

Integrated Short Term Navigation of a Towed Underwater Body*

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I. POSITION OF THE PROBLEM

The following ocean engineering problem is considered. An underwater body, to be called hereafter the *fish*, is towed by a surface ship at the end of a few hundred meters long cable, and uses its sidescan sonars to make observations of the bottom of the sea, a few thousand meters below. To enhance the antenna resolution, a virtual antenna is built up by adding up appropriately the echoes received by the moving physical antenna. To make this SAS (synthetic aperture sonar) experiment efficiently, it is needed to know with an accuracy of a few centimeters, the trajectory of the fish relative to its otherwise unknown initial position, during a few minutes experiment, so that motion compensation can be performed during the synthetic aperture processing.

For this purpose, acceleration measurements provided by an INS (inertial navigation system) located on board of the fish, can be integrated. INS measurements are known to track accurately the high frequency components in the mobile trajectory. However, even on small time intervals, integration of INS measurements can result in unacceptable drifting in position estimates, i.e. poor estimation of low frequency components of the mobile trajectory. To prevent this drifting in position estimates, it is common to use additional measurements which could provide good estimates of the low frequency components of the mobile trajectory. For example, the following additional sensors could be used : (i) a quartz pressure sensor, providing submersion measurements, (ii) an electromagnetic velocity probe, (iii) a magnetic heading sensor, etc. Combining the two complementary type of measurements would usually improve very much the trajectory estimates. This can be performed by means of a Kalman filter, see e.g. Maybeck [1, Chap. 6].

The purpose of this paper is to investigate the possibility of using position estimates of the surface ship, provided by differential GPS (global po-

sitioning system). An additional difficulty is that the satellites involved in GPS cannot provide any position estimates of an underwater body such as a towed fish, but only position estimates of the surface ship. To overcome this difficulty, the idea is to introduce a numerical model for the towing cable, in order to transform GPS position measurements of the surface ship into position and velocity estimates of the towed fish, so as to be able to combine these estimates with INS measurements.

The modelization of the cable-fish system is briefly discussed in Section II.. Numerical results are reported in Section III., which illustrate the interest and feasibility of the proposed hybridation procedure.

II. MODELIZATION OF THE CABLE-FISH SYSTEM

For the sake of simplicity, it is assumed here that the surface ship is heading in a constant fixed direction, during the considered period of time. Moreover, it is assumed that the cable initial profile, and its subsequent deformations are all contained in the vertical plane defined by the surface ship heading direction. The notations in use are introduced in Tables I, II, and III.

By convention, the curvilinear coordinate $s = 0$ corresponds to the fish, and related quantities are indexed with the letter F, e.g. m_F will denote the mass of the fish in the water. Similarly, the curvilinear coordinate $s = L$ corresponds to the towing point on the surface ship, and related quantities are indexed with the letter S, e.g. \vec{r}_S and \vec{v}_S will denote the position and velocity of the surface ship.

Since the cable is short enough (a few hundred meters), it is reasonable to assume that it is *inextensible*, which is expressed by the constraint

$$\left| \frac{\partial \vec{r}}{\partial s} \right| = 1.$$

Also, it is assumed that the cable is *infinitely flexible*, which results in the cable tension to be aligned

*This work was partially supported by IFREMER, under contract no. 92-2-320442.

with the tangent vector, i.e.

$$\vec{T} = T \vec{\tau} .$$

The last modelling assumption is that there is no tangential component of the hydrodynamical drag force per unit length, i.e.

$$\vec{D} = D \vec{n} .$$

The following expression is used for the normal component D of the hydrodynamical drag force per unit length at the current cable point

$$D = -\frac{1}{2} \rho_0 C_d d |v_{\perp}| v_{\perp} , \quad (1)$$

where $v_{\perp} = \vec{v} \cdot \vec{n}$ is the normal component of the velocity at the current cable point, and where the constants are defined in Table I.

Let $\vec{\Sigma}$ denote the sum of all external forces per unit length which are applied at the current cable point, i.e.

$$\vec{\Sigma} = \rho \left[1 - \frac{\sigma \rho_0}{\rho} \right] \vec{g} + \vec{D} .$$

The second law of motion results in the following evolution equation

$$\frac{\partial \vec{T}}{\partial s} + \vec{\Sigma} = \rho \vec{\gamma} , \quad 0 \leq s \leq L .$$

In addition, the position and velocity of the surface ship are given (or measured) at any time, which provides the boundary conditions at $s = L$.

Similarly, let $\vec{\Sigma}_F$ denote the sum of all external forces which are applied to the fish, i.e.

$$\vec{\Sigma}_F = m_F \left[1 - \frac{\rho_0}{\rho_F} \right] \vec{g} + \vec{D}_F ,$$

where \vec{D}_F is the hydrodynamical drag force. The following expression is used for the hydrodynamical drag force \vec{D}_F applied to the fish

$$\vec{D}_F = -\frac{1}{2} \rho_0 C_{d,F} A_F |\vec{v}_F| \vec{v}_F , \quad (2)$$

where both the drag coefficient $C_{d,F}$ and the effective lateral cross-section area A_F are diagonal matrices, and where the constants are defined in Table III, see Ivers and Mudie [3].

The second law of motion results in the following evolution equation

$$\vec{T}_F + \vec{\Sigma}_F = [m_F + m_{a,F}] \vec{\gamma}_F ,$$

where the added mass $m_{a,F}$ is a diagonal matrix. This provides the desired boundary condition at $s = 0$.

L	:	length
d	:	diameter
σ	:	section ($\sigma = \frac{1}{4} \pi d^2$)
s	:	curvilinear coordinate
\vec{r}	:	position
\vec{v}	:	velocity
$\vec{\gamma}$:	acceleration
$\vec{\tau}$:	tangent vector to the cable
\vec{n}	:	normal vector to the cable
ρ	:	mass per unit length
\vec{T}	:	cable tension
\vec{D}	:	hydrodynamical drag force
	:	per unit length
C_d	:	empirical drag coefficient

Table I
Notations for the cable.

\vec{g}	:	gravitational acceleration
ρ_0	:	water mass per unit volume

Table II
Other notations.

\vec{r}_F	:	position
\vec{v}_F	:	velocity
$\vec{\gamma}_F$:	acceleration
m_F	:	mass
$m_{a,F}$:	added mass
ρ_F	:	mass per unit volume
A_F	:	effective lateral
	:	cross-section area
\vec{D}_F	:	hydrodynamical drag force
$C_{d,F}$:	empirical drag coefficient

Table III
Notations for the fish.

L	:	350.0 m
d	:	12.3 mm
ρ	:	0.6 kg/m
C_d	:	1.8
m_F	:	2000 kg
ρ_F	:	1000 kg/m ³
$C_{d,F}$:	0.5

Table IV
Main numerical values for the simulation.

To summarize, the unknown variables in this problem are position \vec{r} , velocity \vec{v} , and cable tension T at each point of the cable.

The model equations are

$$\frac{\partial}{\partial s}(T \vec{\tau}) + \rho \left[1 - \frac{\sigma \rho_0}{\rho}\right] \vec{g} + D \vec{n} = \rho \vec{\gamma}$$

$$\left| \frac{\partial \vec{r}}{\partial s} \right| = 1$$

for $0 \leq s \leq L$.

The boundary conditions at $s = 0$ and $s = L$ are respectively

$$\vec{T}_F + m_F \left[1 - \frac{\rho_0}{\rho_F}\right] \vec{g} + \vec{D}_F = [m_F + m_{a,F}] \vec{\gamma}_F,$$

and

$$\vec{r}_S = \vec{R}, \quad \vec{v}_S = \vec{V},$$

where \vec{R} and \vec{V} are given (or estimated) position and velocity of the surface ship.

Finally, the expressions for the drag coefficients D and \vec{D}_F are given in (1) and (2) respectively.

A numerical approximation scheme for the model equations has been presented in Joannides and LeGland [2].

III. NUMERICAL EXPERIMENT

The following simulation experiment is reported here, which illustrates the interest and feasibility of the proposed hybridation procedure.

The main numerical values for the cable-fish model are given in Table IV. In particular, it should be noticed that the fish is neutrally buoyant, since it has the same mass per unit volume as water. The intuitive effect which is expected from this design, is to make the trajectory of the towed fish as independent as possible from the high frequency vertical oscillations of the surface ship trajectory. This effect can be verified from the numerical simulation reported here.

A. Generation of reference trajectories

Let $v^* = (1.5, 0.0)$ denote some nominal constant velocity (units in m/s).

The horizontal velocity of the surface ship has a low frequency sinusoidal component, with period 60 s and amplitude 0.1 m/s, around the nominal constant velocity 1.5 m/s.

The vertical velocity of the surface ship has a high frequency sinusoidal component, with period

5 s and amplitude 0.628 m/s, around the nominal constant velocity 0 m/s.

At time $t_0 = 0.0$ s, the position and velocity of the surface ship are set to $r_0 = (-0.955, 0.5)$ (units in m), and $v_0 = v^* = (1.5, 0.0)$ (units in m/s), respectively.

The reference trajectory of the fish is obtained by integrating the cable-fish numerical model :

The input (boundary condition at the upper end of the cable) is set to the reference trajectory of the surface ship, obtained by the procedure described above.

At time $t_0 = 0.0$ s, the cable profile is set to the permanent profile associated with the nominal constant velocity $v^* = (1.5, 0.0)$ (units in m/s), with the upper end of the cable (corresponding to the towing point on the surface ship) translated to the position $r_0 = (-0.955, 0.5)$ (units in m) of the surface ship at time t_0 .

B. Generation of INS measurements

From the reference trajectory of the fish, obtained by the procedure described in Subsection A. above, noisy measurements γ_k^{INS} of the acceleration γ_k of the fish at time t_k^{INS} are generated in additive white noise w_k^{INS} , with standard deviation 0.01 m/s^2 (both horizontal and vertical), and period $\Delta^{\text{INS}} = 20 \text{ ms} = 0.02 \text{ s}$.

$$\gamma_k^{\text{INS}} = \gamma_k + w_k^{\text{INS}}$$

$$\Delta^{\text{INS}} = t_{k+1}^{\text{INS}} - t_k^{\text{INS}}$$

C. Integration of INS measurements

The noisy acceleration measurements of the fish, generated by the procedure described in Subsection B. above are integrated to produce INS estimates of the velocity of the fish, according to the integration scheme

$$v_{k+1}^{\text{INS}} = v_k^{\text{INS}} + \Delta^{\text{INS}} \cdot \gamma_k^{\text{INS}}$$

$$= v_k^{\text{INS}} + \Delta^{\text{INS}} \cdot \gamma_k + \Delta^{\text{INS}} \cdot w_k^{\text{INS}}$$

At time t_0^{INS} , the initial condition of the integration scheme is set to the nominal constant velocity $v_0^{\text{INS}} = v^* = (1.5, 0.0)$ (units in m/s).

In general, v_0^{INS} is different from the true velocity of the fish at time t_0^{INS} . This introduces an initial bias in velocity estimates of the fish, which results in a linear drift in position estimates of the fish. This is illustrated by Figures 4 and 6, and by Figures 5 and 7 respectively.

D. Generation of GPS measurements

From the reference trajectory of the surface ship, obtained by the procedure described in Subsection A. above, noisy measurements r_k^{GPS} of the position r_k of the surface ship at time t_k^{GPS} , are generated in addition white noise w_k^{GPS} , with standard deviation 0.2 m (horizontal) and 0.5 m (vertical), and period $\Delta^{\text{GPS}} = 0.6$ s.

$$r_k^{\text{GPS}} = r_k + w_k^{\text{GPS}}$$

$$\Delta^{\text{GPS}} = t_{k+1}^{\text{GPS}} - t_k^{\text{GPS}}$$

E. Kalman filtering of GPS measurements

Smooth inputs are needed for the cable-fish numerical model, with sampling period 0.01 s.

For this purpose, a Kalman filter is used, which combines the model described in Subsection A. above for the velocity of the surface ship (sinusoidal component with known frequency around nominal velocity), and the noisy position measurements of the surface ship, generated by the procedure described in Subsection D. above.

At time $t_0 = 0.0$ s, the initial conditions of the Kalman filter estimates are set to the nominal values, i.e. $\hat{r}_0 = (0.0, 0.0)$ (units in m), and $\hat{v}_0 = v^* = (1.5, 0.0)$ (units in m/s).

F. Generation of GPS-based velocity estimates of the fish

The GPS-based trajectory of the fish is obtained by integrating the cable-fish numerical model :

The input (boundary condition at the upper end of the cable) is set to the Kalman filter position and velocity estimates of the surface ship, obtained by the procedure described in Subsection E. above.

As can be seen from Figures 1, 2 and 3, these estimates become very accurate after a short transient period. Therefore, the initial time for subsequent computations (integration of the cable-fish numerical model, integration and hybridation of INS acceleration measurements) is set to $t_0^{\text{INS}} = 60.0$ s.

At time $t_0^{\text{INS}} = 60.0$ s, the cable profile is set to the permanent profile associated with the constant nominal velocity $v^* = (1.5, 0.0)$ (units in m/s), with the upper end of the cable (corresponding to the towing point on the surface ship) translated to the Kalman filter position estimate of the surface ship at time t_0^{INS}

Results are presented in Figures 8 and 9.

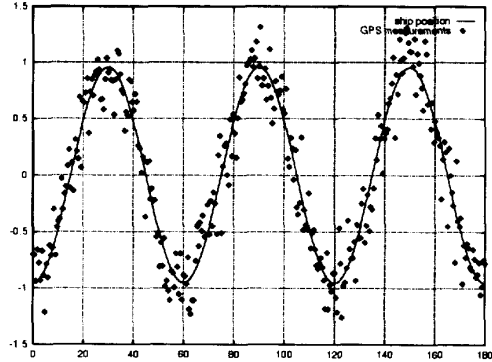


Figure 1. Horizontal position of surface ship : reference trajectory and GPS measurements

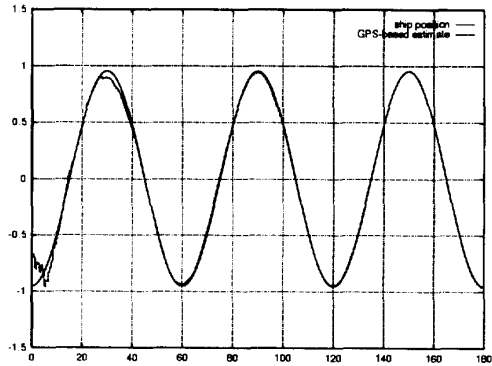


Figure 2. Horizontal position of surface ship : reference trajectory and GPS-based estimate

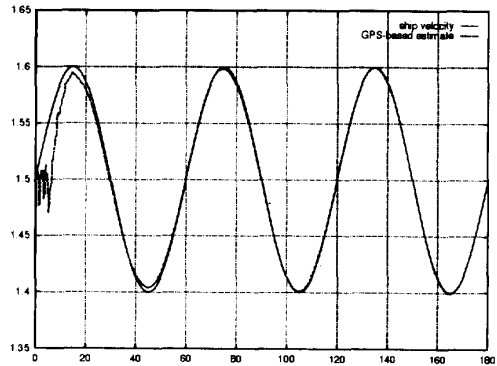


Figure 3. Horizontal velocity of surface ship : reference trajectory and GPS-based estimate

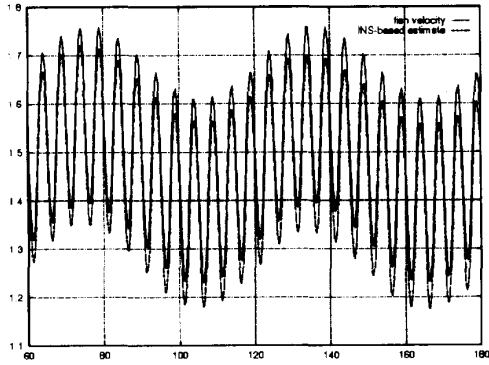


Figure 4. Horizontal velocity of towed fish : reference trajectory and INS-based estimate

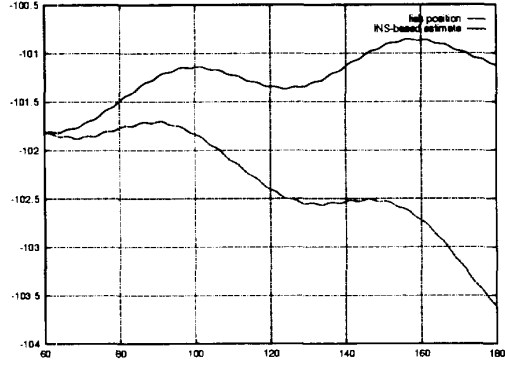


Figure 7. Vertical position of towed fish : reference trajectory and INS-based estimate

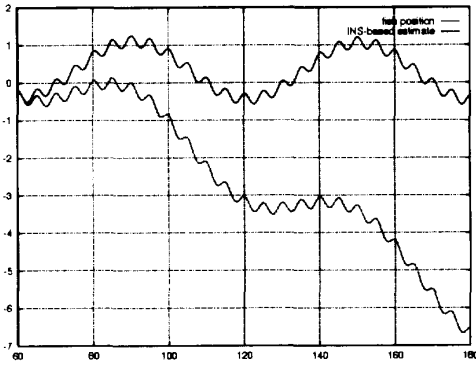


Figure 5. Horizontal position of towed fish : reference trajectory and INS-based estimate

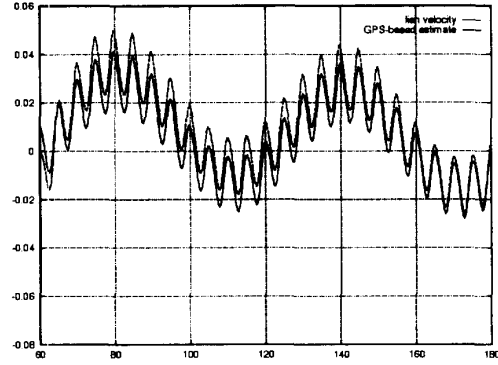


Figure 8. Vertical velocity of towed fish : reference trajectory and GPS-based estimate

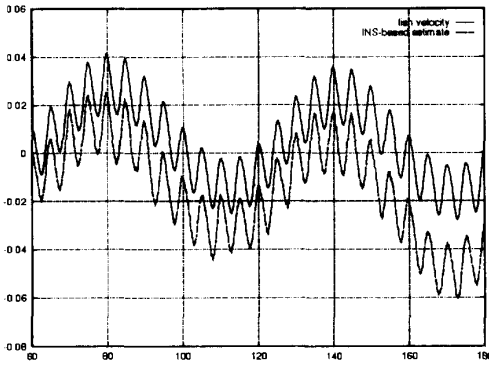


Figure 6. Vertical velocity of towed fish : reference trajectory and INS-based estimate

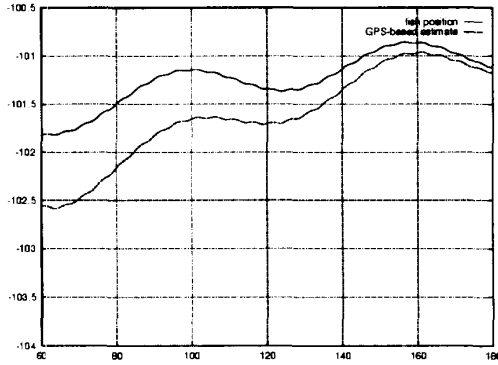


Figure 9. Vertical position of towed fish : reference trajectory and GPS-based estimate

G. Hybridation

To correct the initial bias in velocity estimates of the fish, the INS acceleration measurements are combined with the GPS-based velocity estimates obtained by the procedure described in Subsection F. above, according to the following hybridation procedure

$$\begin{cases} v_{k+1}^{\text{INS}} = \hat{v}_k + \Delta^{\text{INS}} \cdot \gamma_k^{\text{INS}} \\ \hat{v}_{k+1} = \theta_k \cdot v_{k+1}^{\text{INS}} + (1 - \theta_k) \cdot v_{k+1}^{\text{GPS}} \end{cases}$$

which is an adaptive modification of the original integration procedure described in Subsection C. above.

The optimal weighting coefficients θ_k could be obtained by performing a careful sensitivity analysis of the cable-fish numerical model. The result presented in Figure 10 corresponds rather to constant weighting coefficients $\theta_k \equiv \theta$, for a constant value θ which has been selected by try and guess.

IV. CONCLUSION

Although the numerical results presented here are very promising, the following two comments should be made.

- The excellent accuracy of Kalman filter position and velocity estimates of the surface ship is mainly due to the exact knowledge of the mathematical model for the surface ship evolution. This will never be the case in real situation.
- The selection of a constant weighting coefficient obtained by try and guess for the hybridation of INS acceleration measurements and GPS-based velocity estimates is not acceptable. The determination of optimal weighting coefficients is under investigation, and will be presented elsewhere.

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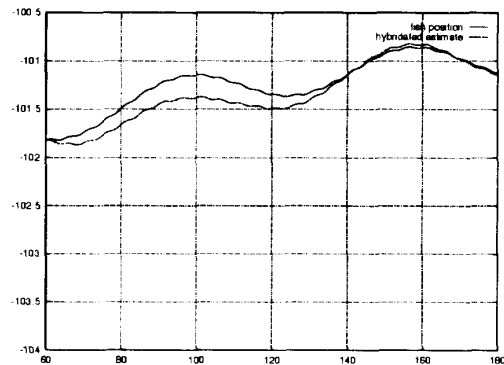


Figure 10. Vertical position of towed fish : reference trajectory and hybrid estimate