Muon Tomography:

Passive detection and imaging using cosmic ray muons

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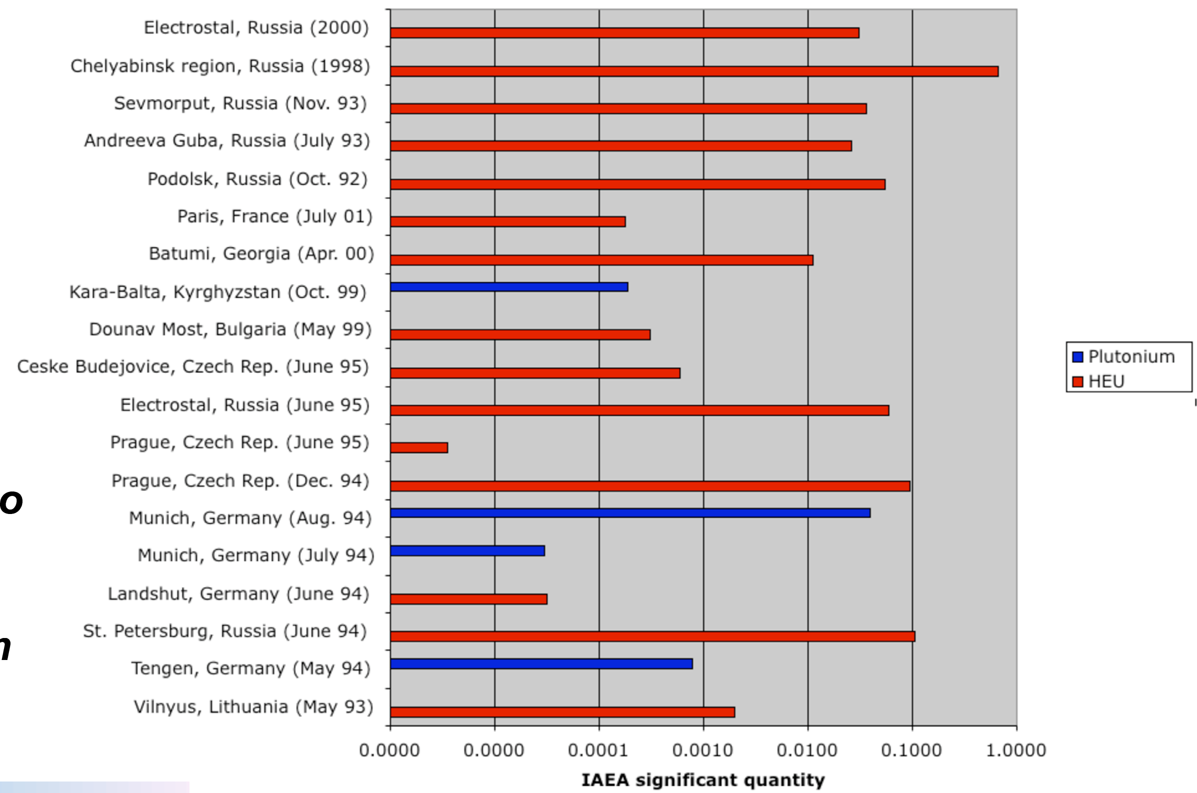
Nuclear smuggling is a clear and present danger

Materials Interceptions

"Law enforcement officials in the US seize only 10 to 40% of the illegal drugs smuggled into the country each year

Russia stops from 2 to 10% of illegally imported goods and illegal immigrants on the border with Kazakhstan"



Los Alamos National Laboratory
October 24, 2002

Stanford Nuclear Smuggling Database: Dynamics and Trends Over the Past Decade

Lyudmila Zaitseva

Center for International Security and Cooperation, Stanford University

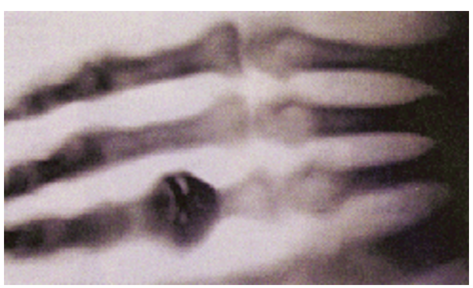
Total = 1.13 IAEA "significant quantities"

(8 kg Pu or 25 kg of U^{235} in HEU)

Active radiography is an established inspection technique

To date, radiography has depended on artificial sources of radiation, which bring with them a risk-benefit tradeoff

1895
First x-ray image
(Mrs. Roentgen's hand)



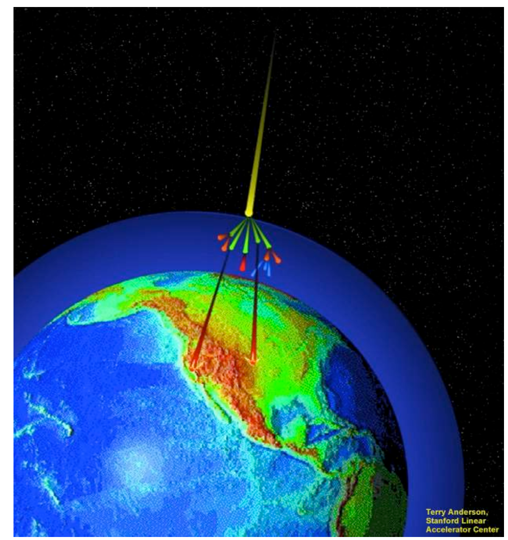
2001
Inspection of truck with American Science and Engineering backscatter x-ray system

IN A TRUCK This backscatter X ray spotted 37 illegal immigrants being smuggled out of Chiapas, Mexico, in a shipment of bananas

Passive Source Radiography: Cosmic Radiation

No artificial radiation means:

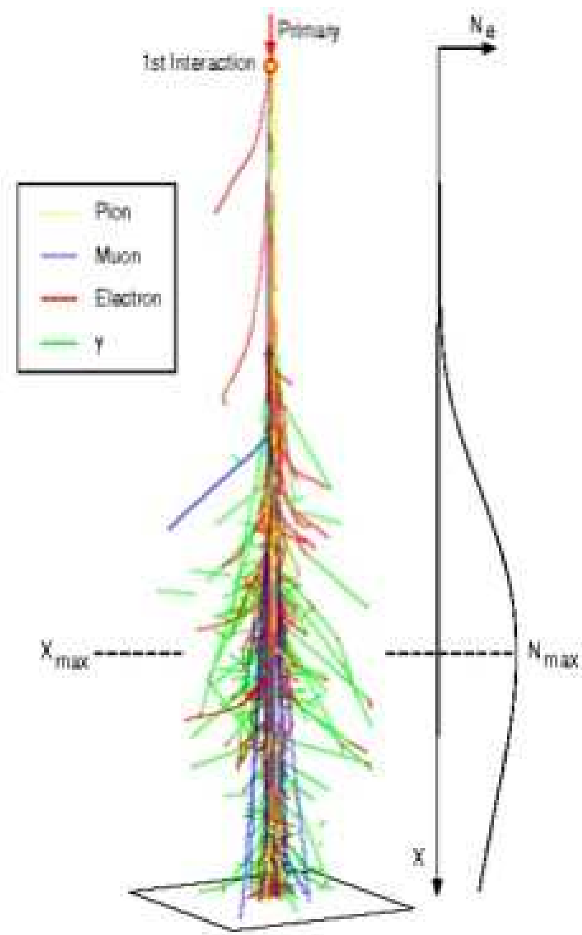
1. Cars and trucks inspection without evacuating the driver
   significant time factor
2. Deployment abroad without local regulatory complications
   Detection at point of origi
3. No radiation signal to set off a salvage trigger
   Minimizes inspection risks.



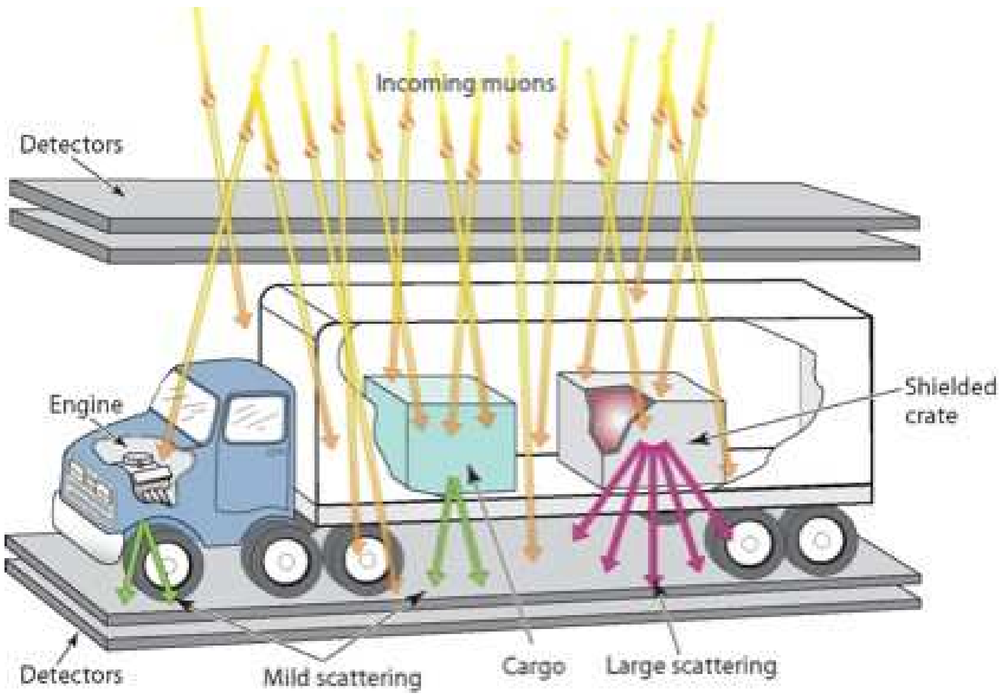
1. Neutrons
2. Neutrinos
3. Electrons
4. Muons
5. Etc.

Cosmic Ray Muons

- As cosmic rays strike upper atmosphere, are broken down into many particle components, dominated by muons.
- Muons have a large penetrating ability, go through tens of meters of rock with low absorption.
- Muons arrive at a rate of 10,000 per square meter per minute (at sea level).



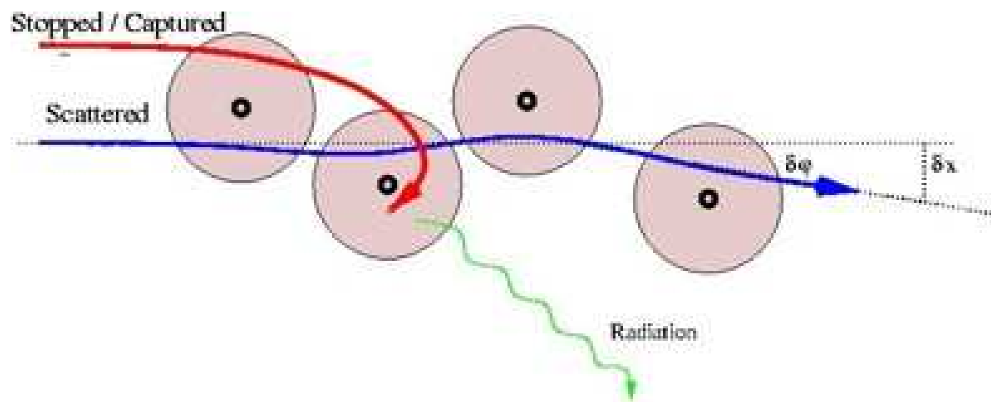
Application: Truck Inspections

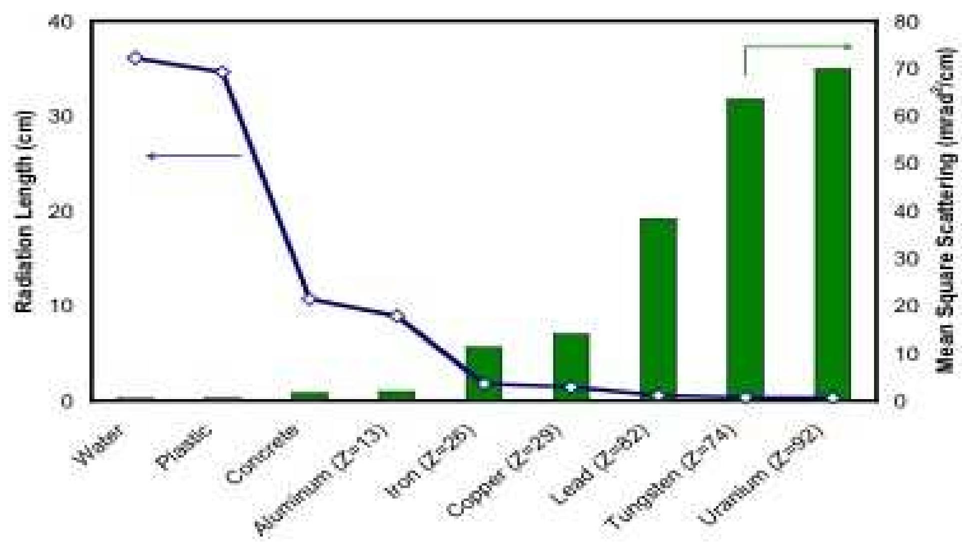


- Penetrating interrogation with no artificial dose
- Prevent illicit movement of nuclear materials
- Used in concert with passive gamma / neutron detection.

Physics

- Coulomb scattering changes the path of muons.
- Some particles are absorbed
- Variance of scattering depends on the material.





Absorption tomography

- Hidden chambers in volcanos (Alvarez, 1970)
- Predicting volcanic erruptions (Tanaka, 2003)

Fig. 1 (top right). The pyramids at Giza. From left to right, the Third Pyramid of Mycerinus, the Second Pyramid of Chephren, the Great Pyramid of Cheops. [© National Geographic Society]

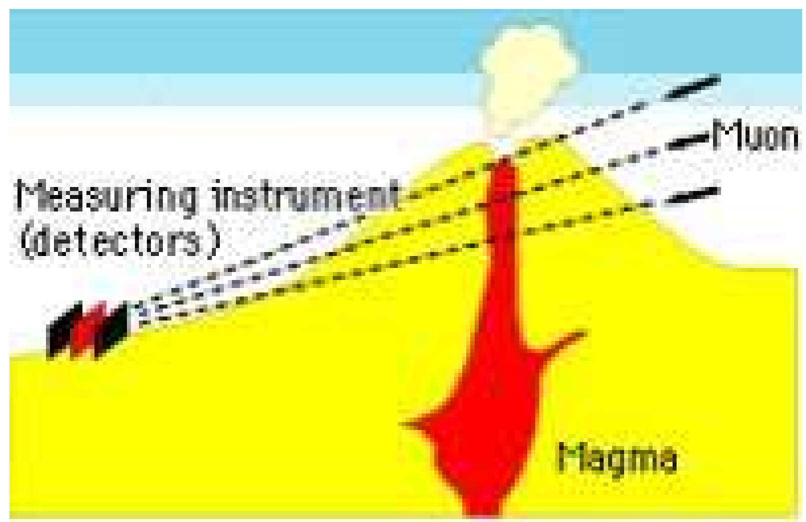
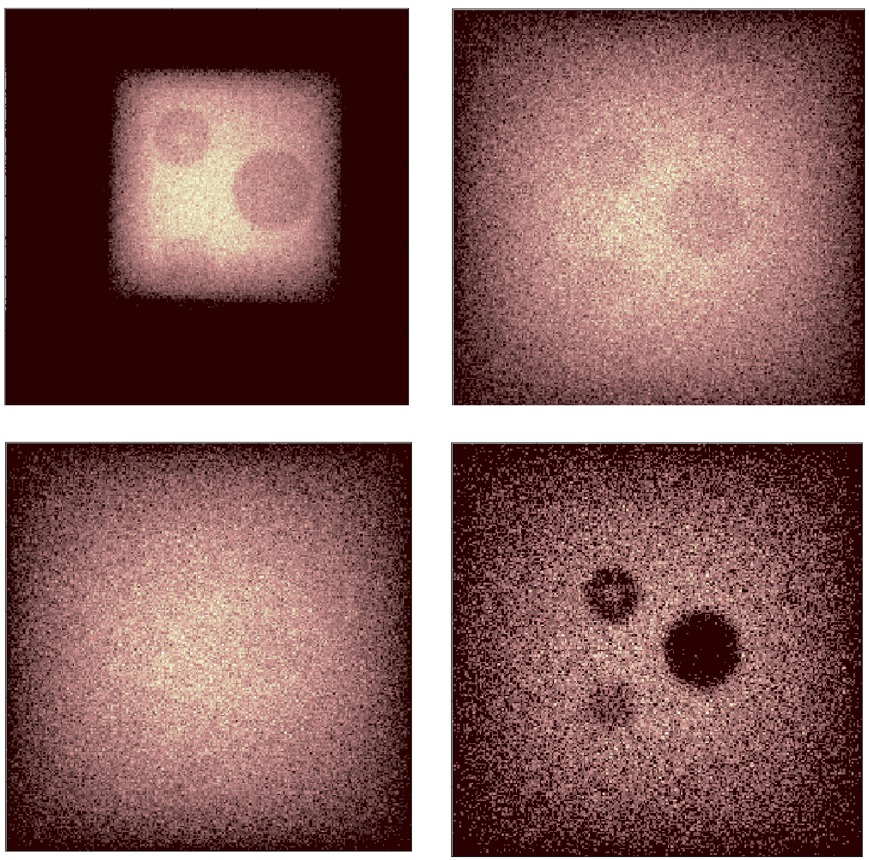


Figure 4: Analyzing the internal structure of a volcanic zone using muons

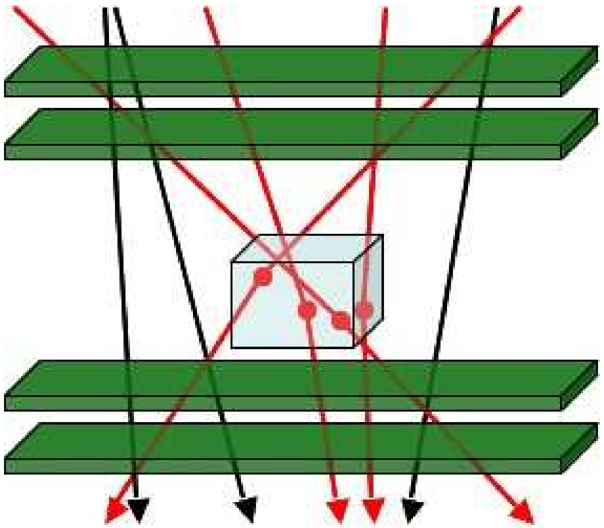
Shadowgrams (from scattering)

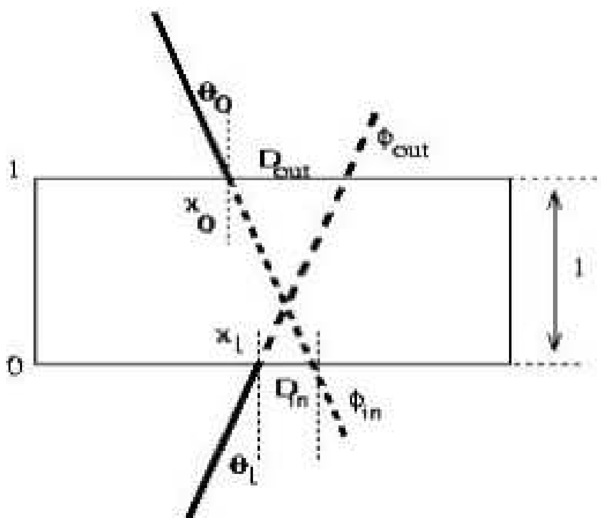


Possible to get shadowgrams from scattering instead of absorption

Proton radiography

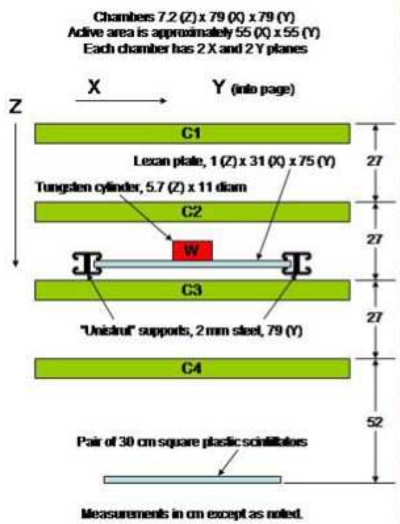
Measuring the scattering

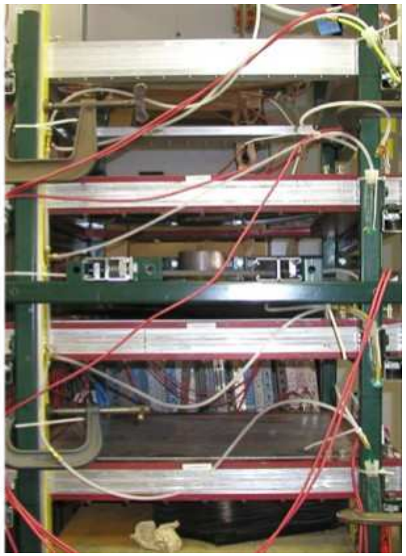


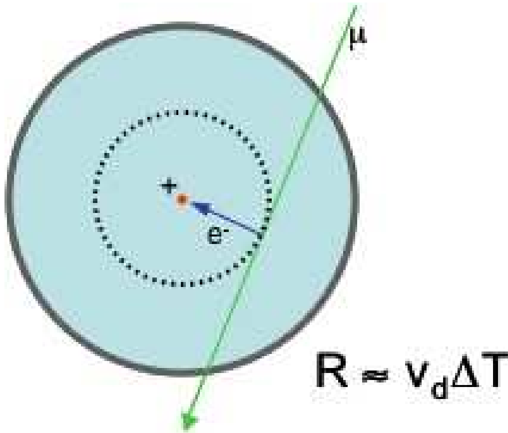


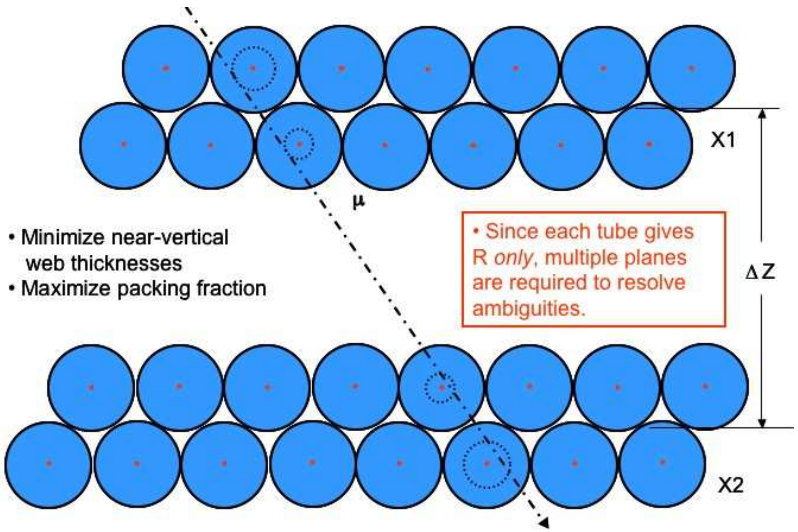
- Tract individual muons
- Measure changes to the paths in and out

Drift tubes and wire chambers









(Idealized) Data

Incident ray: (x_0, θ_0)

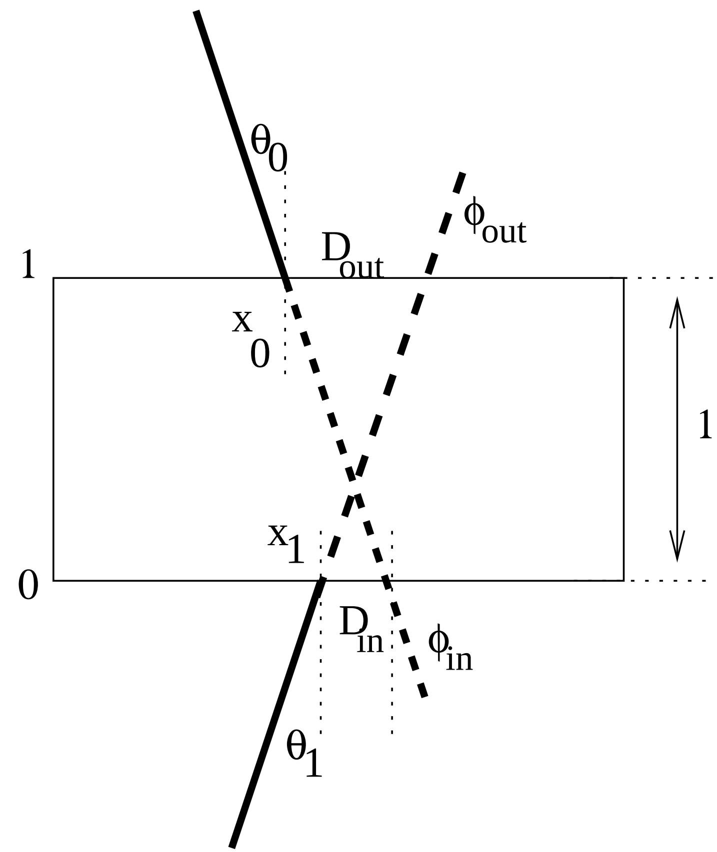
Outgoing ray: (x_1, θ_1)

Strait line $\phi(s)$

$(x_0 + \sin(\theta_0)s, 1 - \cos(\theta_0)s)$

Data (for each muon)

$$\begin{aligned} \Delta &= \theta_1 - \theta_0 \\ D &= x_1 - x_0 - \tan\theta_0 \end{aligned}$$



Statistical Model

$Y = (\Delta, D)$ **conditional** on $I = (\theta_0, x_0)$ is (approximatively) bivariate Gaussian with mean $\mu(\varrho, \phi) = (0, \mu_d)$ and covariance $\Sigma(\varrho, \phi)$ where

$$\begin{aligned}
\mu_d(\varrho, \phi) &= c(\theta_0) \int \|\phi(1) - \phi(s)\| \varrho(\phi(s)) ds \\
\Sigma_{11}(\varrho, \phi) &= \mathbb{E}[\Delta^2] = \frac{1}{p} \int_0^1 \varrho(\phi(s)) ds \\
\Sigma_{12}(\varrho, \phi) &= \mathbb{E}[\Delta D_{in}] = \frac{1}{p} \int_0^1 \|\phi(1) - \phi(s)\| \varrho(\phi(s)) ds \\
\Sigma_{22}(\varrho, \phi) &= \mathbb{E}[D_{in}^2] = \frac{1}{p} \int_0^1 \|\phi(1) - \phi(s)\|^2 \varrho(\phi(s)) ds.
\end{aligned}$$

ϱ: scattering density: parameter of interest

ϕ path of muon (observed)

p: momentum of the muon, unobserved

approximation reasonable for $|\theta_0| \leq \pi/3$.

What we learn from the data

- Signal is (mainly) in the variance;
- To first order, weighted integrals along path ϕ;
- Sampling limited by geometry of detectors;

Questions and Challenges:

ESTIMATION: How well can we reconstruct ϱ?

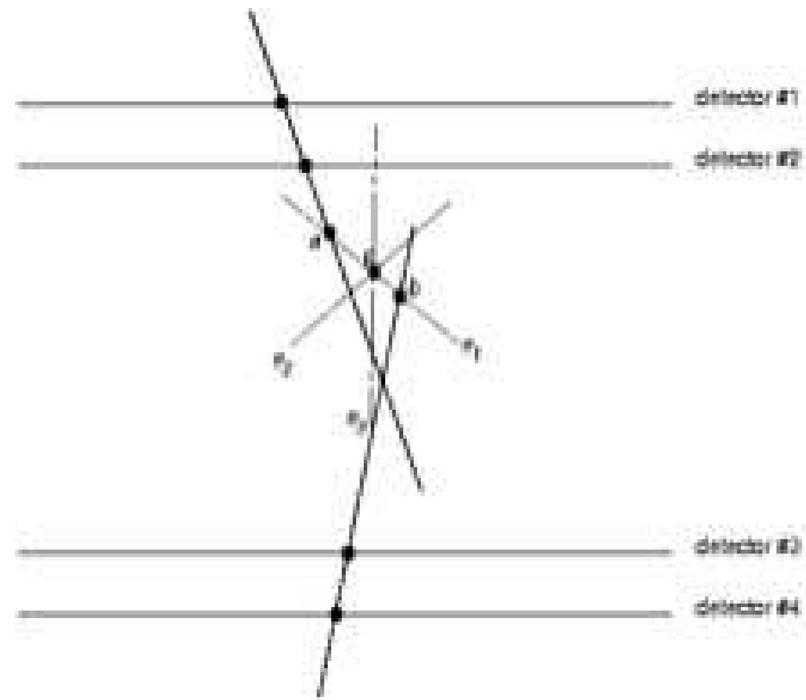
DESIGN: What is the impact of the geometry? Change the difficulty of the inverse problem by changing the geometry.

COMPUTATION Computation in < 1 min; sequential method ok.

Information extraction

- Point of Closest Approach
- Maximum Likelihood
- Support vector (for detection of high-Z regions)

How Good?



Penalized Loglikelihood

Estimator:

$$\hat{\varrho} = \arg\max_{\varrho} - \sum_{i=1}^{n} \log|\Sigma(\varrho, \phi_i)| - Y_i^t \Sigma^{-1}(\varrho, \phi_i) Y_i - \lambda \|\varrho\|^2$$

Difficulty: Infinite dimensional optimization problem.

Solution: Reduce it to a finite dimensional one.

THEOREM. If

$$\int \|\phi_i(1) - \phi_i(s)\|^j \varrho(\phi_i(s)) ds = \int L_{i,j}(s) \varrho(s) ds$$

bounded linear functionals of ϱ, then

$$\hat{\varrho} \in \text{span}\{L_{i,j},\ i = 1, \ldots, n\ j = 0, 1, 2\}$$

Approximate Linear Functionals

K_h kernel, bandwidth h

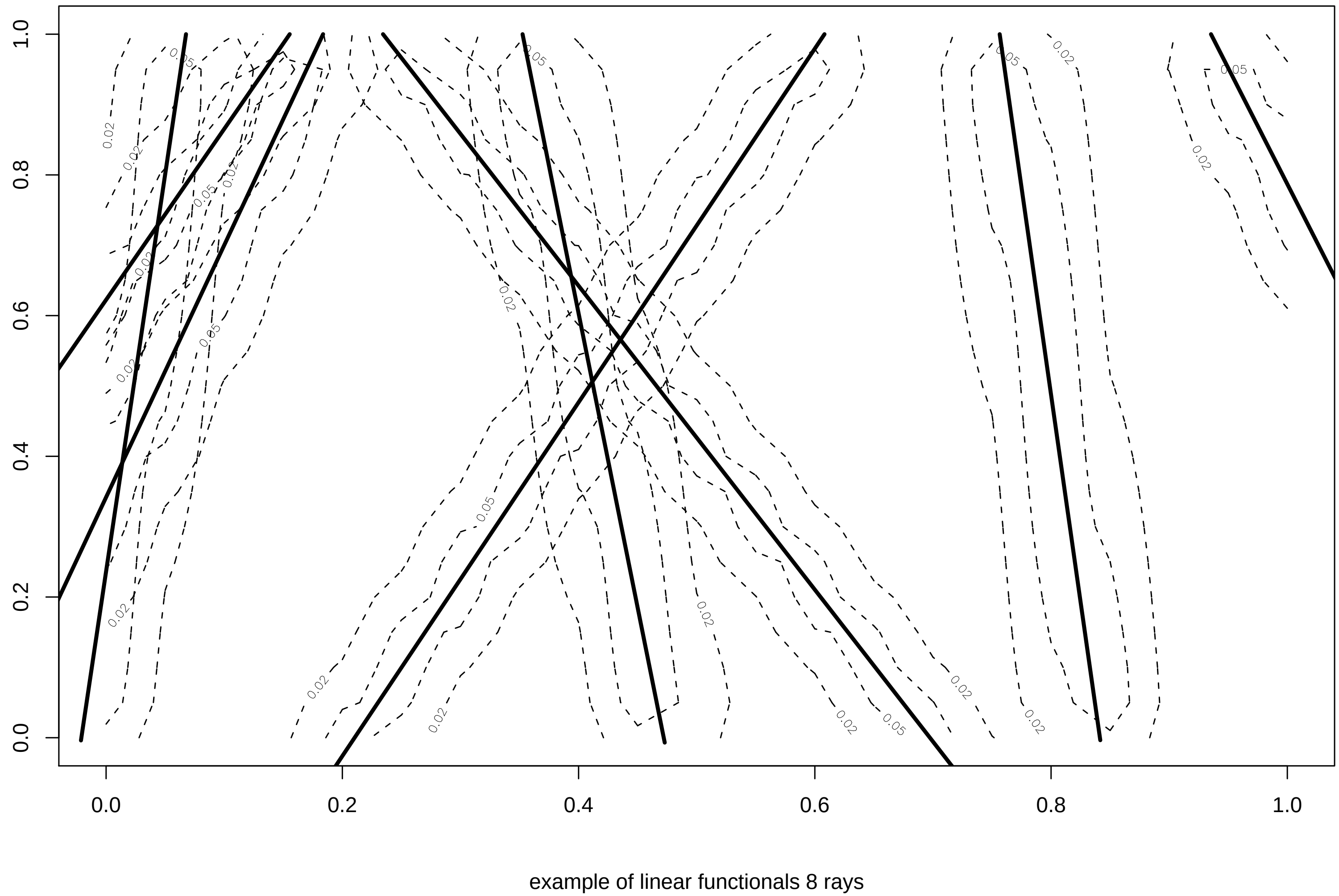
Approximate linear functionals:

$$\begin{aligned}
&\int \|\phi_i(1)-\phi_i(s)\|^j \varrho(\varphi_i(s))ds \\
&\quad = \int_0^1 \|\phi_i(1)-\phi_i(s)\|^j \int \varrho(u) K_h(u-\varphi_i(s))du ds + o(1) \\
&\quad = \int \varrho(u) \left\{ \int_0^1 \|\phi_i(1)-\phi_i(s)\|^j K_h(u-\varphi_i(s))ds \right\} du + o(1)
\end{aligned}$$

Define:

$$L_{i,j} = \int_0^1 \|\phi_i(1)-\phi_i(s)\|^j K_h(u-\varphi_i(s))ds.$$

Example of $L_{i,0}(s)$



Reproducing Kernel Approach

Impose "regularity" on ϱ, lies essentially in a compact set.

ϱ belongs to a r.k.h.s. if exists $K(s,t)$ such that

$$\varrho(s) = \int K(s,t)\varrho(t)dt$$

$$\begin{aligned}
&\int \|\phi_i(1) - \phi(s)\|^j \varrho(\phi_i(s))ds \\
&\quad = \int \|\phi_i(1) - \phi_i(s)\|^j \left\{ \int \mathbb{K}(\phi_i(s), u)\varrho(u)du \right\} ds \\
&\quad = \int \varrho(u) \cdot \left\{ \int \|\phi_i(1) - \phi_i(s)\|^j \mathbb{K}(\phi_i(s), u)ds \right\} du.
\end{aligned}$$

$$L_{i,j}(u) = \int \|\phi_i(1) - \phi_i(s)\|^j \mathbb{K}(\phi_i(s), u)ds.$$

Example

Assume ϱ in space of pixels (piecewise constant)

Basis functions:

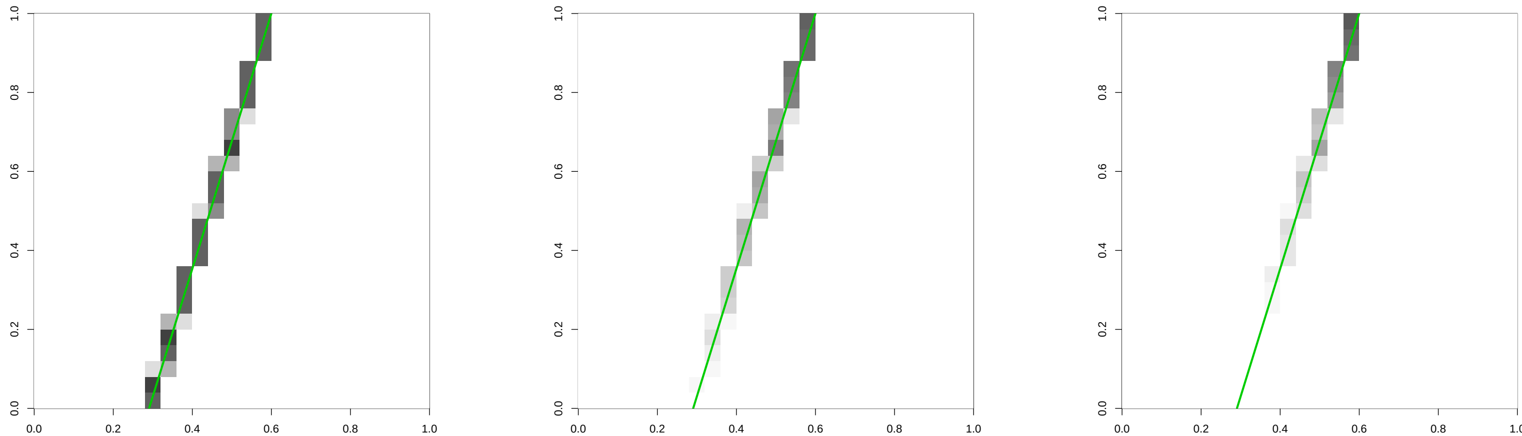
$$\varphi_{i,j}(u) = \begin{cases} 1 & \text{if } u \text{ in voxel } i,j \\ 0 & otherwise. \end{cases}$$

Mercer's Theorem: Reproducing kernel is

$$K(u,v) = \sum_{i,j} \varphi_{i,j}(u)\varphi_{i,j}(v)$$

Example — continued

Linear functional $L_{i,j}(u)$ for pixel example:



Observe how "upper part better" determiner.

Feature, not bug.

What can we learn from this?

Solution lies in $\mathbb{V}_n = \text{span}\{L_{i,j}(u)\ i = 1, \ldots, n\ j = 0, 1, 2\}$

$$\hat{\varrho}(u) = \sum_{i=1}^{n} \sum_{j=0}^{2} \alpha_{i,j} L_{i,j}(u)$$

Negative result:

The part of ϱ orthogonal to $\mathbb{V}_n$ CAN NOT BE ESTIMATED.

Pre-data collection analysis.

Can compare estimates using change in angle alone with those using change in angle and displacement.

Study $\mathbb{V}_n$ and statistical information.

Information and natural basis

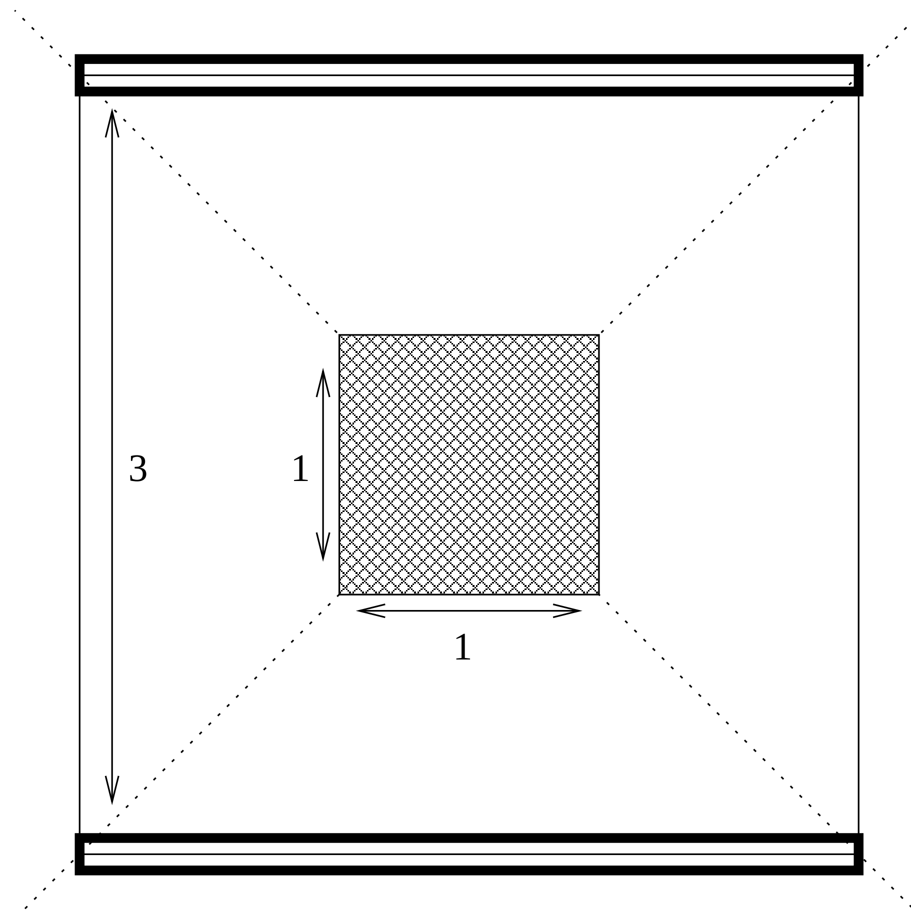
- Want to find basis for span$\{L_{ij}(u)\}$ in increasing difficulty of estimation
- Use characterization to parametrize scattering density $\varrho(u|\alpha) = \sum_{ij} \alpha_{ij} L_{ij}(u)$
- Fisher Information
- Singular value decomposition

Reconstruction using pixels

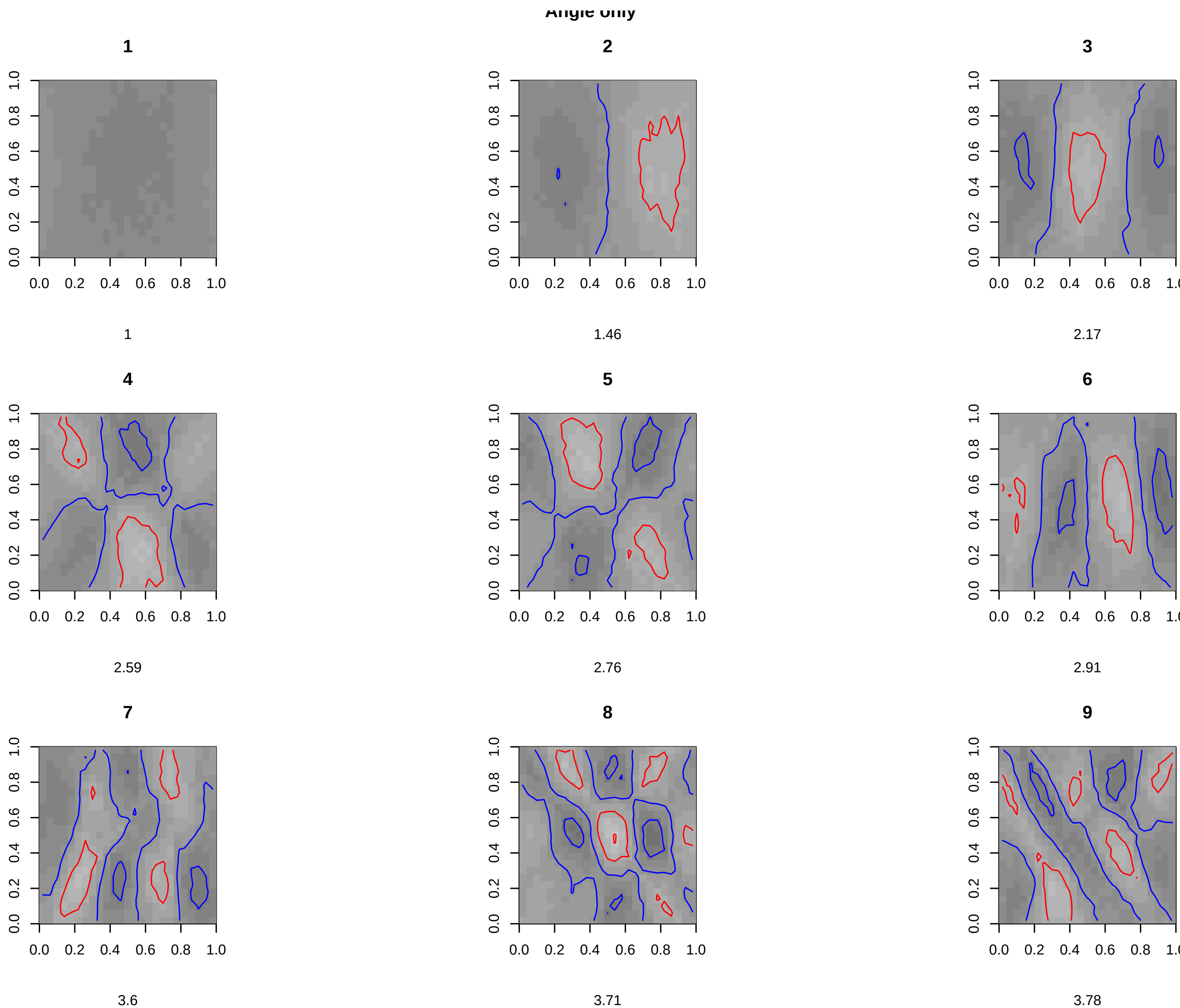
25 x 25 pixelization

Uniform incidence angle

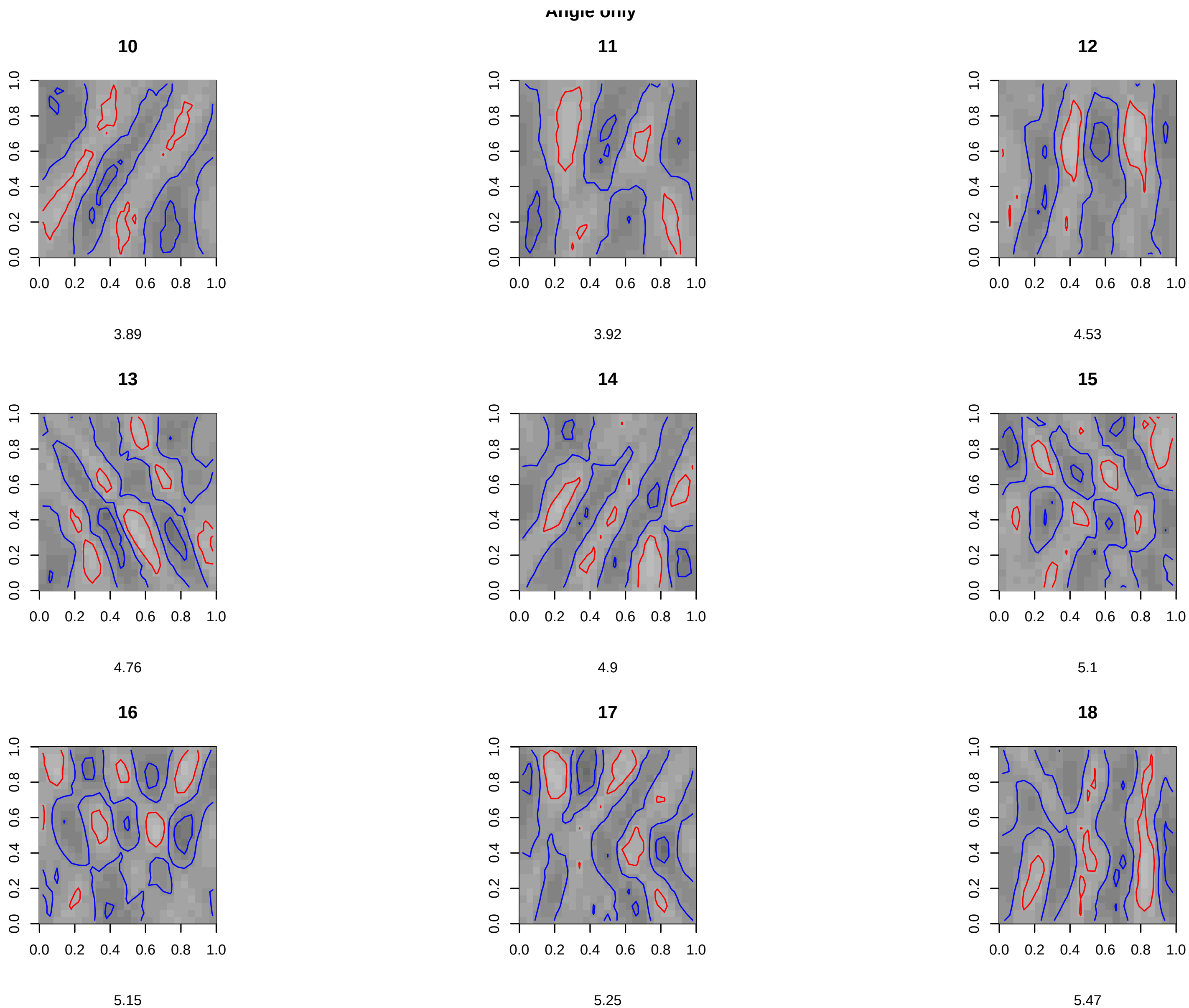
Maximal angle: $[-\pi/4, \pi/4]$



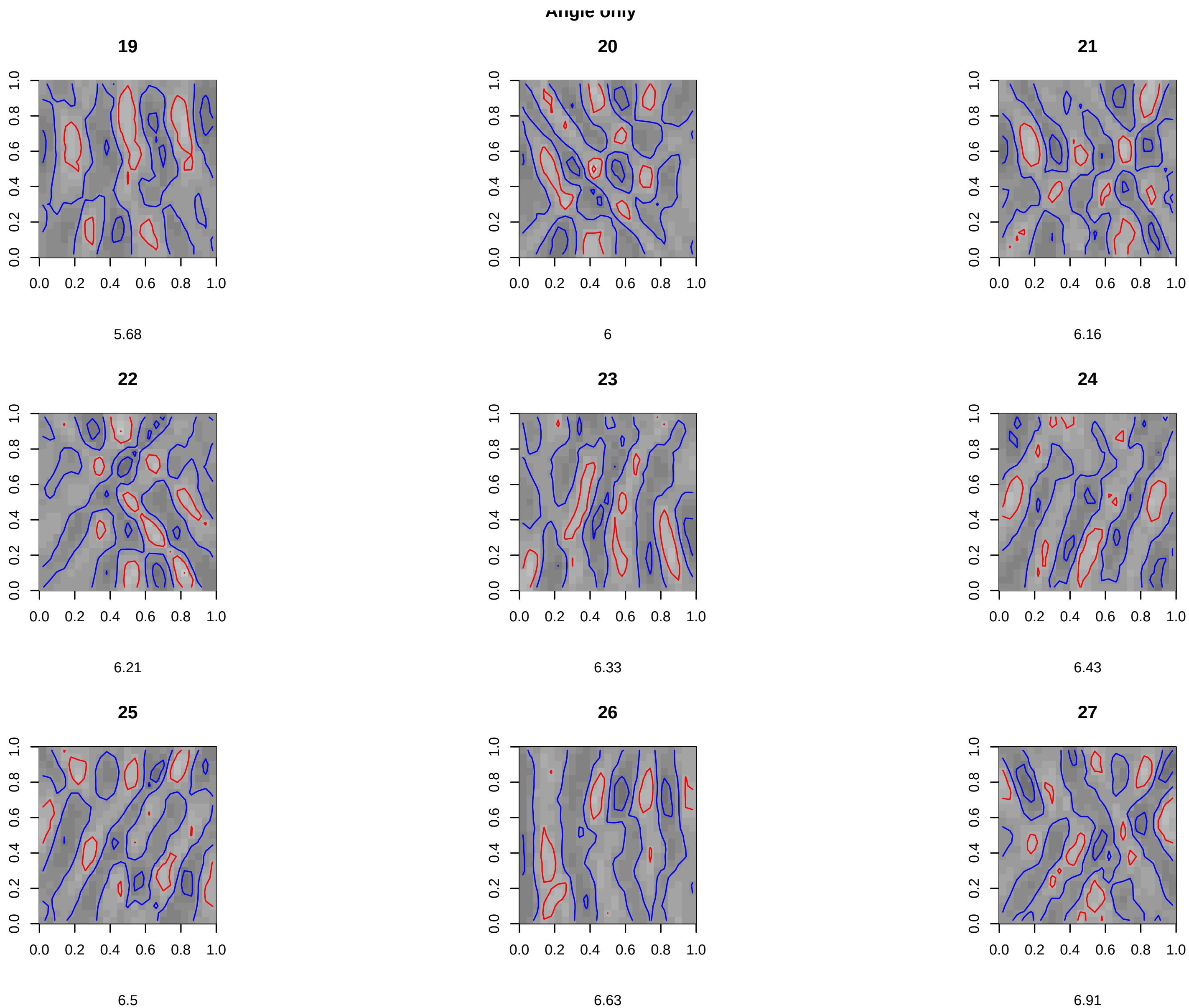
Dictionary (angle alone) 1-9



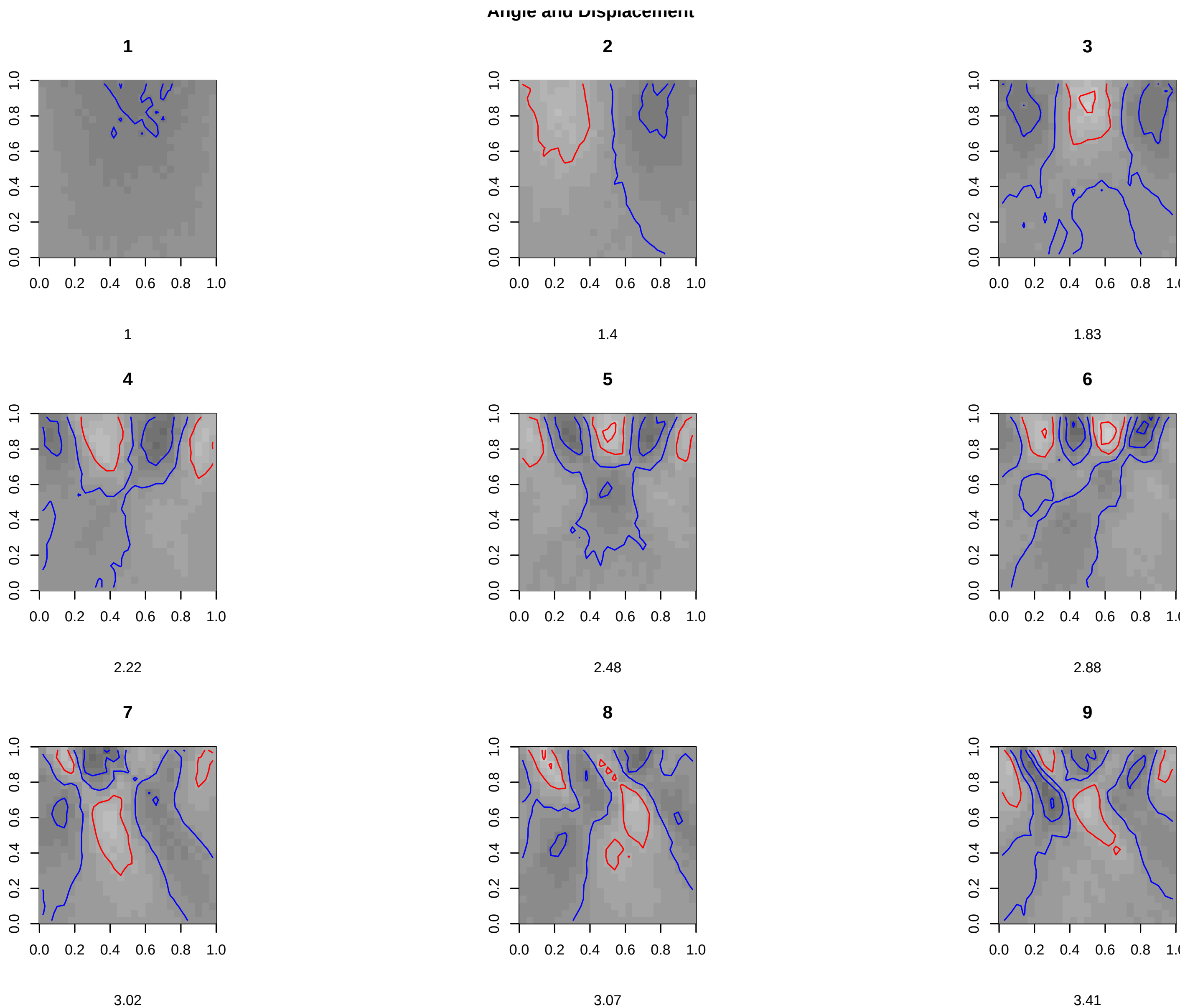
Dictionary (angle alone) 10-18



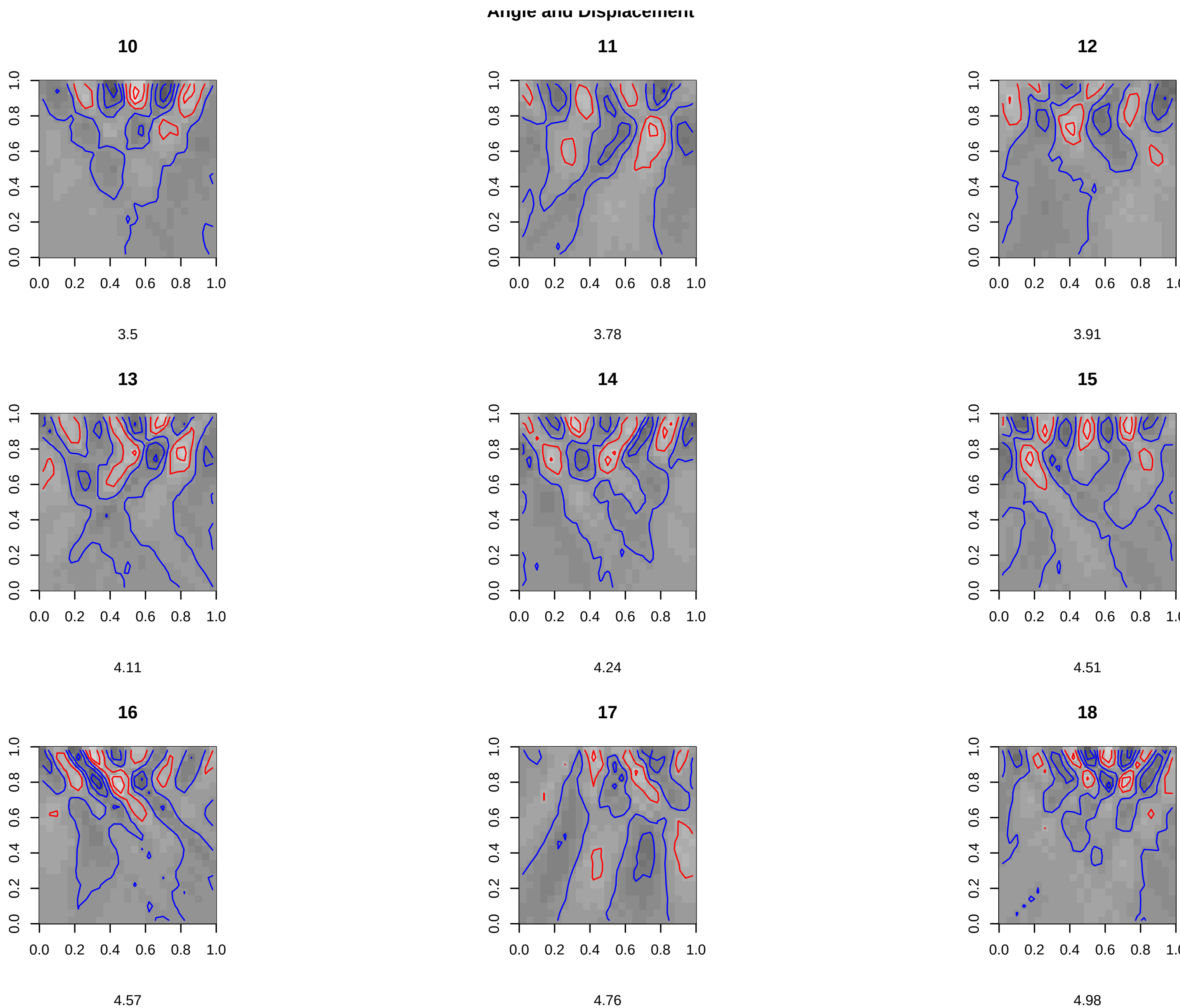
Dictionary (angle alone) 19-27



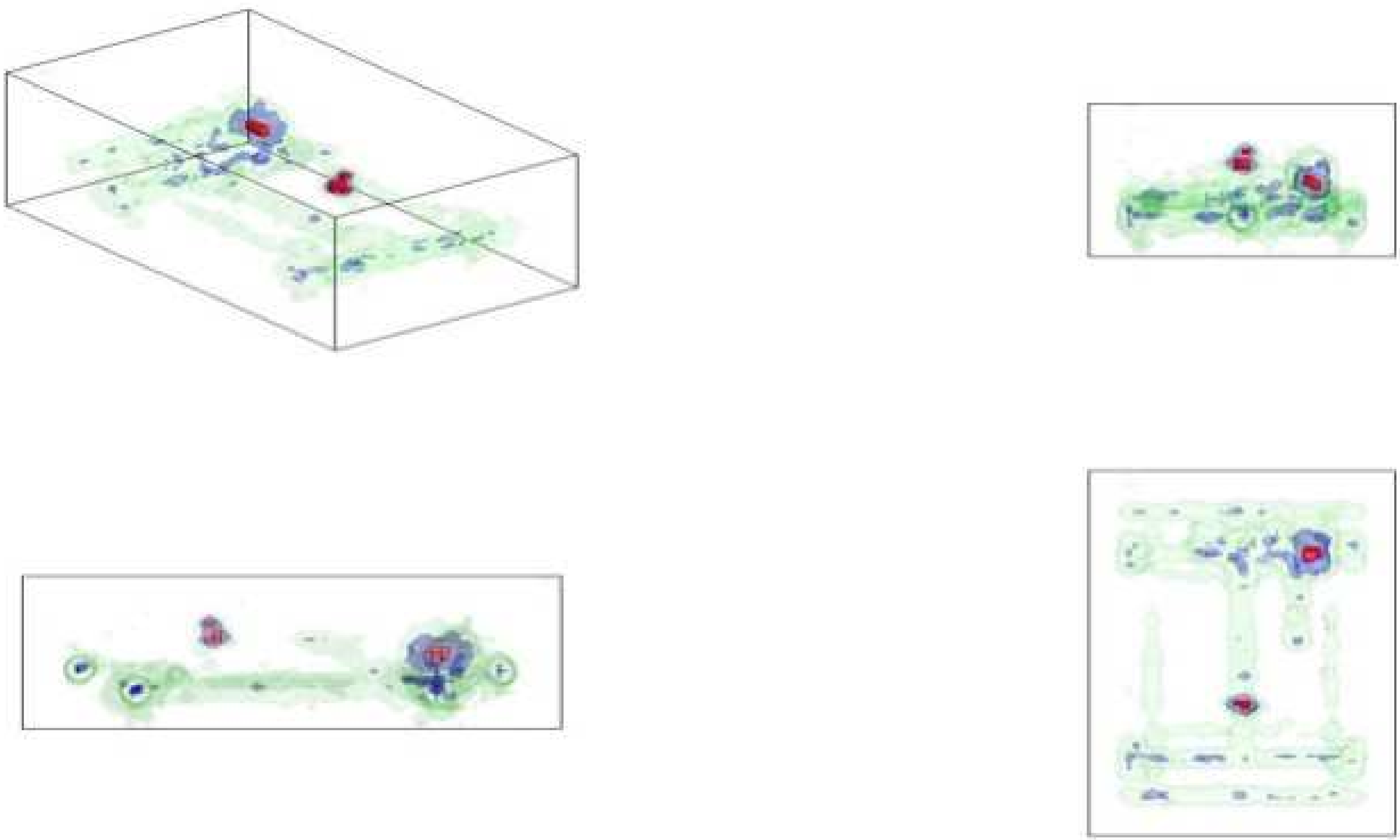
Dictionary (full data) 1-9



Dictionary (full data) 10-18

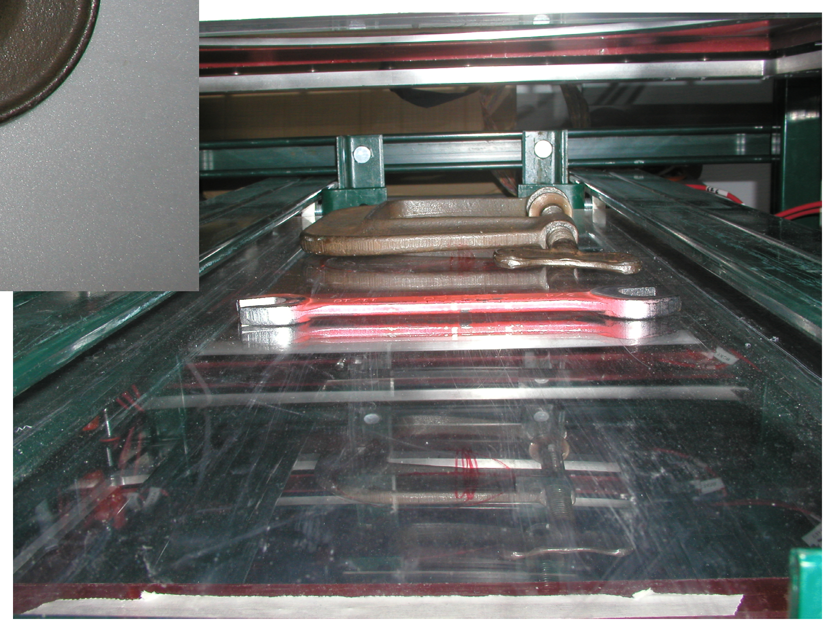


ML Estimation of GEANT Simulation



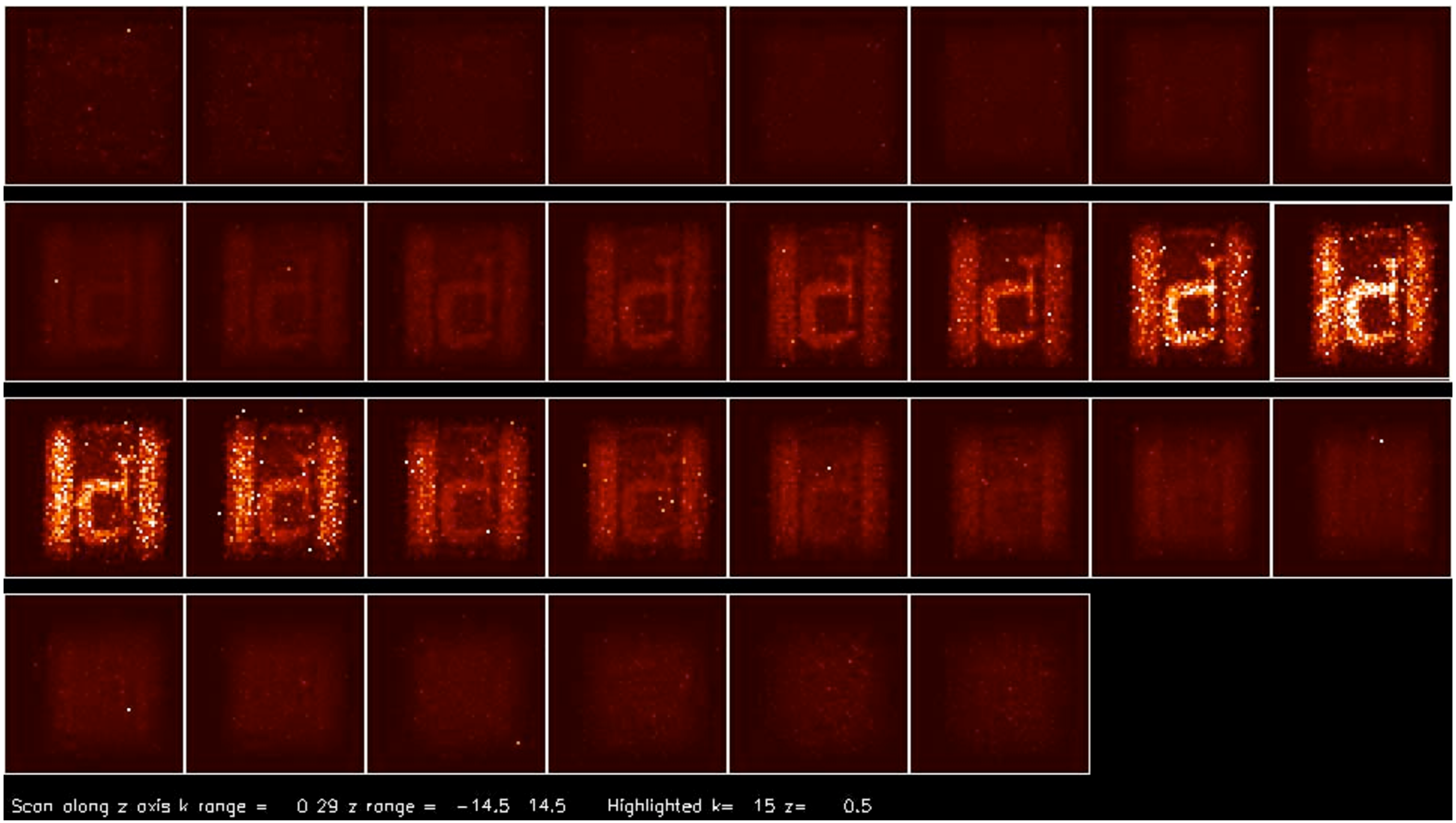
Real Example

Radiograph of another object



Real Example — reconstruction

Clamp in z-projections



Conclusions

Can do nonparametric reconstructions

Limitation in incidence angle should be avoided

Value in using all the data

Can answer: "What can we not recover" and "what is easy to recover" from a given experiment

Can "see" effective pixel size

Find dictionary

Thank You