ is **not** in the "extension".

Example 4.2 (cf ex. 2.1, 2.3, 2.7.a) $W = \{a \vee c\}, D = \{\frac{a:b}{c}\}$.
We cannot get the expected result c, i.e. no extension E contains Lc. Indeed, we have $L(a \vee c)$ (thus $a \vee c$), and $a \Rightarrow Lc$, but this does not give Lc. This is a first serious drawback.

Example 4.3 (cf ex. 2.8) $W = \{\neg c' \vee \neg c\}, D = \{\frac{a:b}{c}, \frac{a:b'}{c'}\}$.
We do not get the unwanted answer $\neg a$. Indeed, from $L(\neg c' \vee \neg c)$, $a \wedge Hb \Rightarrow Lc$ and $a \wedge Hb' \Rightarrow Lc'$, we get $\neg a$ (assuming as usual Hb and Hb'), but we cannot get $L\neg a$.

We do not get the bewildering "shadow contraposition" in this example. However, examples 4.1.b and 4.3 are rather intriguing, because even if we do not get $L\neg a$, we get $\neg a$, which means that a default with $\neg a$ as prerequisite can be used (thus adding $\frac{\neg a:b'}{\neg a}$ will modify the extension!). The following example, where we get a kind of "shadow contraposition", confirms that this can give unwanted results.

Example 4.4 $W = \{\neg a \vee \neg c\}$, $D = \{\frac{a:b}{c}, \frac{\neg a:b'}{c'}\}$.
We get the unwanted result c' (Lc' is in the only extension).

Another modal proposal needs some development. [9, 20, 6]'s proposal has three variants, a modal one, a default logic one and an "extended logic programming" one [5]. A new kind of disjunction (called "effective" or "constructive" in [20]) is defined. $a|b|c$ means informally that we "must have" a, or b, or c. This disjunction is itself a new kind of default. This new disjunction may only appear in the consequent of the defaults, and cannot be combined with the other logical connectors. However, defaults without prerequisite and without justification are allowed, so that $a|b|c$ is indeed $\frac{:}{a|b|c}$ ([6]).

Definition 4.2 [6] $d = \frac{a:b}{c_1|\cdots|c_n}$ is a *disjunctive default*, D is a set of disjunctive defaults. A *disjunctive extension* E of D, is one of the minimal deductively closed sets of sentences E' satisfying the condition: For any default d in D, if $a \in E'$ and $\neg b \notin E$, then for some $i \in \{1, \cdots, n\}$, $c_i \in E'$.

A *disjunctive theorem* is a sentence which belongs to all disjunctive extensions. Note that there is no W here, the authors use defaults such as $\frac{:}{a}$ which they assimilate to elements $a \in W$, to keep the standard writting of default theories.

We give only the flavour of the modal counterpart: $\frac{a:b}{c_1|\cdots|c_n}$ is translated by: $La \wedge \neg L\neg b \Rightarrow Lc_1 \vee \cdots \vee Lc_n$ [6].

Here are some basic examples:

Example 4.5 (cf ex. 2.1, or (11) in [6]). $D{=}\{\frac{a:b}{b}, \frac{c:d}{d}, a|c\}$, $W=\emptyset$. We get two disjunctive extensions $E=Th(a,b)$ and $E'{=}Th(c,d)$, thus $E \cap E'{=}Th((a \wedge b){\vee}(c \wedge d))$ as expected.

Example 4.6 [6] $D{=}\{\frac{:b}{a|b}, \frac{:a}{a|b}\}$, $W{=}\emptyset$. We get two disjunctive extensions $E{=}Th(a)$, $E'{=}Th(b)$, thus $E \cap E'{=}Th(a \vee b)$.

A solution of this kind is satisfactory in that it allows to reason by cases as in example 4.5 without any unwanted contraposition. However, there are drawbacks:

• Such a definition imposes the introduction of a new connector "|", which must live together with the standard "∨". The real starting motivation for introducing reasoning by cases, example 2.1, has not been solved: from $a \vee b$, we do not conclude anything, we need to know $a|b$ instead. And it is not easy to determine whether a disjunctive default theory "entails $a|b$" or not (i.e. whether adding $a|b$ to D is harmless or not). Moreover, $(a|b) \Rightarrow (c|d)$ and $\neg(a|b)$ are not defined.

• Definition 4.2 takes the intersection of all the extensions (note that taking only the minimal sets is useless in definition 4.2 when we are interested only by the intersection of all the extensions). The effect is that the notion of default reasoning is modified. Only "skeptical" or "cautious" reasoning is allowed. This not so minor restriction in the way defaults are used is not always a desirable feature.

5 Conclusion

We have studied the problem of reasoning by cases in default logic. Thanks to basic examples, we have precised what is wanted and what is unwanted. This collection of examples may be used as a benchmark for any proposition of default logic dealing with reasoning by cases. We have provided a simple definition which is a natural modification of Reiter's definition allowing to reason by cases. In order to do this, we started from the classical defaults, taking them by sets. Then we have exhibited some serious drawbacks of this naïve solution, and, trying to eliminate these drawbacks in the most natural way, we ended up with our solution. This solution uses defaults by sets, with some more complications. The good news is that this natural solution is equivalent to a much simpler solution, using the defaults in the standard way, and moreover using only some kind of pre-requisite free defaults.

This definition cannot be implemented as it stands, because of the tests of unprovability. This proposal is to be understood as a precise formalization of an important expressiveness problem in common sense reasoning. It remains to design a practical approach of this ideal. This is the situation existing yet with standard default logic, except that standard default logic cannot be considered as an ideal specification because of its lack of reasoning by cases. Moreover, our proposal using only pre-requisite free defaults, its overall complexity is far inferior to the complexity of standard default logic.

We have also compared our solution to the existing literature, providing new insights. All the proposals we know of, either introduce a new kind of disjunction, or fall prey to the "shadow contraposition" problem.

Deliberately, we wanted to stay as close as possible to standard default logic. Reasoning by cases is a desirable feature even if the notion of commitment is not taken into account. However, our solution could also be applied to some variants of default logic which care about commitment: we have also given it using Poole's formalism.

ACKNOWLEDGEMENTS

This work has been partially sponsored by the EC project DRUMS-II (ESPRIT III BR, project 6156).

REFERENCES

[1] Philippe Besnard. Une procédure décision en logique non monotone. Thèse de l'Université de Rennes I, Sept 1983.

[2] Philippe Besnard, *An Introduction to Default Logic*, Springer Verlag, Heidelberg, 1989.

[3] Philippe Besnard, René Quiniou, and Patrice Quinton, 'A decidable subset of default logic', in *AAAI*, pp. 27–30, Washington, (August 1983). Morgan Kaufmann.

[4] James P. Delgrande and W. Ken Jackson, 'Default Logic Revisited', in *KR'91*, pp. 118–127. Morgan Kaufmann, (1991).

[5] Michael Gelfond and Vladimir Lifschitz, 'Classical Negation in Logic Programs and Disjunctive Databases', *New Generation Computing*, **9**, 365–385, (1991).

[6] Michael Gelfond, Vladimir Lifschitz, Halina Przymusinska, and Mirosław Truszczyński, 'Disjunctive Defaults', in *KR'91*, pp. 230–237. Morgan Kaufmann, (1991).

[7] Kurt Konolige, 'On the relation beween default and autoepistemic logic', *Artificial Intelligence*, **35**, 343–382, (1988).

[8] Vladimir Lifschitz and Thomas Y. C. Woo, 'Answer Sets in General Nonmonotonic Reasoning (Preliminary Report)', in *KR'92*, pp. 603–614, Cambridge, (1992). Morgan Kaufmann.

[9] Fangzhen Lin and Yoav Shoham, 'Epistemic semantics for fixed-point non-monotonic logics', in *TARK-90*, pp. 111–120. Morgan-Kaufmann, (1990).

[10] Witold Łukaszewicz, 'Minimization of abnormality: A simple system for default reasoning', in *ECAI*, pp. 382–388, Brighton, (1986).

[11] Witold Łukaszewicz, 'Considerations on default logic: an alternative approach', *Computational Intelligence*, **4**, 1–16, (1988).

[12] Victor Marek and Mirosław Truszczyński, 'Relating autoepistemic and default logics', in *KR'89*, pp. 276–288, (1989).

[13] Yves Moinard, 'Reasoning by Cases without Contraposition in Default Logic', in *ECAI*, pp. 381–385, 1994.

[14] David L. Poole, 'A logical framework for default reasoning', *Artificial Intelligence*, **36**, 27–47, (1988).

[15] Raymond Reiter, 'A logic for default reasoning', *Artificial Intelligence*, **13**, 81–132, (1980).

[16] Pierre Siegel and Camilla Schwind, 'Hypothesis theory for nonmonotonic reasoning', in *Workshop on Non-standard Queries and Answers*, pp. 189–210, Toulouse, (1–3 july 1991).

[17] Pierre Siegel and Camilla Schwind, 'Modal Semantics for Hypothesis Theory', in *4^{th} Int. Workshop on Nonmonotonic Reasoning*, pp. 200–217, Plymouth, Vermont, (1992).

[18] Pierre Siegel and Camilla Schwind, 'Modal logic based theory for non-monotonic reasoning', *Journal of Applied Non-Classical Logics*, **3**(1), 73–92, (1993).

[19] Mirosław Truszczyński, 'Embedding default logic into modal nonmonotonic logics', in *Proc. of the First Int. Workshop on Logic Programming and Non-monotonic Reasoning*, pp. 151–165, Cambridge, (1991). MIT Press.

[20] Mirosław Truszczyński, 'Modal Interpretations of Default Logic', in *IJCAI*, pp. 393–398, Sydney, (1991). Morgan Kaufmann.

[21] Nic Wilson, 'Default Logic and Belief Functions', in *ESPRIT Project DRUMS RP1 1st Workshop*, Marseille, (February 1990).