Loop Tiling an n-Dimensional Loop 2-Dimensionally

Aditya Gupta
IRISA, Rennes

IRISA

OUTLINE

- Introduction
- Background
- Strategies for 2-Dimensional Tiling of a n-Dimesional Loop
- Mathematical Analysis
- Best Among 2-Dimensional Tiling Strategies
- Comparison with n-Dimensional Strategy
- Limitations, Ongoing And Future Work

Introduction

- Tiling

Introduction

- Tiling
- Problem Specification

for i = 1 to N_1

..

for i = 1 to N_i

..

for i = 1 to N_{n-1}

for i = 1 to N_n

- Tiling Inner Most Two Loops

for i = 1 to N_1

..

for i = 1 to N_{n-2}

for i = 1 to T_1

for i = 1 to T_2

for i = 1 to x_1

for i = 1 to x_2

Previous Work

- Optimal Orthogonal 2-D Iterations (Rumen Andonov & Sanjay Rajopadhye)
- Optimal Orthogonal Tiling (Rumen Andonov, Sanjay Rajopadhye & Nicola Yanev)
- On Tiling As Loop Transformation (Jingling Xue)

Background

Background

BSP Cost Model

- α — Time to execute one Iteration
- β — Synchronization Cost
- τ — Network Bandwidth
- p — Number Of Processors

Background

BSP Cost Model

- α — Time to execute one Iteration
- β — Synchronization Cost
- τ — Network Bandwidth
- p — Number Of Processors

Processor Distribution

- Blocked
- Cyclic

IRISA

Background

BSP Cost Model

- α — Time to execute one Iteration
- β — Synchronization Cost
- τ — Network Bandwidth
- p — Number Of Processors

Processor Distribution

- Blocked
- Cyclic
- $\frac{N_1}{N_2} \geq \frac{(p-1)\alpha}{p\beta}$

Optimal Direction of Projection

IRISA

Optimal Direction of Projection

• Blocked(Along N_1)	Blocked (Along N_2)
$T = \frac{N_1 N_2 \alpha}{p} + (p-1)\beta + N_1 \tau + 2\sqrt{\frac{N_1 N_2 \beta (p-1)\alpha}{p} + N_1 \beta p \tau}$	$T = \frac{N_1 N_2 \alpha}{p} + (p-1)\beta + N_2 \tau + 2\sqrt{\frac{N_1 N_2 \beta (p-1)\alpha}{p} + N_2 \beta p \tau}$

IRISA

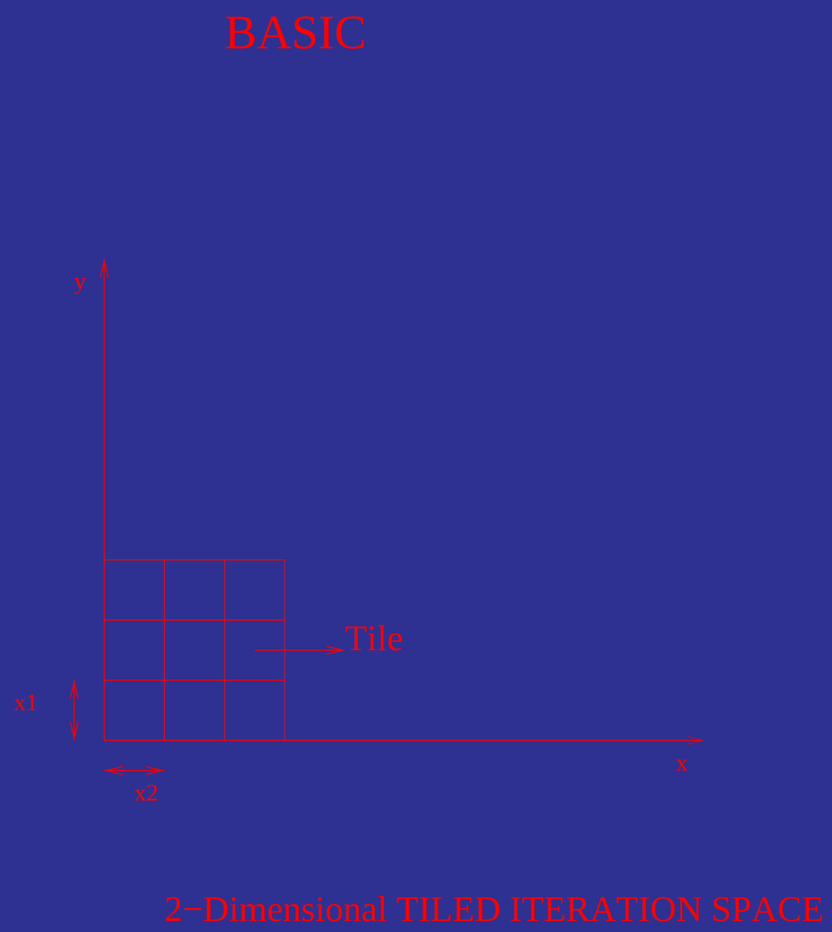
Optimal Direction of Projection

• Blocked(Along N_1) $T = \frac{N_1 N_2 \alpha}{p} + (p-1)\beta + N_1 \tau + 2\sqrt{\frac{N_1 N_2 \beta (p-1)\alpha}{p} + N_1 \beta p \tau}$	Blocked (Along N_2) $T = \frac{N_1 N_2 \alpha}{p} + (p-1)\beta + N_2 \tau + 2\sqrt{\frac{N_1 N_2 \beta (p-1)\alpha}{p} + N_2 \beta p \tau}$
• Cyclic (Along N_1) $T = \frac{N_1 N_2 \alpha}{p} + (p-1)(\beta + \tau) + 2\sqrt{\frac{N_1 N_2 (\beta+\tau)(p-1)\alpha}{p}}$	Cyclic (Along N_2) $T = \frac{N_1 N_2 \alpha}{p} + (p-1)(\beta + \tau) + 2\sqrt{\frac{N_1 N_2 (\beta+\tau)(p-1)\alpha}{p}}$

Strategies for 2-Dimensional Tiling

Strategies for 2-Dimensional Tiling

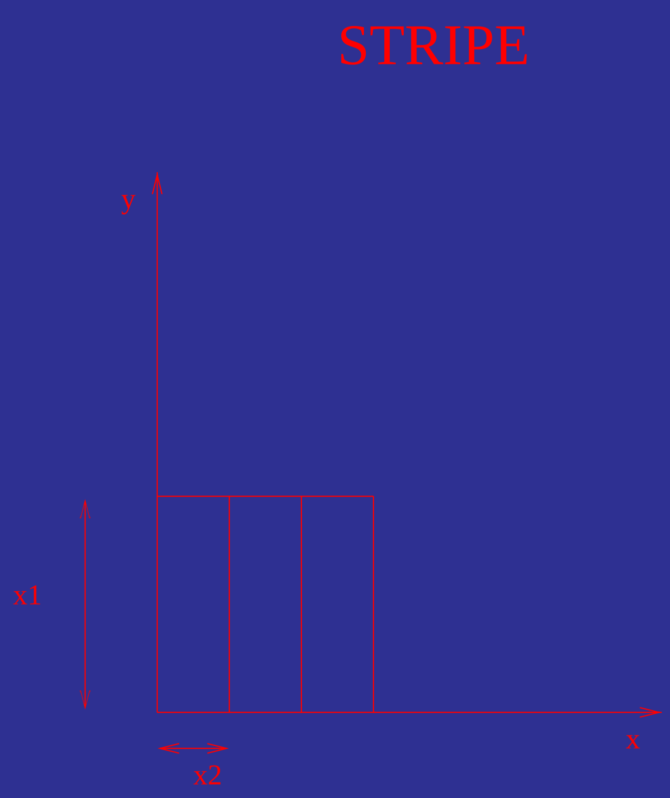
- Basic



-

IRISA

Stripe



- 2−Dimensional ITERATION SPACE

Merge

IRISA

Merge

- The Original Loop Bounds

for i = 1 to N_1

..

for i = 1 to N_i

..

for i = 1 to N_{n-1}

for i = 1 to N_n

- Now

for i = 1 to $N_1 \times N_2 \times \ldots\ldots\ldots \times N_i$

for i = 1 to $N_{i+1} \times N_{i+2} \times \ldots\ldots\ldots \times N_n$

- Now

for i = 1 to $N_1 \times N_2 \times \ldots\ldots\ldots \times N_i$

for i = 1 to $N_{i+1} \times N_{i+2} \times \ldots\ldots \times N_n$

- Notation
  - $W = N_1 \times N_2 \times \ldots\ldots\ldots \times N_n$
  - $M_1 = N_1 \times N_2 \times \ldots\ldots\ldots \times N_i$
  - $M_2 = N_{i+1} \times N_{i+2} \times \ldots\ldots \times N_n$

IRISA

Mathematical Analysis

- Non Linear Discrete Optimization Problem
  - $T(v) = \frac{A}{v} + Bv + C$
  - $v = \sqrt{\frac{B}{A}}, T_{optimal} = C + 2\sqrt{AB}$

IRISA

Mathematical Analysis

- Non Linear Discrete Optimization Problem
  - $T(v) = \frac{A}{v} + Bv + C$
  - $v = \sqrt{\frac{B}{A}}, T_{optimal} = C + 2\sqrt{AB}$
- Basic

Mathematical Analysis

- Non Linear Discrete Optimization Problem
  - $T(v) = \frac{A}{v} + Bv + C$
  - $v = \sqrt{\frac{B}{A}}, T_{optimal} = C + 2\sqrt{AB}$
- Basic
  - Blocked Distribution

IRISA

Mathematical Analysis

- Non Linear Discrete Optimization Problem
  - $T(v) = \frac{A}{v} + Bv + C$
  - $v = \sqrt{\frac{B}{A}}, T_{optimal} = C + 2\sqrt{AB}$
- Basic
  - Blocked Distribution
    - $x_2 = \frac{N_n}{p}, x_1 = \sqrt{\frac{N_{n-1}p(\beta+\tau)}{N_n(p-1)\alpha}}$
    - $T = \frac{W\beta}{N_n x_1} + (p-1)\left(\frac{\alpha N_n}{p} + \tau\right) x_1 + \frac{\alpha W}{p} + (p-1)\beta + \frac{\tau W}{N_n}$

- Cyclic Distribution

IRISA

Cyclic Distribution

- $x_1 = 1, x_2 = \sqrt{\frac{N_{n-1}N_n(\beta+\tau)}{p(p-1)\alpha}}$
- $T = \frac{W(\beta+\tau)}{px_2} + \left[(p-1)\alpha + \left(\frac{\alpha W}{N_n}\right)\right]x_2 + \frac{\alpha W}{p} + \left(p - 1 - \frac{W}{N_n}\right)(\beta+\tau)$

IRISA

- Cyclic Distribution
  - $x_1 = 1, x_2 = \sqrt{\frac{N_{n-1} N_n (\beta+\tau)}{p(p-1)\alpha}}$
  - $T = \frac{W(\beta+\tau)}{p x_2} + \left[(p-1)\,\alpha + \left(\frac{\alpha W}{N_n} \right) \right] x_2 + \frac{\alpha W}{p} + \left(p - 1 - \frac{W}{N_n} \right) (\beta + \tau)$

- Stripe

- Cyclic Distribution
  - $x_1 = 1, x_2 = \sqrt{\frac{N_{n-1} N_n (\beta+\tau)}{p(p-1)\alpha}}$
  - $T = \frac{W(\beta+\tau)}{p x_2} + \left[(p-1)\,\alpha + \left(\frac{\alpha W}{N_n} \right) \right] x_2 + \frac{\alpha W}{p} + \left(p - 1 - \frac{W}{N_n} \right) (\beta + \tau)$
- Stripe
  - $x_1 = N_{n-1}\,, x_2 = \frac{N_n}{p}.$
  - $T = \frac{W\beta}{N_n N_{n-1}} + (p-1) \left(\frac{\alpha N_n}{p} + \tau \right) N_{n-1} + \frac{\alpha W}{p} + (p-1)\,\beta + \frac{\tau W}{N_n}$

- Merge
  - Is Orthogonal Tiling Valid ?

- Merge
  - Is Orthogonal Tiling Valid ?
  - Blocked Distribution

IRISA

- Merge
  - Is Orthogonal Tiling Valid ?
  - Blocked Distribution
    - $x_2 = \frac{M_2}{p}$ and $\frac{M_1}{M_2} \geq \frac{(p-1)\alpha}{p\beta}$
    - $T = \frac{W\alpha}{p} + (p-1)\beta + M_1\tau + 2\sqrt{\frac{W\beta(p-1)\alpha}{p} + M_1\beta p\tau}$
    - As we decrease i we decrease M_1 and T decreases

- Cyclic Distribution

IRISA

- Cyclic Distribution
  - $x_1 = 1$and $\frac{M_1}{M_2} < \frac{(p-1)\alpha}{p\beta}$
  - $T =$
    $$\frac{W\alpha}{p} + (p-1)(\beta+\tau) + 2\sqrt{\frac{W(\beta+\tau)(p-1)\alpha}{p}}$$

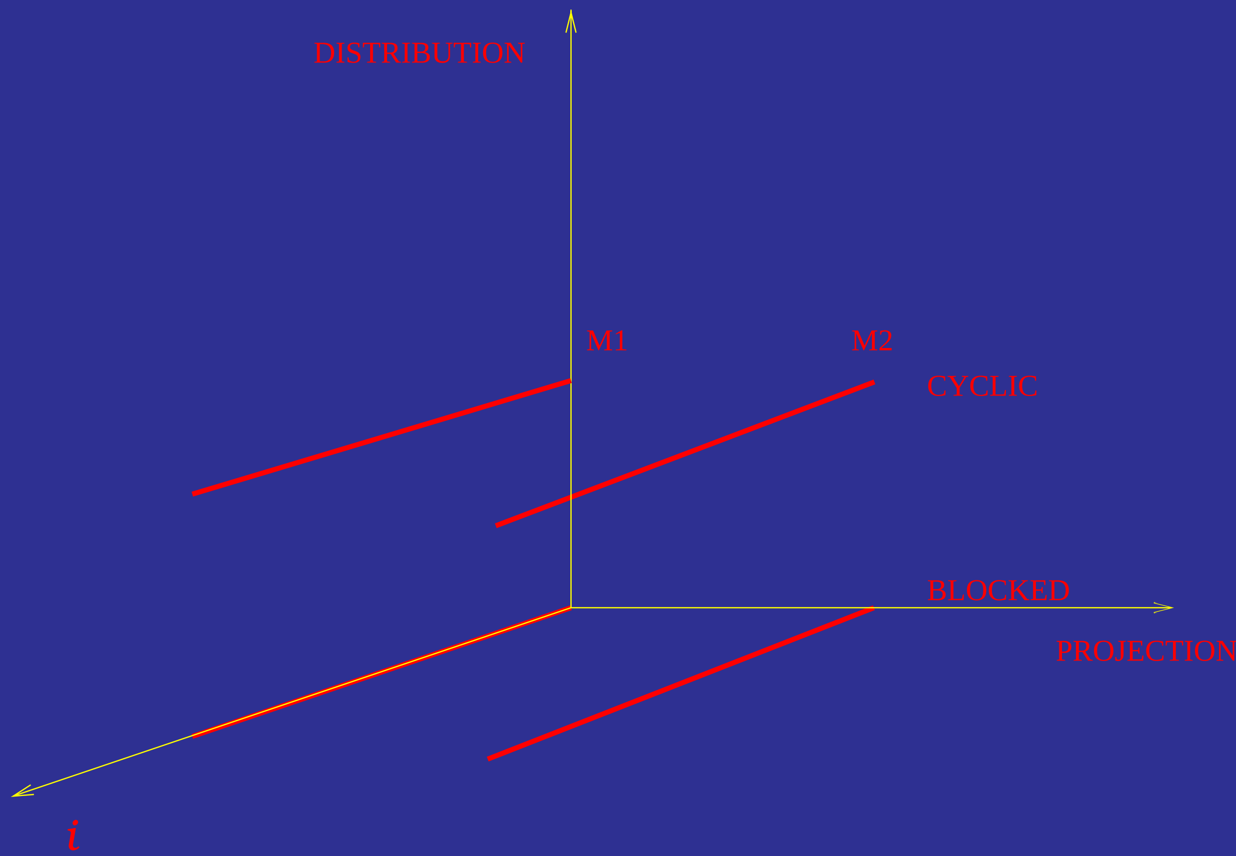
- Cyclic Distribution
  - $x_1 = 1$and $\frac{M_1}{M_2} < \frac{(p-1)\alpha}{p\beta}$
  - $T =$
    $$\frac{W\alpha}{p} + (p-1)(\beta+\tau) + 2\sqrt{\frac{W(\beta+\tau)(p-1)\alpha}{p}}$$
  - Independant of Direction of Projection

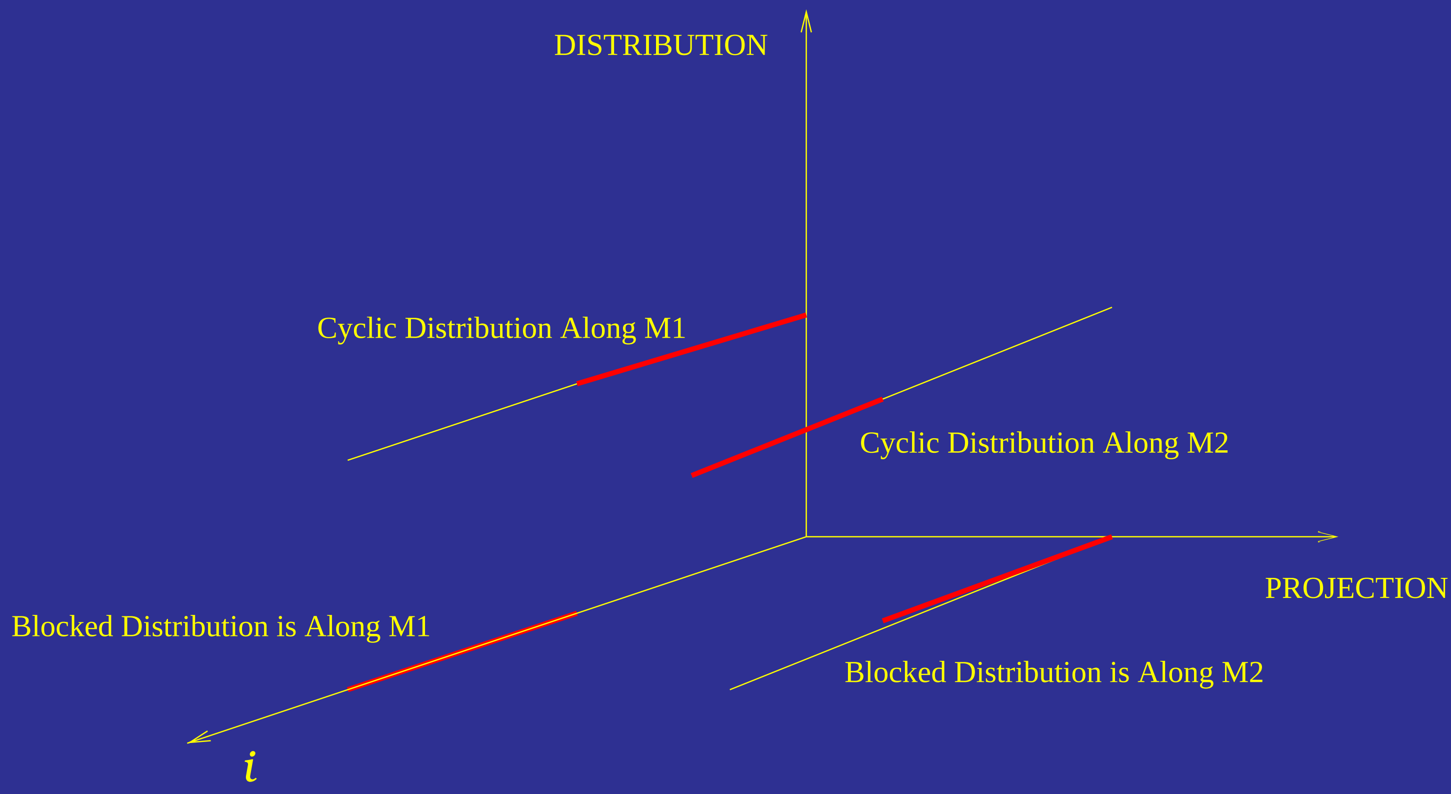
- Cyclic Distribution
  - $x_1 = 1$and $\frac{M_1}{M_2} < \frac{(p-1)\alpha}{p\beta}$
  - $T =$
    $$\frac{W\alpha}{p} + (p-1)(\beta+\tau) + 2\sqrt{\frac{W(\beta+\tau)(p-1)\alpha}{p}}$$
  - Independant of Direction of Projection
  - Independant of Point of Break OR “ i ”

- Where do we break ????

IRISA

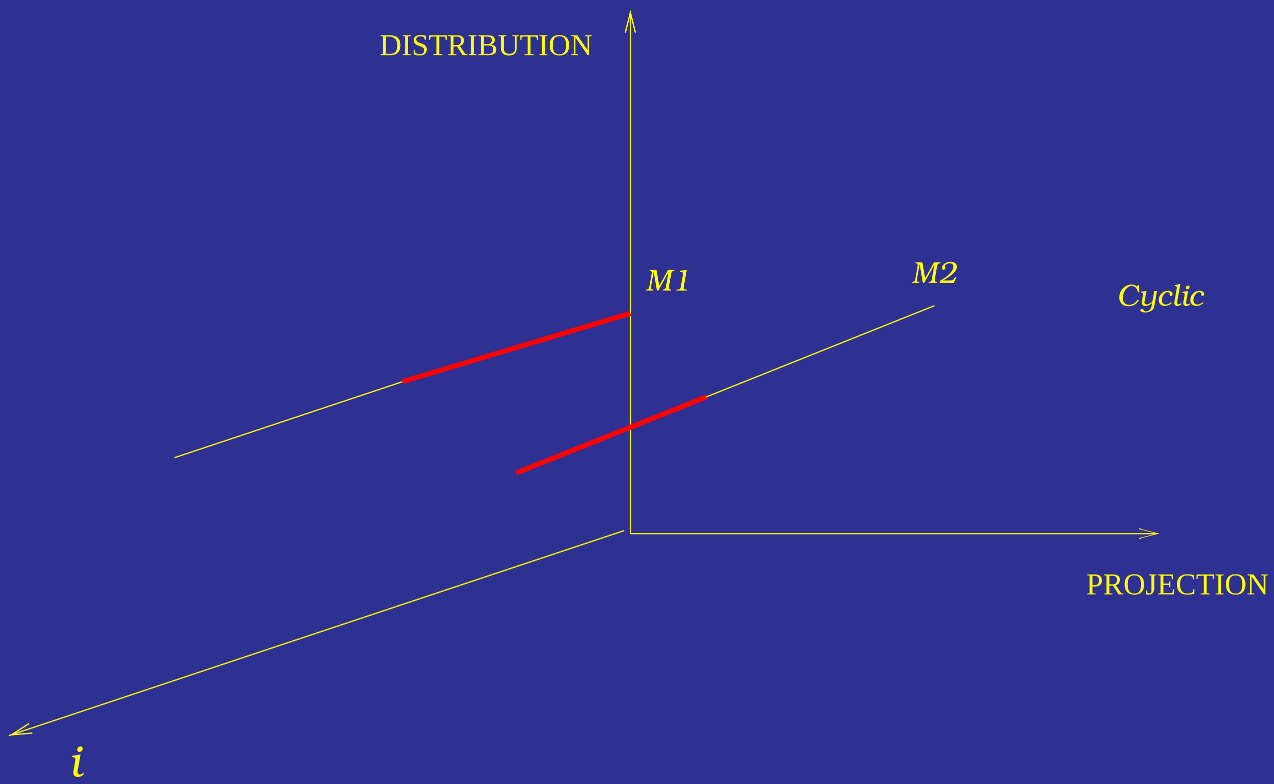
- Where do we break ????





RED --- Optimal Processor Distribution for Different Projections for Varying " i "

IRISA



RED −−− Optimal Solution

RED −−− Optimal Solution

CYCLIC IS OPTIMAL

Comparison(among 2-d)

Comparison(among 2-d)

- Basic V/S Stripe
  - Stripe is BETTER

Comparison(among 2-d)

- Basic V/S Stripe
  - Stripe is BETTER
- Stripe V/S Merge
  - Merge is BETTER

Comparison(among 2-d)

- Basic V/S Stripe
  - Stripe is BETTER
- Stripe V/S Merge
  - Merge is BETTER

MERGE IS BEST AMONG 2-D STRATEGIES

Comparison (with n-d)

- Still Analysing

Comparison (with n-d)

- Still Analysing

BUT

Comparison (with n-d)

- Still Analysing

BUT

- n-Dimensional Tiling should be BETTER

Comparison (with n-d)

- Still Analysing

BUT

- n-Dimensional Tiling should be BETTER
- MERGE is NOT FAR AWAY

Drawbacks in Analysis

Drawbacks in Analysis

- BSP Model

Drawbacks in Analysis

- BSP Model
- Data Locality And Cache Misses

Current Work

Current Work

- Experimental Validation of The Analysis

Future Work

Future Work

- Looking into Oblique Tiling Problems