ASPI

Applications of Interacting Particle Systems to Statistics

Applications Statistiques
des Systèmes de Particules en Interaction

proposition of a research team

INRIA research theme Num C

Optimisation and Inverse Problems for Stochastic or Large–Scale Systems

IRISA / INRIA Rennes

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Contents

1 Team

1.1 ASPI members

- **Head of the research team**
  François Le Gland, DR INRIA
- **Research scientists**
  Fabien Campillo, CR INRIA
  Frédéric Cérou, CR INRIA
- **External member**
  Arnaud Guyader, MC université de Haute–Bretagne
- **PhD student**
  Natacha Caylus (MENRT grant, mathematics)
- **Research team assistant**
  Huguette Béchu, TR INRIA (part–time)

1.2 Biographical sketch

▶ François Le Gland has graduated from École Centrale des Arts et Manufactures in 1978, and he has received a PhD degree in applied mathematics from Université Dauphine in 1981, which he had prepared at INRIA Rocquencourt. He has been hired by INRIA in January 1982, at Rocquencourt until September 1983, when he moved to INRIA Sophia–Antipolis. He has been the vice–head of the research team MEFISTO from this time until September 1993, when he joined IRISA. Since June 1998 he has been the head of the research team SIGMA2. He has held visiting positions in 1986 (three months) in the Department of Electrical Engineering of the University of Maryland at College Park, and in 1993 (five months) in the Department of Mathematics of the University of Southern California.

▶ Fabien Campillo has received a PhD degree in applied mathematics from Université de Provence in 1984. During his military service, he has held a temporary position as *Scientifique du contingent* at DCN–CERDSM in Toulon. He has held a postdoctoral position in 1985 (three months) in the Department of Mathematics of the University of British Columbia. He has been hired by INRIA in October 1986, at Sophia–Antipolis and Marseilles until July 2002, when he joined IRISA. He has been the head of the research team SYSDYS from January 1996 to December 2001, and he has been in charge of *Valorisation and Relations Industrielles* for INRIA Sophia–Antipolis from January 1994 to December 1995.

▶ Frédéric Cérou has received a PhD degree in applied mathematics from Université de Provence in 1995, which he had prepared at INRIA Sophia–Antipolis. During his military service, he has held a temporary position as *Scientifique du contingent* at ALCATEL–ALSTHOM Recherche in Marcoussis. He has held a postdoctoral position in 1995 (three months) in the Department of Mathematics of the University of Edinburgh. He has been hired by INRIA in September 1995, at Sophia–Antipolis and Marseilles until July 1999, when he joined IRISA. He has held visiting positions in 1996 (three months) and 1999 (six months) in the Department of Civil Engeneering and Operations Research of Princeton University.

► Arnaud Guyader has obtained the *Agrégation* degree in mathematics in July 1997, and he has received a PhD degree in signal processing and telecommunications from Université de Rennes 1 in April 2002, which he had prepared at IRISA. From September 2001 to June 2003 he has held a temporary ATER position at Université de Haute Bretagne (UHB, Rennes), and since September 2003 he is *Maître de conférence* in the Department of Mathematics of UHB.

1.3 Historical background

The activity at IRISA in the domain of particle methods can be traced back to the beginning of 1997, with first contacts with Pierre Del Moral (LSP, Laboratoire de Statistiques et Probabilités, Université Paul Sabatier, Toulouse) and with Christian Musso (ONERA, Chatillon). These early contacts resulted in a first research project *Méthodes particulaires et filtrage non–linéaire*, which started at the beginning of 1998 with the support of CNRS, within the programme *Modélisation et simulation numérique*, and coordinated by Pierre Del Moral. Within this first project, several workshops have been organized in Toulouse (December 1997), in Rennes (June 1998), in Cambridge (December 1999) and in Paris (June 2001). In particular, the Cambridge workshop has been the occasion of first contacts with Arnaud Doucet (CUED, Cambridge University Engineering Department), Éric Moulines and Olivier Cappé (ENST, Paris) and Christian Robert (CEREMADE, Centre de Recherche en Mathématiques de la Décision, Université Dauphine, Paris), and has partly contributed to the decision of Patrick Pérez to join Microsoft Research in Cambridge for the four subsequent years, where he started his activities on particle methods in computer vision.

The motivation and impetus to build a research group at IRISA in the domain of particle methods dates to the end of 2001, when a new research project *Chaînes de Markov cachées et filtrage particulaire* started with the support of CNRS, within the multidisciplinary programme *MathSTIC*, and coordinated by François LeGland and Éric Moulines. A further project AS 67 *Méthodes particulaires*, started at the end of 2002 with the continuing support of CNRS as a DSTIC *action spécifique*, and coordinated by Olivier Cappé and François LeGland, with the objective of drawing a picture of the research in France in the domain of particle methods, and of federating the research efforts within the scientific community, in relation with the industrial world. In this respect and with the additional support of the GDR ISIS, a two–day meeting on applications of particle filtering in signal processing and in computer vision has been co–organized by Jean–Pierre LeCadre and François LeGland in December 2002, and a one–day meeting on applications of particle filtering in tracking and in digital communications, has been co–organized by Olivier Cappé and François LeGland in December 2003. Another outcome of these two events has been the organization of training sessions and seminars on particle filtering in response to demand from industrial partners (SAGEM, CNES and EDF R&D).

Over this period, two PhD theses devoted to particle methods have been completed or are still in progress at IRISA within the SIGMA2 research team (and four more within the VISTA research team).

In view of this brief historical background, it appears that IRISA has gained a prominent position in the domain of particle methods, and the proposition of a new research team devoted to the design, analysis and implementation of particle methods should be seen as an opportunity to confirm and strenghten this position.

2 Overall objectives

The scientific objectives of the ASPI research team are the design, analysis and implementation of interacting Monte Carlo methods, or particle methods, with focus on

- statistical inference in hidden Markov models (state and parameter estimation),
- risk evaluation.

The whole problematic is multidisciplinary, not only because of the many scientific and engineering areas in which particle methods are used, especially in positioning, navigation and tracking, visual tracking, mobile robotics, etc. but also because of the diversity of the scientific communities which have already contributed to establish the foundations of the field :

- target tracking,
- interacting particle systems,
- empirical processes,
- genetic algorithms (GA),
- hidden Markov models and nonlinear filtering,
- Bayesian statistics,
- Markov chain Monte Carlo (MCMC) methods.

The ASPI research team will essentially carry methodological research activities, rather than activities oriented towards a single application area, with the objective to obtain generic results with high potential for applications, and to bring these results (and other results found in the literature) until implementation on a few appropriate examples, through collaboration with industrial partners. The research team is clearly positioned within the former 4B research programme of INRIA, and within the Num C research programme in the new organization.

3 Scientific background

The objective here is to explain how interacting Monte Carlo methods differ from classical Monte Carlo methods, and to introduce the general and extremely fruitful framework of Feynman–Kac formulas, which will be constantly refered to later in Section 4, in the presentation of the research programme.

3.1 Monte Carlo methods

Monte Carlo methods are numerical methods that are widely used in situations where (i) a stochastic (usually Markovian) model is given for some underlying process, and (ii) some quantity of interest should be evaluated, that can be expressed in terms of the expected value of a functional of the process trajectory, or the probability that a given event has occurred. Numerous examples can be found, e.g. in financial engineering (pricing of options and derivative securities) [4], in performance evaluation in communication networks (probability of buffer overflow), in statistics of hidden Markov models (state estimation, evaluation of contrast and score functions), etc. Very often in practice, no analytical expression is available for the quantity of interest, but it is possible to simulate trajectories of the underlying process. The idea behind Monte Carlo methods is to generate independent trajectories of the underlying process, and to use as an approximation (estimator) of the quantity of interest the average of the functional over the resulting independent sample. For instance, if

$$\mu_n(f) = \mathbb{E}[f(X_n)] \ ,$$

where $(X_n \, , \, n \geq 0)$ denotes a Markov chain with transition kernels $(Q_n(x, dx') \, , \, n \geq 1)$ and initial probability distribution $\mu_0(dx)$, then

$$\mu_n(f) \approx \mu_n^N(f) = \frac{1}{N} \sum_{i=1}^N f(\xi_n^i) \qquad \text{i.e.} \qquad \mu_n \approx \mu_n^N = \frac{1}{N} \sum_{i=1}^N \delta_{\xi_n^i} \ ,$$

where $(\xi_n^i \, , \, i = 1, \cdots, N)$ is an N–sample whose common probability distribution is precisely μ_n, which can be easily achieved as follows : independently for any $i = 1, \cdots, N$

$$\xi_0^i \sim \mu_0(dx) \qquad \text{and} \qquad \xi_k^i \sim Q_k(\xi_{k-1}^i, dx') \ ,$$

for any $k \geq 1$. By the law of large numbers, the above estimator $\mu_n^N(f)$ converges to $\mu_n(f)$ as the size N of the sample goes to infinity, with rate $1/\sqrt{N}$ and the asymptotic variance can be estimated. To reduce the asymptotic variance of the estimator, many variance reduction techniques are routinely used, among which importance sampling can be defined as follows : for any given importance decomposition

$$\mu_0(dx) = W_0^*(x) \, \mu_0^*(dx) \qquad \text{and} \qquad Q_k(x, dx') = W_k^*(x, x') \, Q_k^*(x, dx') \ ,$$

for any $k \geq 1$, it holds

$$\mu_n(f) = \mathbb{E}[f(X_n)] = \mathbb{E}^*[f(X_n) \prod_{k=0}^n W_k^*(X_{k-1}, X_k)] \ , \tag{IS}$$

with $W_0(x, x') = W_0(x')$, hence the alternative Monte Carlo estimator

$$\mu_n^N(f) = \frac{\displaystyle\sum_{i=1}^N [f(\xi_n^i) \prod_{k=0}^n W_k^*(\xi_{k-1}^i, \xi_k^i)]}{\displaystyle\sum_{i=1}^N [\prod_{k=0}^n W_k^*(\xi_{k-1}^i, \xi_k^i)]} = \sum_{i=1}^N w_{0:n}^i \, f(\xi_n^i) \qquad \text{i.e.} \qquad \mu_n^N = \sum_{i=1}^N w_{0:n}^i \, \delta_{\xi_n^i} \ ,$$

where independently for any $i = 1, \cdots, N$

$$\xi_0^i \sim \mu_0^*(dx) \qquad \text{and} \qquad \xi_k^i \sim Q_k^*(\xi_{k-1}^i, dx') \ ,$$

for any $k \geq 1$, i.e. independent trajectories are generated under an alternate wrong model, and are weighted according to their likelihood for the true model. For a given test function f, there are some adequate choices of the importance decomposition, for which the asymptotic variance of the alternative Monte Carlo estimator is smaller than the asymptotic variance of the original Monte Carlo estimator.

However, running independent Monte Carlo simulations can lead to very poor results, because trajectories $\xi_{0:n}^i = (\xi_0^i, \cdots, \xi_n^i)$ are generated *blindly*, and only afterwards is the corresponding weight $w_{0:n}^i$ evaluated, which can happen to be negligible in which case the corresponding trajectory is not going to contribute to the estimator, i.e. computing power has been wasted.

3.2 Interacting Monte Carlo methods

A recent and major breakthrough [64, 2], [3], a brief mathematical presentation of which is given in Section 3.3 has been the introduction of interacting Monte Carlo methods, also known as sequential Monte Carlo (SMC) methods, in which a whole (possibly weighted) sample also called a system of particles, is propagated in time, where the particles

- explore the state space under the effect of a mutation mechanism which mimics the evolution of the underlying process,
- and are replicated or terminated, under the effect of a selection mechanism which automatically concentrates the particles, i.e. the available computing power, into regions of interest of the state space.

In full generality, the underlying process is a Markov chain, whose state space can be finite, continuous (Euclidean), hybrid (continuous / discrete), constrained, time varying, pathwise, etc., the only condition being that it can easily be *simulated*. The very important case of a sampled continuous–time Markov process, e.g. the solution of a stochastic differential equation driven by a Wiener process or a more general Lévy process, is also covered.

In the special case of particle filtering, originally developed within the tracking community, the algorithms yield a numerical approximation of the optimal filter, i.e. of the conditional probability distribution of the hidden state given the past observations, as a (possibly weighted) empirical probability distribution of the system of particles. In its simplest version, introduced in several different scientific communities under the name of *interacting particle filter* [89], *bootstrap filter* [40], *Monte Carlo filter* [46] or *condensation* (conditional density propagation) algorithm [44], and which historically has been the first algorithm to include a redistribution step, the selection mechanism is governed by the likelihood function : at each time step, a particle is more likely to survive and to replicate at the next generation if it is consistent with the current observation. The algorithms also provide as a by–product a numerical approximation of the likelihood function, and of many other contrast functions for parameter estimation in hidden Markov models, such as the prediction error or the conditional least–squares criterion.

Particle methods are currently being used in many scientific and engineering areas :

- positioning, navigation, and tracking [41],
- multiple target or multiple object tracking [42, 43], visual tracking [44], [82, 72],
- mobile robotics [65],
- risk evaluation and simulation of rare events [39], [86],
- biological and molecular dynamics,
- audio signal processing [34, 55],
- digital communications [95], [18, 57, 59, 60], [93].

Other examples of the many applications of particle filtering can be found in the contributed volume [3] and in the special issue [11], which contains in particular the tutorial paper [12], and in the forthcoming textbook [9] devoted to applications in target tracking. Applications of sequential Monte Carlo methods to other areas, beyond signal and image engineering, e.g. to genetics, and molecular dynamics, can be found in [6]. Further information can be found in the following few web sites :

- *Sequential Monte Carlo Methods and Particle Filtering*, at Cambridge University, see `http://www-sigproc.eng.cam.ac.uk/smc/`.
- *Monte Carlo Localization (MCL)*, on particle filters in mobile robotics, by Dieter Fox at University of Washington, see `http://www.cs.washington.edu/ai/Mobile_Robotics/mcl/`.
- *Filtrage Particulaire*, in French, with a set of pedagogical demos (MATLAB scripts), by Fabien Campillo at IRISA, see `http://www.irisa.fr/sigma2/campillo/site-pf/`.

The lecture notes [1] gives a very accessible introduction to the mathematical aspects of particle methods, the survey article [64] presents a synthesis of the many results obtained by the authors, which is further elaborated in the textbook [2].

Particle methods are very easy to implement, since it is sufficient in principle to simulate independent trajectories of the underlying process. The whole problematic is multidisciplinary, not only because of the already mentioned diversity of the scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have contributed to establish the foundations of the field : target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods.

3.3 General framework : Particle approximation of Feynman–Kac flows

The following abstract point of view, developed and extensively studied by Pierre Del Moral [64, 2], has proved to be extremely fruitful in providing a very general framework to the design and analysis of numerical approximation schemes, based on systems of branching and / or interacting particles, for nonlinear dynamical systems with values in the space of probability distributions, associated with Feynman–Kac formulas of the form

$$\mu_n(f) = \frac{\gamma_n(f)}{\gamma_n(1)} \qquad \text{where} \qquad \gamma_n(f) = \mathbb{E}[f(X_n) \prod_{k=0}^{n} g_k(X_k)] \ , \tag{FK}$$

where $(X_n \, , \, n \geq 0)$ denotes a Markov chain with (possibly) time dependent state spaces $(E_n \, , \, n \geq 0)$ and with transition kernels $(Q_n(x, dx') \, , \, n \geq 1)$, and where the nonnegative potential functions $(g_n \, , \, n \geq 0)$ play the role of selection functions.

Feynman–Kac formulas (FK) naturally arise whenever importance sampling is used, as seen from (IS) above : this applies for instance to simulation of rare events, see Section 4.2, to filtering, i.e. to state estimation in hidden Markov models (HMM), see Section 4.3, etc.

Clearly, the flow $(\gamma_n \, , \, n \geq 0)$ satisfies the dynamical system

$$\gamma_n(f) = \gamma_{n-1}(Q_n(g_n \, f)) = \gamma_{n-1}(R_n \, f) \ ,$$

with the nonnegative kernel $R_n(x, dx') = Q_n(x, dx') \, g_n(x')$, and the flow $(\mu_n \, , \, n \geq 0)$ of associated normalized measures (i.e. probability distributions) satisfies the dynamical system

$$\mu_n(f) = \frac{\mu_{n-1}(Q_n(g_n \, f))}{\mu_{n-1}(Q_n \, g_n)} = \bar{R}_n(\mu_{n-1})(f) \qquad \text{where} \qquad \bar{R}_n(\mu) = \frac{\mu \, R_n}{\mu \, R_n(1)} \ ,$$

which can be decomposed in the following two steps

$$\mu_{n-1} \longmapsto \eta_n = \mu_{n-1} \, Q_n \longmapsto \mu_n = g_n \cdot \eta_n \ ,$$

i.e.

$$\mu_{n-1}(dx) \xrightarrow{\text{mutation}} \eta_n(dx') = \mu_{n-1} \, Q_n(dx') = \int_{E_{n-1}} \mu_{n-1}(dx) \, Q_n(x, dx')$$

$$\xrightarrow{\text{weighting}} \mu_n(dx') = (g_n \cdot \eta_n)(dx') = g_n(x') \, \eta_n(dx') \ / \eta_n(g_n) \ .$$

Conversely, the normaling constant $\gamma_n(1)$, hence the unnormalized flow $(\gamma_n \, , \, n \geq 0)$ as well, can be expressed in terms of the normalized flow : indeed $\gamma_n(1) = \eta_0(g_0) \cdots \eta_{n-1}(g_{n-1})$. To solve these equations *numerically*, and in view of the basic assumption that it is *easy* to *simulate* r.v.'s according to the probability distributions $(Q_n(x, dx') \, , \, n \geq 1)$, i.e. to mimic the evolution of the Markov chain, the original idea behind particle methods consists of looking for an approximation of the probability distribution μ_n in the form of a (possibly weighted) empirical probability distribution associated with a system of particles :

$$\mu_n \approx \mu_n^N = \sum_{i=1}^{N} w_n^i \, \delta_{\xi_n^i} \qquad \text{with} \qquad \sum_{i=1}^{N} w_n^i = 1 \ .$$

The approximation is completely characterized by the set $\Sigma_n = (\xi_n^i, w_n^i \, , \, i = 1, \cdots, N)$ of particle positions and weights, and the algorithm is completely described by the mechanism which builds Σ_k from Σ_{k-1}. The simplest version of the algorithm could be described as follows :

- the mutation step could not be applied *exactly*, because the exact expression

$$\mu_{k-1}^N \, Q_k(dx') = \sum_{i=1}^N w_{k-1}^i \; Q_k(\xi_{k-1}^i, dx') \ ,$$

is not tractable, and it is replaced by the particle approximation

$$\mu_{k-1}^N \, Q_k \approx \eta_k^N = \frac{1}{N} \, \sum_{i=1}^N \delta_{\xi_k^i} \ ,$$

where $(\xi_k^i \, , \, i = 1, \cdots, N)$ is precisely an N–sample with common probability distribution $\mu_{k-1}^N \, Q_k(dx')$, which can easily be achieved as follows : independently for any $i = 1, \cdots, N$

$$\tau_{k-1}^i \sim (w_{k-1}^1, \cdots, w_{k-1}^N) \qquad \text{and} \qquad \widehat{\xi}_{k-1}^i = \xi_{k-1}^{\tau_{k-1}^i} \ , \tag{R}$$

which means that a particle ξ_{k-1}^i from the set Σ_{k-1} with a higher weight w_{k-1}^i is more likely to be selected than other particles with a smaller weight, and

$$\xi_k^i \sim Q_k(\widehat{\xi}_{k-1}^i, dx') \ ,$$

which is easy by assumption,

- the weighting step is applied *exactly* to the approximation η_k^N, which yields

$$g_k \cdot \eta_k^N = \mu_k^N = \sum_{i=1}^N \frac{g_k(\xi_k^i)}{\displaystyle\sum_{j=1}^N g_k(\xi_k^j)} \; \delta_{\xi_k^i} = \sum_{i=1}^N w_k^i \, \delta_{\xi_k^i} \ ,$$

i.e. each particle ξ_k^i receives a weight w_k^i which is proportional to the selection function evaluated at the particle position.

The algorithm yields a numerical approximation of the probability distribution μ_n as the weighted empirical probability distribution μ_n^N associated with a system of particles, and many asymptotic results have been proved as the number N of particles (sample size) goes to infinity :

- convergence in $\mathbb{L}^p$ [27, 24, 25],
- convergence of empirical processes indexed by classes of functions [26],
- uniform convergence in time [24], [49, 50],
- central limit theorem [23, 77], [91],
- propagation of chaos [27],
- large deviations principle [22],
- moderate deviations principle [90], etc.

using techniques coming from applied probability (interacting particle systems, empirical processes [10]), see e.g. the survey article [64] or the forthcoming textbook [2], and references therein. Beyond this simplest (bootstrap) version of the algorithm, many algorithmic variations have been proposed [32], and are commonly used in practice :

- In the redistribution step, sampling with replacement could be replaced with other redistribution schemes so as to reduce the variance (this issue has also been addressed in genetic algorithms).
- To reduce the variance and to save computational effort, it is often a good idea not to redistribute the particles at each time step, but only when the weights $(w_k^i\,,\, i=1,\cdots,N)$ are too much uneven.
- Even with interacting Monte Carlo methods, it could happen that some particle ξ_k^i generated in one time step has a negligible weight $g_k(\xi_k^i)$. If this happens for too many particles in the sample $(\xi_k^i\,,\, i=1,\cdots,N)$, then computer power has been wasted, and it has been suggested to use importance sampling again in the mutation step, i.e. to let particles explore the state space under the action of an alternate wrong mutation kernel, and to weight the particles according to their likelihood for the true model, so as to compensate for the wrong modeling.

Most of the results proved in the literature assume that particles are redistributed (i) at each time step, and (ii) using sampling with replacement. Studying systematically the impact of these algorithmic variations on the convergence results is still to be done, and is part of the research programme of the ASPI research team, see Section 4.4.

4 Detailed research programme

Four main research directions, which are described with some details below, have been isolated under the following headings

- particle approximation for linear tangent Feynman–Kac flows,
- simulation of rare events,
- simulation–based methods for statistics of HMM,
- and algorithmic issues.

The first three are directions where the ASPI research team has a leading position within the contributors to the domain of particle methods, and the last direction is proposed with the objective of assessing on solid ground arguments if possible, and not only on whatever convincing simulation experiments, some of the many algorithmic variations that have been proposed either here or elsewhere as well.

4.1 Particle approximation for linear tangent Feynman–Kac flows

In situations where the underlying Markov model depends on an unknown parameter, i.e. where both the transition kernels $(Q_n(x, dx') \, , \, n \geq 1)$ and the selection functions $(g_n(x) \, , \, n \geq 0)$ depend on the parameter in a smooth sense, one would like to compute numerically the derivative of the Feynman–Kac flows $(\gamma_n \, , \, n \geq 0)$ or $(\mu_n \, , \, n \geq 0)$ w.r.t. the parameter, so as to get sensitivity information. Applications could be found in financial engineering, where derivatives (greeks) of options prices w.r.t. volatility, initial condition, etc. are extremely important quantities to be estimated. Another whole field of application is statistics of HMM, see Section 4.3, e.g. parameter estimation using minimum contrast estimators, or residual (score function) generation for monitoring dynamical systems subject to potential small changes.

Earlier ASPI contribution :

Assume that the statistical model is dominated, i.e. there exists another probability distribution $\mathbb{P}^0$, independent of the parameter, such that $\mathbb{P} \ll \mathbb{P}^0$ with Radon–Nikodym derivative

$$\frac{d\mathbb{P}}{d\mathbb{P}^0}\Big|_{\mathcal{F}_n} = \prod_{k=0}^{n} \Lambda_k \; .$$

Then

$$\gamma_n(f) = \mathbb{E}[f(X_n) \prod_{k=0}^{n} g_k(X_k)] = \mathbb{E}^0[f(X_n) \prod_{k=0}^{n} \Lambda_k \prod_{k=0}^{n} g_k(X_k)] \; ,$$

and taking derivative w.r.t. the parameter yields

$$\partial\gamma_n(f) = \mathbb{E}[f(X_n) \sum_{k=0}^{n} [\Xi_k + s_k(X_k)] \prod_{k=0}^{n} g_k(X_k)] \; ,$$

where

$$\Xi_k = \partial \log \Lambda_k \qquad \text{and} \qquad s_k(x) = \partial \log g_k(x) \; ,$$

for any $k \geq 1$. Introducing the signed kernel $\Gamma_k(x, dx')$ defined by

$$\Gamma_k \, f(x) = \int_{E_{n-1}} \Gamma_n(x, dx') \, f(x') = \mathbb{E}[f(X_n) \, \Xi_n \mid X_{n-1} = x] \; , \tag{AC}$$

for any $k \geq 1$, then clearly the linear tangent flow $(\partial\gamma_n \, , \, n \geq 0)$ satisfies the dynamical system

$$\partial\gamma_n(f) = \partial\gamma_{n-1}(Q_n(g_n \, f)) + \gamma_{n-1}(\Gamma_n(g_n \, f)) + \gamma_{n-1}(Q_n(g_n \, s_n \, f)) \; ,$$

and the linear tangent flow $(w_n \, , \, n \geq 0)$ for the normalized measures $w_n = \partial\mu_n = \partial(\gamma_n/\gamma_n(1))$ satisfies the dynamical system

$$\begin{aligned} w_n(f) \; &= \; \frac{w_{n-1}(Q_n(g_n \, f)) + \mu_{n-1}(\Gamma_n(g_n \, f)) + \mu_{n-1}(Q_n(g_n \, s_n \, f))}{\mu_{n-1}(Q_n \, g_n)} - a_n \, \frac{\mu_{n-1}(Q_n(g_n \, f))}{\mu_{n-1}(Q_n \, g_n)} \\ &= \; \bar{W}_n(\mu_{n-1}, w_{n-1})(f) \; , \end{aligned}$$

where the nonlinear operator $\bar{W}_n(\cdot, \cdot)$ is defined by

$$\bar{W}_n(\mu, w) = \frac{(w\, Q_n + \mu\, \Gamma_n + (\mu\, Q_n)\, s_n)\, g_n}{\mu(Q_n\, g_n)} - a_n\, \frac{(\mu\, Q_n)\, g_n}{\mu(Q_n\, g_n)} \ ,$$

and where the scalar a_n (depending on μ and w) is uniquely determined by the condition $\bar{W}_n(\mu, w)(1) = 0$.

It follows from the domination assumption that $w_n \ll \mu_n$ for any $n \geq 1$, and the idea here consists of looking for a joint approximation of the probability distribution μ_n and of the signed measure w_n in the form of two weighted empirical probability distributions associated with a unique system of particles :

$$\mu_n \approx \mu_n^N = \sum_{i=1}^N w_n^i\, \delta_{\xi_n^i} \qquad \text{and} \qquad w_n \approx w_n^N = \sum_{i=1}^N \rho_n^i\, w_n^i\, \delta_{\xi_n^i} \ ,$$

with

$$\sum_{i=1}^N w_n^i = 1 \qquad \text{and} \qquad \sum_{i=1}^N \rho_n^i\, w_n^i = 0 \ .$$

Plugging the weighted particle approximations μ_{k-1}^N and w_{k-1}^N into the nonlinear operators $\bar{R}_k(\cdot)$ and $\bar{W}_k(\cdot, \cdot)$, and using the same ideas from auxiliary particle filter as described in Section 4.4, see also [16], results in different joint particle approximation schemes, depending on the assumptions on the linear tangent kernel $\Gamma_k(x, dx')$. Notice that representation (AC) holds if and only if the linear tangent kernel $\Gamma_k(x, dx')$ is absolutely continuous w.r.t. the Markov transition kernel $Q_k(x, dx')$, i.e.

$$\Gamma_k(x, dx') = I_k(x, x')\, Q_k(x, dx') \ , \qquad \text{with} \qquad I_k(x, x') = \mathbb{E}[\Xi_k \mid X_{k-1} = x, X_k = x'] \ ,$$

and the following three different cases can be considered (with decreasing level of generality)

Case A (AC) holds, and it is easy to simulate jointly (X_k, Ξ_k) given $X_{k-1} = x$.

Case B (AC) holds, and an explicit expression is available for the density

$$I_k(x, x') = \mathbb{E}[\Xi_k \mid X_{k-1} = x, X_k = x'] \ .$$

Case C the Markov transition kernel $Q_k(x, dx')$ is dominated, i.e.

$$Q_k(x, dx') = q_k(x, x')\, Q_k^0(x, dx') \qquad \text{hence} \qquad \Gamma_k(x, dx') = \partial \log q_k(x, x')\, Q_k(x, dx') \ ,$$

and an explicit expression is available for the density $q_k(x, x')$ and for its logarithmic derivative w.r.t. the parameter.

In the special case of linear tangent filtering, a theoretical framework has been proposed in [80], based on earlier results [62], to analyze some of these joint particle approximation schemes.

Research objectives :

- Study asymptotic properties, as the number N of particles goes to infinity, of the various joint particle approximation schemes for linear tangent Feynman–Kac formulas : convergence in $\mathbb{L}^p$, central limit theorem, etc.

- Obtain stability results for the linear tangent Feynman–Kac flow, w.r.t. the initial condition and w.r.t. the model, and obtain uniform error estimates for the various joint particle approximation schemes, following and adapting the techniques of [47, 50].

- Design alternate joint particle approximation schemes for flows of bounded signed measures, e.g. using the Jordan decomposition into positive and negative parts, and study their asymptotic properties.

- Find general conditions for the existence of a probabilistic representation such as (AC) for the linear tangent kernel, e.g. using Malliavin calculus techniques [35, 36], [92].

- Study simulation–based approximation schemes, e.g. based on importance sampling, to compute the density
$$I_k(x, x') = \mathbb{E}[\Xi_k \mid X_{k-1} = x, X_k = x'] \ ,$$
in cases where no explicit expression is available. Such cases include for instance sampled continuous–time Markov process, e.g. the solution of a stochastic differential equation driven by a Wiener process or a more general Lévy process. This would imply also to incorporate a time discretization scheme and to study its effect on the overall algorithm.

- Extend the above results to a more general class of problems than (FK), where the nonlinear flow $(\mu_n \, , \, n \geq 0)$ in the space of probability distributions, is described by
$$\mu_n(f) = \bar{R}_n(\mu_{n-1})(f) \qquad \text{where} \qquad \bar{R}_n(\mu) = \frac{\mu \, R_n}{\mu \, R_n(1)} \ ,$$
and where $R_n(x, dx')$ is an *arbitrary* nonnegative finite kernel.

4.2 Simulation of rare events

Management and real–time control of large complex systems is facing a combined evolution towards a greater spatial distribution of sensors, decisions, etc. and an increasing concern for safety issues. In all domains where risk evaluation needs to be taken into account, being able to evaluate the probability that a rare event occurs, that would be critical for the safe behaviour of the system, is a difficult but important question. This includes for instance distributed control of large communication, computer and power networks, or advanced air trafic management, as considered in the IST HYBRIDGE project, see Section 6.1.

In mathematical terms, the problem deals with risk evaluation in a hybrid stochastic dynamical system, with a modular description, and the proposed solutions are based on risk decomposition techniques which exploit modularity, and interacting Monte Carlo methods for an efficient implementation of the algorithms.

An alternative method to the classical *importance sampling* is the *importance splitting* method, in which a sequence of increasingly rare events is defined, and a selection mechanism is introduced where sample paths for which an intermediate event holds true split / branch into several offsprings, while sample paths for which none of the intermediate events hold true are terminated [97], [79, 68, 38, 39], [73]. This selection mechanism allows to generate many sample paths for which the rare event holds true, and to evaluate statistics of such sample paths.

Earlier ASPI contribution :

In a preliminary work [86], the *importance splitting* algorithm has been cast into the general framework of particle approximation of Feynman–Kac formulas. To be more specific, let

$$T_B = \inf\{t \geq 0 \, : \, X_t \in B\} \ ,$$

denote the first time when the process $X = \{X_t \, , \, t \geq 0\}$ hits the critical set B. It may happen that most of the realizations of X never reach the set B, which makes the corresponding rare event probabilities extremely difficult to estimate

$$\mathbb{P}(T_B \leq T) \qquad \text{and} \qquad \text{Law}(X_t \, , \, 0 \leq t \leq T_B \mid T_B \leq T) \ , \tag{RE}$$

where T is either a deterministic finite time, or a $\mathbb{P}$–almost surely finite stopping time, for instance the hitting time of a recurrent set $R \subset S$. The second case covers the situation of a particle trying to escape from a region packed with obstacles, before it hits and gets killed by one of the obstacles.

With any decreasing sequence

$$B = B_m \subset \cdots \subset B_1 \subset B_0 \ .$$

of level sets, it is possible to associate (i) an event–driven Markov chain $(\mathcal{X}_n \, , \, n = 0, \cdots , m)$, taking values in the path space

$$E = \bigcup_{t' \leq t''} \mathbb{D}([t', t'']) \ ,$$

and defined by

$$\mathcal{X}_n = (X_t \, , \, T_{n-1} \wedge T \leq t \leq T_n \wedge T) \qquad \text{where} \qquad T_n = \inf\{t \geq 0 \, : \, X_t \in B_n\} \ ,$$

is the first time when the process X hits the intermediate level set B_n, and (ii) selection functions $(g_n(x) \, , \, n = 0, \cdots , m)$ defined for each $x = (x_t \, , \, t' \leq t \leq t'')$ in the path space E by

$$g_n(x) = 1_{(x_{t''} \, \in \, B_n)} \ .$$

This modeling yields

$$\mathbb{E}[f(X_t \, , \, T_{n-1} \wedge T \leq t \leq T_n \wedge T) \, 1_{(T_n \, \leq \, T)} \,] = \mathbb{E}[f(\mathcal{X}_n) \prod_{k=0}^{n} g_k(\mathcal{X}_k) \,] = \gamma_n(f) \ ,$$

which clearly is in a form of a Feynman–Kac formula (FK) as introduced in Section 3.3, and in particular

$$\mathbb{P}[T_n \leq T] = \mathbb{E}[\prod_{k=0}^{n} g_k(\mathcal{X}_k) \,] = \gamma_n(1) = \prod_{k=0}^{n} \eta_k(g_k) \ ,$$

which after all is nothing but the straightforward formula

$$\mathbb{P}[T_n \leq T] = \prod_{k=0}^{n} \mathbb{P}[T_k \leq T \mid T_{k-1} \leq T] \; .$$

More generally

$$\mathbb{E}[f_n(X_t \, , \, 0 \leq t \leq T_n \wedge T) \; 1_{(T_n \; \leq \; T)} \,] = \mathbb{E}[f(\mathcal{X}_0, \cdots, \mathcal{X}_n) \, \prod_{k=0}^{n} g_k(\mathcal{X}_k) \,] \; ,$$

which provides information about trajectories that eventually hit the critical set B_n. From this Feynman–Kac representation, it is easy to design particle approximation schemes for the rare event probabilities (RE), and the selection mechanism takes the following form : particles that do not manage to reach the next level before time T are removed, and the other successful particles are replicated, so as to make sure that the same, or at least a significant, number of particle is alive at each generation (the n–th generation being there a cloud (or swarm) of particles trying to reach the n–th level set). The tremendous benefit of this modeling, is that all the theoretical results for particle approximation of general Feynman–Kac flows are available for free.

Research objectives :

- Another very similar method is the RESTART algorithm [75, 74, 83, 56], which seems to have less solid mathematical foundations. Our objective there would be to translate the RESTART algorithm into the formalism of Feynman–Kac flows. An interpretation in terms of interacting and branching particle systems could be more appropriate there.

- Design algorithms that would combine *importance sampling* and *importance splitting*, and study their asymptotic properties. Sample paths would be generated under a proposal probability distribution for which the intermediate events are not so rare, and those sample paths for which an intermediate event holds true would be given a number of offsprings related to their weight, i.e. to the Radon–Nikodym derivative of the proposal probability distribution w.r.t. the true probability distribution. In other words, among all the sample paths for which an intermediate event holds true, those which are closest to a typical sample path from the true probability distribution would be given more offsprings than others.

- On the *importance sampling* side, choice of the proposal Markov model, based on asymptotic results for the particle approximation of the rare event probabilities (RE), especially on a central limit theorem. If a parametrized family of proposal Markov models is used, sensitivities and their particle approximation could be used to optimize within the family.

- On the *importance splitting* side, choice of the levels defining the intermediate less rare events, and choice of the number of such levels. For a given amount of computational power, would it be more efficient to use many particles and few levels, or the other way round, less particles but more levels, the objective being to make sure that some particle at last reach the next level, so as to avoid the extinction of the particle system.

- Consider branching / selection mechanism associated with more general stopping times than simply hitting times of intermediate level sets.
- Specialize the general results to the case of stochastic differential equations driven by a Wiener process or a more general Lévy process, with values in a hybrid continuous / discrete state space. The target application here is the evaluation of collision risk in air traffic management (ATM), as considered in the IST HYBRIDGE project, see Section 6.1.

4.3 Simulation–based methods for statistics of HMM

Hidden Markov models (HMM) form a special case of partially observed stochastic dynamical systems, in which the state of a Markov process (in discrete or continuous time, with finite or continuous state space) should be estimated from noisy observations. These models are very flexible, because of the introduction of latent variables (non observed) which allows to model complex time dependent structures, to take constraints into account, etc. In addition, the underlying Markovian structure makes it possible to use numerical algorithms (particle filtering, Markov chain Monte Carlo methods (MCMC), etc.) which are computationally intensive but whose complexity is rather small. HMM are widely used in various applied areas, such as speech recognition, alignment of biological sequences, tracking in complex environment, modeling and control of networks, digital communications, etc.

In HMM, also known as state–space models, several equivalent expressions are available for the likelihood function, which can be seen as a marginal / integral of the complete data likelihood function (for the hidden states and the observations jointly), or as a Feynman–Kac integral.

To be specific, assume that conditionally on the hidden states $\{X_k\,,\,k \geq 0\}$, the observations $\{Y_k\,,\,k \geq 0\}$ are mutually independent, and that the conditional probability distribution of Y_k depends only on the hidden state X_k at the same time instant, where by definition

$$\mathbb{P}[Y_k \in dy' \mid X_k = x'] = g_k(x', y')\, \lambda_k^F(dy') \ ,$$

which introduces

$$\Psi_k(x') = g_k(x', Y_k) \ ,$$

as a measure of the adequation of the state x' with the observation Y_k. Using the Feynman–Kac formula

$$\gamma_n(f) = \mathbb{E}[f(X_n) \prod_{k=0}^{n} \Psi_k(X_k)] \ ,$$

it is possible to estimate recursively the hidden state X_n given the observations $(Y_0, \cdots, Y_n)$, in view of the following relation

$$\mu_n(f) = \frac{\gamma_n(f)}{\gamma_n(1)} = \mathbb{E}[f(X_n) \mid Y_0, \cdots, Y_n] \ ,$$

but it is also possible to get, as a by–product, the following expression for the likelihood function

$$\mathbb{E}[\prod_{k=0}^{n} \Psi_k(X_k)\,] = \gamma_n(1) = \prod_{k=0}^{n} \eta_k(\Psi_k) \ . \qquad \text{(LF)}$$

Many other contrast functions, e.g. the prediction error, could be expressed as well in terms of the filter $(\mu_k\,,\,k \geq 0)$ or the predictor $(\eta_k\,,\,k \geq 0)$.

This expression (LF) is obviously not explicit, which means that numerical approximation schemes should be used to evaluate the likelihood function for any possible value of the unknown parameters. A related popular approach is the EM algorithm, an iterative method to find numerically the maximum of the likelihood function, where at each iteration of the algorithm an auxiliary function is evaluated, which is also given by a (more complicated) integral formula and which ideally could be easily maximized w.r.t. the parameters. Whether direct maximization of the likelihood function should be preferred to the EM algorithm has been discussed in [14]. Among the various classes of numerical approximation schemes to evaluate the likelihood function in state–space models, Monte Carlo methods have emerged during the last decade as the main and most efficient :

- Monte Carlo maximum likelihood (MCML) has been proposed in [37, 67] and further analyzed in [15], see also [84],
- for completeness, Monte Carlo EM (MCEM) and stochastic approximation EM (SAEM) have been proposed and analyzed in [58] and [28], respectively.

All these algorithms rely on importance sampling and on generating independent samples from the joint posterior distribution of the hidden state sequence given the observations. In particular the MCML formulation allows to approximate the likelihood function globally, i.e. to obtain a Monte Carlo approximation which is smooth w.r.t. the parameter, thus answering an objection raised in [101, 54]. Using interacting Monte Carlo methods could yield a more efficient numerical approximation scheme to evaluate the likelihood function.

Earlier ASPI contribution :

To maximize the likelihood function w.r.t. the parameter, most of the proposed Monte Carlo methods rely on external numerical procedures, and it would be very useful to have also numerical approximation schemes to compute the score function, i.e. the derivative of the log–likelihood function w.r.t. the parameter. Obviously, this would rely on a numerical approximation of the linear tangent filter, i.e. the derivative of the optimal filter w.r.t. the parameter.

A major contribution of the ASPI research team is precisely to propose and to study several joint particle approximation schemes for the optimal filter and the linear tangent filter [62], which exploit the absolute continuity of the linear tangent filter w.r.t. the optimal filter, valid under a mild assumption on the model, see Section 4.1. These joint particle approximation schemes have been applied to design parameter estimation algorithms [85, 80], [33], including recursive parameter estimation algorithms.

In the simple case of HMM with finite state space, asymptotic properties of the maximum likelihood estimator (MLE) and of other minimum contrast estimators have been obtained during the PhD thesis of Laurent Mevel [98] as the number of observations goes to infinity. The key idea there is to express the contrast function, e.g. the log–likelihood function

$$\ell_n = \frac{1}{n} \sum_{k=0}^{n} \log \eta_k(\Psi_k) \;,$$

and its derivative w.r.t. the parameter, as an additive functional (actually a time average) of an extended Markov chain consisting of (i) the hidden state, (ii) the observation, (iii) the prediction filter, and if needed (iv) its derivative w.r.t. the parameter. Assuming that the underlying finite–state Markov chain is primitive, it is proved that the extended Markov chain is geometrically ergodic [47, 48], from which asymptotic properties (consistency, asymptotic normality, etc.) of the minimum contrast estimators follow. Recursive versions of the estimators have been studied as well. These results have been recently extended by other authors to HMM with general state–space [29] and to AR models with hidden Markov regime [30].

Research objectives :

- Obtain a smooth Monte Carlo approximation of the likelihood function, that could then be differentiated w.r.t. the parameter. A possible idea would be to use the generic algorithm proposed in [76, 49], see Section 4.4, within the MCML formulation. This would improve the implementation of order $O(N^2)$ proposed in [69].
- For observations on a finite time horizon, study the asymptotic behaviour, as the number N of particles goes to infinity, of the various interacting particle implementations of the MCML estimator to the MLE.
- Develop a complete theory of statistical inference, including parameter estimation, hypotheses testing, validation and model selection, monitoring, etc., for models where hidden states and observations form jointly a Markov chain (which generalize HMM, AR models with hidden Markov regime, etc.). Large time asymptotic results will rely on some kind of forgetting property for the filter and its derivative w.r.t. the parameter, which is part of the research programme of the ASPI research team, see Section 4.1. In particular, working with the filter will allow to make use of stability results [50] which improve upon earlier stability results [47], [24] obtained for the predictor.
- Alternatively, small noise asymptotics could also be used, especially for continuous–time models [5], where it is rather easy to obtain interesting explicit results, in terms close to the language of nonlinear deterministic control theory. Preliminary results have been obtained in [45], [70] for partially observed diffusion processes, which will be generalized to other classes of models, e.g. stochastic differential equations driven by a general Lévy process, with values in a hybrid continuous / discrete state space.
- For observations in an infinite time horizon, study the asymptotic properties, as the number N of particles and the number n of observations go to infinity simultaneously, of the various interacting particle implementations of the minimum contrast estimators to the true value of the parameter.

4.4 Algorithmic issues

Even though particle methods, and especially particle filtering methods, could be credited for major advances and successful application in many areas, there is a common agreement that many algorithmic improvements could still be made. In the recent past, significant improvements have been made :

- In designing algorithms to propose new particles which are approximately distributed according to the posterior probability distribution, in situations where the prior probability distribution and the likelihood function do not match well, e.g. using the auxiliary particle approach [52], using interacting Metropolis–Hastings samplers [63] or sequential Monte Carlo samplers [88], which provides an interacting particle implementation of the annealed importance sampling algorithm [51], etc.

- In using dimension reduction (Rao–Blackwell) techniques, by taking advantage of the special structure of some models, e.g. conditionally linear Gaussian models [96], [17], and others models [31], [99].

- In adding a (kernel) regularization step to the classical particle methods, as a response to *degeneracy of particle positions* and to *degeneracy of particle weights*, which are two undesirable phenomena which are known to occur when the state noise or the observation noise are too small. The corresponding regularized particle filters (RPF, L2RPF) have been actively studied at ONERA [71], during the PhD thesis of Nadia Oudjane [100], and some asymptotic results have been obtained in [50].

Earlier ASPI contribution :

The following generic algorithm has been recently proposed in [76, 81], following the auxiliary particle approach introduced by [52] : plugging the weighted particle approximation

$$\mu_{k-1} \approx \mu_{k-1}^N = \sum_{i=1}^N w_{k-1}^i \, \delta_{\xi_{k-1}^i} \ ,$$

into the nonlinear operator $\bar{R}_k(\cdot)$ for the normalized Feynman–Kac flow, and using an *arbitrary* importance decomposition

$$R_k(x, dx') = Q_k(x, dx')\, g_k(x') = W_k(x, x')\, P_k(x, dx') \ ,$$

into a mutation kernel $P_k(x, dx')$ and a weighting function $W_k(x, x')$, yields (up to normalizing constant)

$$\bar{R}_k(\mu_{k-1}^N)(dx') \propto \sum_{i=1}^N w_{k-1}^i \; R_k(\xi_{k-1}^i, dx') = \sum_{i=1}^N \underbrace{\frac{w_{k-1}^i}{\pi_k^i} \; W_k(\xi_{k-1}^i, x')}_{r_{k-1}^i(x')} \; \underbrace{\pi_k^i \; P_{k-1}(\xi_{k-1}^i, dx')}_{m_{k-1}^i(dx')} \ ,$$

which can be interpreted as the marginal of an unnormalized joint probability distribution on the product space $\{1, \cdots, N\} \times E_k$. Importance sampling in the product space yields the following weighted particle approximation

$$\sum_{i=1}^N \frac{r_{k-1}^{\tau_k^i}(\xi_k^i)}{\displaystyle\sum_{j=1}^N r_{k-1}^{\tau_k^j}(\xi_k^j)} \; \delta_{(\tau_k^i, \, \xi_k^i)} \ ,$$

where $((\tau_k^1, \xi_k^1), \cdots, (\tau_k^N, \xi_k^N))$ is precisely an N–sample in the product space, with common probability distribution $(m_{k-1}^1(dx'), \cdots, m_{k-1}^N(dx'))$, which can easily be achieved as follows : independently for any $i = 1, \cdots, N$

$$\tau_k^i \sim (\pi_k^1, \cdots, \pi_k^N) \qquad \text{and} \qquad \widehat{\xi}_{k-1}^i = \xi_{k-1}^{\tau_k^i} \ ,$$

which means that a particle ξ_{k-1}^i from the set Σ_{k-1} with a higher weight π_k^i is more likely to be selected than other particles with a smaller weight, and

$$\xi_k^i \sim P_k(\widehat{\xi}_{k-1}^i, dx') \ ,$$

independently for any $i = 1 \cdots N$, and marginalizing yields

$$\bar{R}_k(\mu_{k-1}^N) = \frac{\mu_{k-1}^N \ R_k}{(\mu_{k-1}^N \ R_k)(E)} \approx \sum_{i=1}^N \frac{r_{k-1}^{\tau_k^i}(\xi_k^i)}{\displaystyle\sum_{j=1}^N r_{k-1}^{\tau_k^j}(\xi_k^j)} \ \delta_{\xi_k^i} = \mu_k^N$$

This generic algorithm generalizes many of the known and currently used particle approximation schemes, and provides a unified framework to obtain asymptotic convergence results. Notice that introducing the discrete importance weights $\pi_k = (\pi_k^1, \cdots, \pi_k^N)$ makes it possible to decide which fraction of the importance weight allocated to a particle goes to selection, and which fraction remains to weighting.

Research objectives :

In addition to the classical particle methods, for which many asymptotic results are available, see Section 3.3, some very efficient algorithmic variations have been proposed, some of which coming from other scientific communities such as artificial intelligence (AI), genetic algorithms (GA), etc., and the objective here is to assess these algorithmic variations on solid ground arguments.

- Study the effect of adaptive resampling, in which resampling occurs only when a criterion (effective number of particles, entropy of the weights distribution, etc.) exceeds a given threshold.
- Study the effect of alternative redistribution schemes, some of which could be essentially deterministic.
- Modify the redistribution scheme, e.g. using niching techniques as in [61], so as to track all the modes of a multimodal probability distribution.
- Adapt the sample size (number of particles), e.g. as in the sequential particle filter (SPF) [50], in the adaptive particle filter (KLD–sampler) [66], etc.
- Provide guidelines as to which importance decomposition to use, i.e. how to chose the mutation kernel $P_k(x, dx')$ and the discrete importance weights $(\pi_k^i \ , \ i = 1, \cdots, N)$ to be used in the selection step, so as to improve the quality of exploration of the state space. For instance, a mutation kernel depending on the observations could be used.

- Study alternative algorithms in which the selection step is implemented by independent branching mechanisms, and control the random size (cardinality) of the particle system.
- Revisit the different smooting algorithms that have been proposed in the literature, none of which is really satisfactory.
- For state estimation (filtering) in models with a special structure, such that conditionally upon some components of the hidden state and upon the observations, the estimation of the other components has an explicit solution (e.g. in terms of a Kalman filter), a Rao–Blackwell approach has been systematically used, where the filter is approximated by the empirical probability distribution of a system of particles of reduced dimension, with a Kalman filter associated with each particle [96], [31], [99]. In a preliminary work [87], the problem has been interpreted in terms of Feynman–Kac flows in a random environment, and the solution as a system of Kalman filters in interaction. Deriving asymptotic properties, especially central limit theorems, for the Rao–Blackwell particle approximation, would make it possible to assess on a solid ground its performance as compared with the full particle approximation of the same problem.
- Design and analyze particle filters for non standard state estimation problems, e.g. in models with noise–free observations, for Markov models with algebraic constraints, for models where it is easy to simulate jointly the state and the observation but where no explicit expression is available for the likelihood function, etc.

5 Software and demos

To illustrate the fact that particle filtering algorithms are efficient, easy to implement, and extremely visual and intuitive by nature, several demos have been programmed by Fabien Campillo, with the corresponding MATLAB scripts available on the site `http://www.irisa.fr/sigma2/campillo/site-pf/`. This material has proved very useful in training sessions and seminars that have been organized in response to demand from industrial partners (SAGEM, CNES and EDF R&D). This effort will be continued within the ASPI research team. At the moment, the following three demos are available :

▶ Navigation of an aircraft using altimeter measurements and elevation map of the terrain : a noisy measurement of the terrain height below the aircraft is obtained as the difference between (i) the aircraft altitude above the sea level (provided by a pression sensor) and (ii) the aircraft altitude above the terrain (provided by an altimetric radar), and is compared with the terrain height in any possible point (read on the elevation map). In this demo, a cloud (swarm) of particles explores multiple possible trajectories according to some raw model, and are replicated or discarded depending on whether the terrain height below the particle (i.e. at the same horizontal position) matches or not the available noisy measurement of the terrain height below the aircraft.

▶ Tracking a dim point target in a sequence of noisy images. In this *track–before–detect* demo, a point, which cannot be detected in a single image of the sequence, can be automatically tracked in a sequence of noisy images.

► Positioning and tracking in the presence of obstacles. In this interactive demo, presented by Simon Maskell (QinetiQ and CUED) at a GDR ISIS event co–organized by François Le Gland and Jean–Pierre Le Cadre in December 2002, several stations (the number and locations of which are chosen interactively) try to position and track a mobile from noisy angle measurements, in the presence of obstacles (walls, tunnels, etc., the number, locations and orientations of which are also chosen interactively), which make the mobile temporarily invisible from one or several stations. This nonlinear filtering problem in a complex environment, with many constraints, would be practically impossible to implement using Kalman filters.

6 Applications and industrial contracts

6.1 IST project HYBRIDGE

The members of the ASPI research team contribute to the project *Stochastic Analysis and Distributed Control of Hybrid Systems*, see http://www.nlr.nl/public/hosted-sites/hybridge/, coordinated by Henk Blom (NLR, National Aerospace Laboratory, Amsterdam), and supported from January 2002 to December 2004 by the european commission within the IST programme, under the project number IST–2001–32460. The partners of the project are NLR (Netherlands), Cambridge University (United Kingdom), Universita di Brescia (Italy), Universita dell'Aquila (Italy), Twente University (Netherlands), National Technical University of Athens (NTUA, Greece), Centre d'Études de la Navigation Aérienne (CENA, France), Eurocontrol Experimental Center (EEC), AEA Technology (United Kingdom), BAe Systems (United Kingdom), INRIA Rennes (France).

The general objective of the project is to contribute to the management and real–time control of large complex systems which combine spatial distribution of sensors and decisions, and safety critical issues. The target application is to advanced air traffic management, but the contributions of the project could later be applied to other complex systems such as large communication, computer and power networks. The contribution of the members of the ASPI research team to this project concerns the work package on modeling accident risks with hybrid stochastic systems, and the work package on risk decomposition and risk assessment methods, and their implementation using conditional Monte Carlo methods. This problem has motivated our work on combined importance splitting and importance sampling algorithms for the simulation of rare events, and their implementation using interacting particle systems, see Section 4.1.

6.2 Collaboration with EDF R&D

The objective of this work, to be supported by a one–year research contract under discussion with EDF R&D, is to estimate parameters of various multi–factor models for electricity spot price, from observed futures prices that are traded on the market. This problem fits into the general framework of parameter estimation in hidden Markov models. In some special cases an algorithmically simple (although non explicit) solution can be found, for instance if the factors are modelled as Ornstein–Uhlenbeck processes driven by a Brownian motion : in this case Kalman filtering can be used to estimate the hidden state process (model factors) and to compute the likelihood function, which can subsequently be maximized numerically w.r.t. the parameters.

In practice however, the traded futures contracts specify delivery over a whole time interval, hence the resulting futures prices are no longer linear functions (on a logarithmic scale) of the model factors, and in principle Kalman filtering cannot be used.

Another objective of this study is to introduce more realistic multi–factor models for electricity spot price, where for instance factors are modelled as Ornstein–Uhlenbeck processes driven by a Lévy process [13].

For parameter estimation in such models, an approach based on Kalman filtering, to estimate the hidden state process and to compute the likelihood function, seems hopeless and alternative more sophisticated approaches should be considered. Simulation–based methods, including joint particle approximation schemes developped in the ASPI research team for the filter and the linear tangent filter, see Section 4.1, will be implemented in this work.

6.3 Collaboration with SAGEM

Global positioning system (GPS) provides useful additional information to stand alone inertial navigation systems (INS) which are known to be subject to a drift phenomenon, but on the other hand tracking the GPS signal is very sensitive to jamming, and a trade–off must be found. The objective of this work is to design anti–jamming procedures for combined INS / GPS navigation, possibly based on nonlinear filtering techniques and on their implementation using particle systems. This work should start within a PhD thesis with the support of a CIFRE grant, currently under evaluation.

6.4 Other application : Tracking mobiles in a cellular network

Position location of users in a cellular network is becoming an important issue [94], [78], [53], and there are many potential applications of mobile positioning. This includes for instance : localization of emergency calls, cellular planning, hand–over prediction, mobile yellow pages, position dependent billing, position dependent advertising, etc.

Several sources of information can be used for this purpose. To anticipate and handle hand–overs, the base stations (BTS) are broadcasting signals that are received also in the neighbouring adjacent cells, and in a urban area, it can happen that a mobile receives signals from four to five BTS, which makes it possible to determine first the distance of the mobile to each BTS, e.g. using differential time of arrival (DTOA), and then to localize the mobile using hyperbolic triangulation. An attenuation map relative to one or several stations could also be used, if available. All these observations could then be combined with a raw dynamical model for the motion of the mobile, which makes the whole problem fit into a Bayesian state estimation problem (filtering), similar to terrain aided navigation. Recent contacts with Bernard Uguen (IETR / INSA Rennes) are going to be very useful in providing a simulated attenuation map, produced from a standard evelation map and a model of signal propagation.

7 Participation to academic research networks

7.1 IHP research network DYNSTOCH

The members of the ASPI research team participate in the european research network *Statistical Methods for Dynamical Stochastic Models*, see http://www.math.ku.dk/~michael/dynstoch/, coordinated by Michael Sørensen (Københavns Universitet), and supported from September 2000 to August 2004 by the european commission within the IHP program, under the project number HPRN–CT–2000–00100. For information, this network was a follow–up of the european research network *Statistical Inference for Stochastic Processes*, coordinated by Jean Jacod (Université Pierre et Marie Curie, Paris), and supported from January 1994 to December 1996 by the european commission within the HCM program, under the project number CHRX–CT–92–0078. The nine research teams participating in the current network are : Københavns Universitet (Denmark), Universiteit van Amsterdam (Netherlands), Humboldt Universität zu Berlin (Germany), Albert Ludwigs Universität Freiburg (Germany), Universidad Politécnica de Cartagena (Spain), Helsingin Yliopisto (Finland), University College London (United Kingdom), LADSEB / CNR (Italy), Université Pierre et Marie Curie (France). The contribution of the members of the ASPI research team within the French team of the network (PMA, Laboratoire de Probabilités et Modèles Aléatoires, Université Pierre et Marie Curie and Université Diderot, Paris), is focused on large time and / or small noise asymptotic statistics of HMM with finite or continuous state space, and their particle implementation.

The proposal of a follow–up Marie Curie research training network DYNSTOCH, coordinated by Peter Spreij (Universiteit van Amsterdam), has been submitted to the November 2003 call of the FP6, with INRIA Rennes as a research team on its own, and with four additional research teams : MTA / SZTAKI (Hungary), Universiteit Gent (Belgium), Ruprecht Karls Universität Heidelberg (Germany), and Linköpings Universitet (Sweden). Modelling and identification (parameter estimation) of financial time series using state–space models driven by a Lévy process, and simulation–based methods for numerical implementation of statistical procedures in state–space models have been explicitly included in the workplan of the network, and are part of the research programme of the ASPI research team, see Section 6.2.

7.2 TMR research network ERNSI

The members of the former SIGMA2 research team have participated in the european research network *System Identification*, see http://www.cwi.nl/~schuppen/ernsi/tmrsi.html, coordinated by Jan van Schuppen (CWI, Amsterdam), and supported from March 1998 to February 2003 by the european commission within the TMR program, under the project number FMRX–CT–98–0206. For information, this network was a follow–up of the european research network *System Identification*, already coordinated by Jan van Schuppen (CWI, Amsterdam), and supported from July 1992 to June 1995 by the european commission within the *Science* program, under the project number SC1*–CT–92–0779. The nine research teams participating in the network were : CWI (Netherlands), Technische Universität Wien (Austria), Université Catholique de Louvain (UCL, Belgium), INRIA Sophia–Antipolis (France), IRISA / Université de Rennes 1 (France), Cambridge University (United Kingdom), LADSEB / CNR and Università degli Studi di Padova (Italy), KTH (Sweden), and Linköpings Universitet (Sweden). The contribution of the members of the ASPI research team in this terminated network has been focused on particle filtering and hidden Markov models (HMM).

The proposal of a follow–up Marie Curie research training network ERNSI+, coordinated by Bo Wahlberg (KTH, Stockholm), has been submitted to the November 2003 call of the FP6, with two additional research teams : MTA / SZTAKI (Hungary), and Czech Technical University (Czech Republic). A research project on simulation–based methods, including SMC methods, for parameter estimation in state–space models, has been included in the workplan of the network, with a participation from partners in Cambridge University (Arnaud Doucet) and Linköping University (Fredrik Gustafsson).

7.3 CNRS MathSTIC project

Since December 2001, François Le Gland has been coordinating with Éric Moulines (ENST Paris) the project *Chaînes de Markov cachées et filtrage particulaire*, within the inter–departmental CNRS programme *MathSTIC*. The following events have been organized to impulse the research activity in this domain :

- a two–day kick–off meeting at ENST Paris, January 2002, see http://www.irisa.fr/sigma2/hmm-stic/kick-off/,
- an invited session on particle filtering at the *Journées du groupe MAS* of SMAI (Société de Mathématiques Appliquées & Industrielles) in Grenoble, September 2002,
- a two–day *Journées thématiques* session on applications of particle filtering in tracking and computer vision, with the additional support of the GDR ISIS in Paris, December 2002, see http://www.irisa.fr/sigma2/hmm-stic/isis/.

Researchers from the six following CNRS research units (UMR) have been participating in this project : LSP / Université Paul Sabatier, Toulouse, CEREMADE / Université Dauphine, Paris, LMO / Université Paris–Sud, Orsay, LMC / IMAG, Grenoble, LTCI / ENST Paris, IRISA, Rennes (research teams SIGMA2 and VISTA), together with researchers from ONERA, Chatillon.

7.4 CNRS DSTIC action spécifique

Since September 2002, François Le Gland is coordinating with Olivier Cappé (ENST Paris) a project (action spécifique) AS 67 *Méthodes particulaires*, see http://www.tsi.enst.fr/~cappe/aspart/, supported by the STIC department of CNRS, and promoted by the RTP 24 *Mathématiques de l'information et des systèmes*, with the objective of drawing a picture of the research in France in the domain of particle methods, and of federating the research efforts within the scientific community, in relation with the industrial world.

A one–day workshop has been organized on applications of particle filtering, at ENST Paris, December 2003, see http://www.irisa.fr/sigma2/as67/, with the joint support of the AS 67 and of the GDR ISIS.

7.5 Other CNRS actions

▶ The ASPI research team has been rather active recently within the GDR ISIS, see http://gdr-isis.org/, especially through the organization of meetings, see Sections 7.3 and 7.4.

▶ The GDR GRIP, see http://acm.emath.fr/grip/, is concerned with interacting particle systems from the double point of view of (i) analyzing their macroscopic integro–differential equations, and their numerical approximation schemes, some of which are expressed in terms of random or deterministic systems of particles, and (ii) studying the asymptotic properties of random interacting particle systems. Even though the ASPI research team is considering interacting particle systems from still another point of view, directed towards applications in statistics, it is certainly a good idea to participate in this action.

▶ The ASPI research team is considering its participation in the GDR MOMAS, see https://mcs.univ-lyon1.fr/MOMAS/, especially within the working group on probabilistic methods, coordinated by Antoine Lejay, from the OMEGA research team in Nancy. Potential collaboration would be around the use of interacting Monte Carlo methods to simulate the probability that a pollutant within a fractured medium eventually reaches the fractures network, a rare but critical event.

8 Positioning and interaction

8.1 Within IRISA

▶ The VISTA research team is very active in particle filtering, with contributions in target tracking problems by Jean–Pierre Le Cadre, partly through a continuing collaboration with Christian Musso (ONERA, Chatillon), in tracking points or extended rigid objects in a background fluid motion by Étienne Mémin, in collaboration with Bernard Delyon (IRMAR, Rennes), and in visual tracking by Patrick Pérez (with Microsoft Research, Cambridge until February 2004). Already existing contacts with this research team are going to continue and progress in the following years.

▶ The ARMOR research team, especially Bruno Tuffin and Gerardo Rubino, has a long experience and many contributions to simulation of rare events, using both importance sampling and importance splitting, with applications to communication networks.

8.2 Within INRIA

▶ Financial engineering is an area where Monte Carlo methods are widely used, e.g. for evaluation various options prices and their derivatives (greeks) w.r.t. volatility, initial condition, etc. and for general risk evaluation problems. Contacts with the MATHFI research team started two years ago on the occasion of the renewal of the Lyapunov Institute project *Mathématiques Financières* coordinated by Agnès Sulem. These contacts should be strengthened, and also with the OMEGA (Sophia Antipolis) research team.

▶ Even though interacting particle systems per se, and their limiting macroscopic equations from physics, are not explicitely mentioned in its research programme, the ASPI research team shares common scientific interest with the OMEGA (Nancy and Sophia Antipolis) research team on this topic, and is considering possible collaboration within the two GDR MOMAS and GRIP.

▶ Interacting Monte Carlo methods are sometimes formulated in terms of genetic algorithms, and several algorithmic issues are addressed within the two communities, which means that fruitful contacts could be established with the TAO research team.

8.3 In France

The continuing support of CNRS, through the already mentioned programmes *Modélisation et simulation numérique* (SPM Department), *MathSTIC* (joint initiative of SPM and STIC departments) and *DSTIC actions spécifiques* (STIC department), has been very fruitful to organize meetings, stimulate scientific cooperation, provide visibility at the national level, and identify the most active research teams in the domain. The question of further formalizing the existing cooperation between

- Pierre Del Moral, Pascal Lezaud (also with CENA, Toulouse) and Laurent Miclo (LSP, Toulouse),
- Éric Moulines, Olivier Cappé and Jamal Najim (LTCI / ENST, Paris),
- the SIGMA2 and VISTA research teams (IRISA, Rennes),

and to associate smaller groups or isolated but active researchers, namely Christian P. Robert and Arnaud Guillin (CEREMADE, Paris), Randal Douc (CMAP, Palaiseau), Manuel Davy (IRCCyN, Nantes), and Jean-Yves Tourneret (TeSA / ENSEEIHT, Toulouse), within the proposition of an EPML has been considered, in the conclusions of the AS 67.

▶ Above all, the scientific collaboration with Pierre Del Moral has been extremely motivating and fruitful, as explained in Section 1.3. Bringing the general and abstract formulation in terms of Feynman–Kac formulas, see Section 3.3, is a major contribution to the domain, which makes it possible to bring similar algorithmic solutions to otherwise apparently very different applications, such as target tracking (and more generally, filtering), simulation of rare events, molecular simulation, etc. All opportunities to formalize these longstanding exchanges with the LSP at Université Paul Sabatier, Toulouse will be considered by the ASPI research team.

▶ A very active research group, with strong background in MCMC methods and in statistical inference in HMM, and in their applications to Bayesian statistics and to signal processing, is informally organized around the LTCI at ENST Paris. Already existing contacts, essentially with Éric Moulines, Olivier Cappé, Christian P. Robert and Randal Douc, have been very fruitful and are going to continue and progress in the following years.

▶ Target tracking, especially related with defence applications, is undoubtedly the area where industrial applications of particle filtering are the more advanced. Even though it is not always possible, for obvious reasons, to get a precise information about the real achievements in this domain, it is nevertheless clear that very active academic contributors are Christian Musso (ONERA, Chatillon), Jean–Pierre Le Cadre (IRISA, Rennes), Gérard Salut and André Monin (LAAS, Toulouse), and that industrial partners are to be found in DIGINEXT, THALÈS (several divisions), SAGEM, etc., with funding from the DGA. The ASPI research team is not directly involved at the moment in target tracking applications, but is certainly willing to be more present and active there.

8.4 In Europe

▶ One of the most active research team in particle filtering can undoubtedly be found in the Department of Engineering of Cambridge University : Arnaud Doucet especially has made very important contributions to theoretical, algorithmic and application–oriented issues. He has organized several workshops, meetings, invited sessions in international conferences, served as guest editor of several special issues of international journals, co–edited a contributed volume [3], etc. Jako Vermaak has contributions in audio and video signal processing, and has returned to CUED after a period at Microsoft Research Cambridge, where he was a close collaborator of Patrick Pérez. Finally, Simon Maskell has a partial appointment with QinetiQ (a spin–off of DERA), and has just completed a PhD thesis in target tracking and multisensor management. The ASPI research team is willing to strenghten its already existing contacts with this research team. A closely related group of people can be found in the Department of Statistics of the University of Bristol, where Christophe Andrieu and Nicolas Chopin are active contributors with a strong background in MCMC methods.

▶ The high potential of particle filtering for positioning, navigation, and tracking applications has been recognized in the Department of Electrical Engineering of Linköping University, where Fredrik Gustafsson is leading a group with important contributions and industrial connections through the ISIS competence center. The ASPI research team is willing to develop closer contacts with this research team.

▶ Early individual contributors to the approximation of continous time filtering equations using branching and interacting particle systems [20, 19, 21], are Dan Crişan from the Department of Mathematics of Imperial College, and Terry J. Lyons from the Mathematics Institute of Oxford University.

▶ Algorithmic contributions and contributions to statistical inference using particle methods in models driven by Lévy processes have been obtained by Neil Shephard from Nuffield College in Oxford University, and Michael Pitt from the Department of Economics of the University of Warwick.

▶ Another active but isolated contributor is Hans–Rüdi Künsch from the Seminar für Statistik at ETH Zürich, with both algorithmic and theoretical contributions, and strong background in statistics for state–space models.

8.5 World–wide

▶ One of the most active contributor to sequential Monte Carlo methods, with a strong background in Bayesian statistics and a wide range of research interests, is Jun Liu from the Department of Statistics of Harvard University, who has made very important contributions to algorithmic issues and to applications, especially in genetics and computational biology.

▶ One of the more structured research team in particle filtering is the MITACS project *Prediction in Interacting Systems* (PINTS), coordinated by Michael Kouritzin from the Department of Mathematical and Statistical Sciences of the University of Alberta, and geographically distributed over Canada, with contributions to theoretical and algorithmic issues, and to applications in target tracking, in partnership with Lockheed Martin.

▶ Another contributor is Petar M. Djurić from the Department of Electrical and Comuting Engineering of the State University of New York at Stony Brooks, who is very active in applications of particle methods to signal processing.

▶ At the international level, the most active research teams in target tracking can be found in the United Kingdom, at QinetiQ (a spin–off of DERA) with David J. Salmond and Simon Maskell, and in Australia, at DSTO in Adelaide, with Sanjeev M. Arulampalam, Branko Ristić and Neil J. Gordon (formerly with QinetiQ).

▶ Very interesting contributions to algorithmic issues which are addressed here in Section 4.4, are given in other scientific communities (artificial intelligence, evolutionary computation, genetic algorithms, mobile robotics, etc.) with which the ASPI research team has so far no contact at all. An effort should be made to fill this gap and try to set contacts with some of the most active contributors, Dieter Fox from Department of Computer Science and Engineering of the University of Washington, Nando de Freitas from the Department of Computer Science of the University of British Columbia, and Sebastian Thrun from the Computer Science Department of Stanford University.

9 Teaching, training and PhD supervision

Since the academic year 1994 / 95, François Le Gland is giving a course on Kalman filter and hidden Markov models, at the DEA STIR (Signal, Télécommunications, Images et Radar) of Université de Rennes 1.

François Le Gland and Fabien Campillo have given a four–days training course on particle filtering at SAGEM, Argenteuil in the summer of 2004, and have been invited to give a one–day seminar on particle filtering at CNES, Toulouse in November 2003. François Le Gland has been invited to participate in a one–day seminar on particle methods at EDF R&D, Clamart in March 2004. The ASPI research team is also involved in the development of an advanced risk management course, as part of the IST HYBRIDGE project, see Section 6.1.

Further cooperation with the industrial partners should take the form of a PhD thesis with the support of a CIFRE grant submitted by SAGEM and under evaluation, and a research contract with EDF R&D under discussion, see Sections 6.3 and 6.2.

Since October 2002, the ASPI research team has set a weekly internal working group on applications of interacting particle systems to statistics, see `http://www.irisa.fr/sigma2/aspi/gt.html`.

François Le Gland has supervised the PhD thesis of Nadia Oudjane [100], jointly with Christian Musso (ONERA, Chatillon), with contributions in tracking applications, in algorithmic issues [71] (regularized particle filter (RPF), progressive correction), in stability of the filter w.r.t. the initial condition and w.r.t. the model, and in error estimates uniform in time for particle approximation schemes [50], and in the proposition of a general robustification principle [49].

Since October 2002, François Le Gland is supervising the PhD thesis of Natacha Caylus, who is studying asymptotic properties of linear tangent Feynman–Kac formulas, and convergence issues ($\mathbb{L}^p$ estimates, central limit theorem, etc.) for various joint particle approximation schemes.

10 Visibility

10.1 Animation, organization of workshops

Since December 2001, François Le Gland has been coordinating two CNRS projects with Éric Moulines (ENST Paris) and with Olivier Cappé (ENST Paris), see Sections 7.3 and 7.4 respectively. He has organized or co–organized :

- a workshop on particle filtering at IRISA, Rennes in June 1998, see `http://www.irisa.fr/sigma2/legland/workshop98/`,
- the kick–off meeting of the *MathSTIC* project *Chaînes de Markov cachées et filtrage particulaire*, at ENST Paris in January 2002, see `http://www.irisa.fr/sigma2/hmm-stic/kick-off/`,
- a two–day *Journées thématiques* session on applications of particle filtering in tracking and computer vision, at IHP, Paris in December 2002, see `http://www.irisa.fr/sigma2/hmm-stic/isis/`,
- a one–day workshop on applications of particle filtering, at ENST Paris in December 2003, see `http://www.irisa.fr/sigma2/as67/`.

10.2 Invited talks, contributions, etc.

François Le Gland and Frédéric Cérou have been invited to give

- a talk in the workshop on *Theoretical and Practical Aspects of Particle Filtering*, organized in Cambridge, December 1999 by Arnaud Doucet.

François Le Gland has been invited to give

- a talk in the workshop on *Particle Systems and Filtering*, organized at IHP in Paris, June 2001 by René Carmona, Pierre Del Moral and Jean Jacod,
- a talk in the summer school *Méthodes de Monte Carlo pour l'Inférence Statistique*, organized at CIRM in Luminy, September 2001 by Éric Moulines and Christian P. Robert,
- a talk at the SMAI *Journées du groupe MAS* in Grenoble, September 2002, in a special session on Monte Carlo methods and particle filters organized by Éric Moulines,

- a talk at the IFAC / IFORS symposium on *System Identification* (SYSID) in Rotterdam, August 2003, in a special session on particle filters in system identification, organized by Fredrik Gustafsson,
- a plenary talk on particle filtering at the French 19th symposium *Traitement du Signal et des Images* (GRETSI) in Paris, September 2003,
- a poster presentation at the IEEE workshop on *Statistical Signal Processing* (SSP) in Saint Louis, September / October 2003, in a special session on sequential Monte Carlo methods, organized by Patrick J. Wolfe,
- a talk at the joint 6th World Congress of the Bernoulli Society and 67th Annual Session of the IMS in Barcelona, July 2004, in a special session on applications of particle filtering in statistics, organized by Arnaud Doucet,
- a contribution to the volume *Sequential Monte Carlo Methods in Practice* [3] edited by Arnaud Doucet, Nando De Freitas and Neil J. Gordon,
- a contribution to the volume [7] edited by José–Luis Menaldi, Edmundo Rofman and Agnès Sulem in honor of Alain Bensoussan on the occasion of his 60th birthday,
- a talk at the workshop *Stochastic Theory and Control* in Lawrence, October 2001 and a contribution to the volume [8] edited by Boženna Pasik–Duncan in honor of Tyrone E. Duncan on the occasion of his 60th birthday.

10.3 Visits, seminars, etc.

François Le Gland has given a talk in the seminar

- of the MATHFI research team at CERMICS, Marne la Vallée, March 2002,
- of the Laboratoire de Probabilités et Statistiques, INRA / Université de Montpellier 2, February 2004,

and he has been invited

- by René Carmona to the Department of Operations Research and Financial Engineering (ORFE) of Princeton University in October 2001, where he has given one lecture in the course *Interacting particle approximations of nonlinear filtering problems* given there by Pierre Del Moral.
- by Hans–Rüdi Künsch to the Seminar für Statistik of ETH Zürich in October 2003, where he has given a talk in the seminar of statistics.

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Acronyms

AS	Action Spécifique
CIFRE	Convention Industrielle de Formation par la Recherche
EPML	Équipe–Projet Multi–Laboratoire
GDR	Groupe de Recherche
RTP	Réseau Thématique Pluridisciplinaire
STIC	Sciences et Technologies de l'Information et de la Communication
UMR	Unité Mixte de Recherche

CENA	Centre d'Étude de la Navigation Aérienne
CEREMADE	Centre de Recherche en Mathématiques de la Décision
CIRM	Centre International de Rencontres Mathématiques
CNES	Centre National d'Études Spatiales
CUED	Cambridge University Engineering Department
CWI	Centrum voor Wiskunde en Informmatica
DERA	Defence and Evaluation Research Agency
DGA	Délégation Générale pour l'Armement
DSTO	Defence Science and Technology Organisation
EDF	Électricité de France
ENSEEIHT	École Nationale Supérieure d'Électrotechnique, d'Électronique, d'Informatique, d'Hydraulique et des Télécommunications
ENST	École Nationale Supérieure des Télécommunications
GRIP	Groupe de Recherche Interactions de Particules
IHP	Institut Henri Poincaré
IMS	Institute of Mathematical Statistics
INRA	Institut National de Recherche Agronomique
IRCCyN	Institut de Recherche en Communications et en Cybernétique de Nantes
IRMAR	Institut de Recherche Mathématique de Rennes
ISIS	Information, Signal, Images et Vision
ISIS	Information Systems for Industrial Control and Supervision
ISM	Institute of Statistical Mathematics
LMO	Laboratoire de Mathématiques d'Orsay
LSP	Laboratoire de Statistique et Probabilités
LTCI	Laboratoire de Traitement et de Communication de l'Information
MITACS	Mathematics of Information Technology and Complex Systems
MOMAS	Modélisation Mathématique et Simulations Numériques liées à la Gestion des Déchets Nucléaires
NLR	Nationaal Lucht– en Ruimtevaartlaboratorium (National Aerospace Laboratory)
ONERA	Office National d'Études et de Recherches Aérospatiales
PINTS	Prediction in Interacting Systems
PMA	Laboratoire de Probabilités et Modèles Aléatoires
SMAI	Société de Mathématiques Appliquées & Industrielles
STIR	Signal, Télécommunications, Images et Radar
UvA	Universiteit van Amsterdam
UHB	Université de Haute Bretagne

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