Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Computer Networks 47 (2005) 907–921
Computer Networks
www.elsevier.com/locate/comnet

A mathematical analysis of the cumulus pricing scheme

Y. Hayel *, B. Tuffin

IRISA/INRIA, Campus Universitaire de Beaulieu, 35042 Rennes Cedex, France

Received 30 October 2002; received in revised form 1 October 2004; accepted 28 October 2004
Available online 20 November 2004

Responsible Editor: U. Krieger

Abstract

One important task in current and future communication networks is to define a suitable pricing scheme. It is then preferable to formulate a mathematical model, so that parameters will be optimized and important properties such as fairness or truthful anticipated load revelation (or incentive compatibility) will be verified. In this paper we study a simple and promising scheme called the cumulus pricing scheme, which can address service differentiation and scalability among other issues. Based on a mathematical model, we determine values for optimizing the provider's revenue, which happens under the constraint that each user has an incentive to reveal its anticipated load. This has led to a small variation of the initial model from the literature as in the modelling, cumulus points are translated into financial terms, and measurements induce a cost as well.

Keywords: Cumulus pricing scheme; Network management; Quality of service

1. Introduction

The Internet is experiencing a tremendous growth of its traffic and a transformation of its architecture. The current flat-rate pricing scheme, adopted by most Internet Service Providers (ISPs), is an incentive to over-use the network which, in conjunction with the increasing number of subscribers, drives to congestion, which in turn reduces the quality of service (QoS). A usage-based pricing scheme would overcome this drawback. Similarly, the future network architecture will have to respond to different QoS requirements of different types of applications. Architectures such as Diff-Serv [5] deal with this problem, but an adapted pricing scheme has to be associated with it, lest a user always chooses the service class providing the best QoS. Devising a new pricing scheme is the subject of extensive research; the reader is

* Corresponding author. Tel.: +33 2 99847134.
E-mail addresses: yhayel@irisa.fr (Y. Hayel), btuffin@irisa.fr (B. Tuffin).

1389-1286/$ - see front matter
doi:10.1016/j.comnet.2004.10.009

invited to see [2–4,6,11,18,19] for introductory or overview papers listing the existing schemes.

We deal in this paper with the so-called cumulus pricing scheme (CPS) [10,12,16,17]. In this scheme, a contract is negotiated between the ISP and the users. Then, during periods of time, the utilization is measured and (positive or negative) cumulus points are awarded, depending on how far the user seems to be from the contract specifications. Threshold values of consumption fluctuations are used in order to award the cumulus points. After a while, the contract is renegotiated if the total number of cumulus points exceeds a given value. In addition, at a given time, extra-fees can be charged.

The CPS is promising due to its simplicity and adaptability (to DiffServ architecture for instance), in part because it is applied at the edge of the network and because it is close to flat-rate pricing. In this paper, we study the CPS from a mathematical point of view. Our contributions are the following:

- We take a deeper look at the economic model by studying, from the user and the network points of view, the benefits in terms of the threshold values. In our model, each cumulus point is translated into financial terms, as well as is the measurement procedure for the network.
- We then show, by mathematical analysis, that the initial CPS creates an incentive to "cheat", by not revealing the anticipated resource consumption.
- By a slight modification of the model, we give a sufficient condition on threshold values under which truthful revelation is the most profitable strategy for the users. Remarkably, our sufficient condition is independent of the user's utility function, which represents his valuation of the required service, and is hardly known in practice.
- Then, under this constraint, we present some elements helping the provider with the choice of the threshold values in order to maximize its revenue. In a general setting, we claim that analytical results are intractable to obtain; we then use a simulated annealing algorithm. Nevertheless, in some particular cases, like with symmetric or linear thresholds, we have been able to obtain explicit results.

The paper is organized as follows. In Section 2 we describe and analyze the CPS: Section 2.1 presents the model given in the literature; Section 2.2 gives some additional definitions of provider's revenue and user's overall level of satisfaction that will be helpful in our analysis; Section 2.3 shows that the CPS does not provide an incentive to reveal the real anticipated network consumption. Based on this result, we slightly modify the original CPS model in Section 3 by penalizing every over-use of the network. We then give some conditions over threshold values under which each user has no incentive to cheat. Under this condition, Section 4 works on setting threshold values maximizing the provider's revenue. Some numerical illustrations of our results are given in Section 5 and the conclusions and directions for future research are presented in Section 6.

2. The cumulus pricing scheme (CPS)

In Section 2.1, we describe the cumulus pricing scheme as devised in [10,13,12,16,17]. The truthful anticipated consumption revelation property is studied in Section 2.3.

2.1. Model presentation

The CPS is characterized by a feedback mechanism that operates at different time-scales. First, over a long time-scale, a contract is negotiated between the user and the provider, defining a flat-rate pricing. Over a short time-scale, the actual consumption is monitored and, over a medium time-scale, the cumulative user behavior is reported back through a feedback mechanism (the so-called cumulus points) indicating how far the consumption is from the service requirement specification. The accumulation of points over the long time-scale can then result in a renegotiation of the flat-rate contract, and so on.

Formally, define $V(t)$ as the actual resource consumption at time t (i.e., short time-scale) and let x be the stated expected requirement (as defined in the contract). The time interval is decomposed in measurement periods $[t_i, t_{i+1}]$ (i.e., the medium time-scale) ($i \geqslant 1$), so that the over- or under-utilization over period $[t_i, t_{i+1}]$ is

$$\Delta_i = \int_{t_i}^{t_{i+1}} V(t)\,\mathrm{d}t - x(t_{i+1} - t_i).$$

At the end of each period, some cumulus points are assigned by the ISP according to the following rule. Let N be the maximum number of points that can be assigned and define θ_n, $n = -N, \ldots, N$ with $\theta_{\pm(N+1)} = \pm\infty$ ($\theta_0 = 0$) as thresholds such that $C_i := C(\Delta_i)$ positive points are assigned if

$$0 \leqslant \theta_{C_i} \leqslant \Delta_i < \theta_{C_i+1},$$

and $C_i := C(\Delta_i)$ negative points are assigned if

$$\theta_{C_i-1} < \Delta_i \leqslant \theta_{C_i} \leqslant 0.$$

Fig. 1 illustrates this cumulus points assignment when we consider four positive and two negative thresholds.

As a reaction rule, the provider decides to renegotiate the contract after the monitoring period K as soon as

$$\left|\sum_{i=1}^{K} C_i\right| \geqslant \Theta,$$

that is, as soon as the sum of cumulus points assigned since the start of the contract reaches a threshold Θ [12].

In order to establish the contract, the provider has to define a tariff function $p(x)$ per resource unit, when the user is requiring x resource units

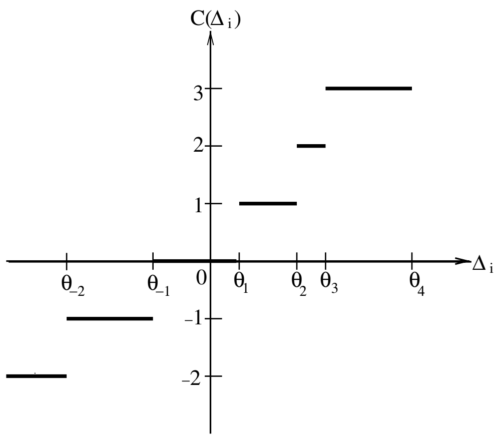

Fig. 1. Cumulus points assignment in terms of the deviation Δ_i from the expected resource consumption.

[12]. The total charge as defined in the contract will then be $c(x) = xp(x)$. Moreover, an extra-fee is charged after accumulating a sufficient number of (positive) cumulus points. Assume that a user having required x actually consumes $\xi = x + \Delta$ (with $\Delta > 0$). The extra-fee is based on the *estimation* δ of the over-use Δ for the period during which cumulus points have been accumulated (meaning that $\delta = \overline{X} - x$ where $\overline{X}$ is the standard estimator of ξ). For convenience, function c is used to define this additional charge, which is then $c(\delta)$. The compound charge is then $c(x) + c(\delta)$.

In [12], the authors are presented the following properties that function c has to satisfy in order to provide correct economic incentives. First, let

$$\Psi(x, \xi) = c(\xi) - (c(x) + c(\xi - x))$$

be the penalty function, corresponding to the overcharge for not telling the right requirement ξ when the actual consumption is x. Functions p, c and Ψ are required to verify the following properties:

(1) $\forall x > 0$, $p(x) > 0$ and $p(x)$ is monotonically decreasing (economy of scale property).
(2) $\forall x > 0$, $c(x) = xp(x)$ is monotonically increasing (increasing cost property).
(3) if $x \neq \xi$, $\Psi(x, \xi) < 0$ and $\Psi(x, \xi) = 0$ if $x = \xi$ (truthful declaration incentive: a customer is penalized from not stating correctly his resource consumption).
(4) $\Psi(x, x + \delta)$ is monotonically decreasing in δ (this property ensures that the absolute value of penalty increases with the deviation from the expected requirement).
(5) $|\Psi(x, \xi)| < |\Psi(\beta x, \beta\xi)| \leqslant \beta|\Psi(x, \xi)|$ for $\beta > 1$ (scaling property: similar relative error yields a higher penalty for higher bandwidth requirements, and the penalty does not grow more than linearly with the scaling factor).

For instance, $c(x) = \sqrt{x}$ fulfills these requirements [12]. In the next subsections, our model will extend this set of properties by adding the cumulus points in the user's revenue/cost.

With respect to the current Internet flat-rate pricing scheme, the CPS needs to measure the resources that are actually used. The precision of these measurements has to be good enough in order to limit the probability of wrongly assigning cumulus points. Assuming that $V(t)$ is a stochastic process in steady-state, from the usual central limit theorem (with an unknown variance) using l independent measurements, at confidence level α, we can obtain a confidence interval with half-width $\varepsilon_{\alpha,l} = q_\alpha \widehat{S}/\sqrt{l}$ where $\widehat{S}$ is the unbiased estimator of the variance of the stochastic process $V(t)$ and q_α is the $(1 - \alpha/2)$ quartile of the normal distribution. In order to make the model more or less independent of the measurement method, any two neighboring thresholds need to be at least at distance $2\varepsilon_{\alpha,l}$ [12].

2.2. Mathematical definitions of revenues

In this subsection, we define the network revenue and the user level of satisfaction. Our goal is to carry out a deeper analysis of CPS than in the previous subsection, by fixing threshold values to those optimizing the provider's revenue under the constraint that customers have an incentive to reveal their anticipated consumption. Before formally expressing the network revenue and user level of satisfaction, let us enumerate their components.

We assume that each cumulus point (positive or negative) has an economic effect on the user and the provider, which could be either direct, by assigning a charge for each point, or indirect, since (for instance) a customer will gain or be penalized from each cumulus point in the renegotiation procedure. We hope that this assumption (with respect to the general model defined in the previous subsection) will provide some elements when defining threshold values. The economic impact of each point is expressed by a parameter γ, meaning that we assume it to be linear. Thus, depending on the sign of δ, the difference between the actual use and the contract, and whether you are a customer or the provider, the positive or negative cumulus points are translated into a positive or negative financial impact $\gamma C(\delta)$.

Another cost for the network comes from the measurement sample size l. As we have seen above, in order to make the model more or less independent of the measurement method, l must satisfy the following condition [12]:

$$\forall i \in \{-N, \ldots, N-1\}, \quad \theta_{i+1} - \theta_i \geqslant 2\varepsilon_{\alpha,l}, \tag{1}$$

where $\varepsilon_{\alpha,l} = q_\alpha \widehat{S}/\sqrt{l}$, as defined in the previous subsection. Condition (1) gives a minimum distance between two neighboring thresholds, leading to the following condition over l:

$$l \geqslant \frac{4\widehat{S}^2 q_\alpha^2}{\min(\theta_{i+1} - \theta_i)^2}. \tag{2}$$

Introducing a parameter β representing the cost of a single measurement and assuming that the sampling cost is linear with respect to l gives a measurement cost βl.

We also set a last cost for the provider which depends on the measurement error estimation. Indeed, it seems relevant to penalize the provider with this kind of "ethical" cost from the precision of the model. We suppose that this cost is linear with the length of the confidence interval $2\varepsilon_{\alpha,l} = 2\widehat{S}q_\alpha/\sqrt{l}$. So we use the function F_{net} to express the network costs (the measurement cost plus the ethical cost) depending on the sample size l,

$$F_{\text{net}}(l) = \beta l + 2\mu \frac{\widehat{S} q_\alpha}{\sqrt{l}}.$$

The network benefits also come from the total charge $c(x)$ for the demand in the SLA and the extra-fee $c(\delta)$ from the over-consumption if the specific renegotiation threshold Θ is reached (where $\delta = \sum_{i=1}^{k} \delta_i$ is the sum of the deviations from the stated consumption up to period k) and, of course, the cumulus points if the user under-estimates his consumption. So, for the kth monitoring period between two renegotiation, the network revenue for a user is expressed in the following definition:

Definition 1. The expression of the network revenue, during the kth monitoring period, for user who requires x but consumes $x + \delta_k$ is

$$G_{\text{net}}(x, \delta_k, \theta, k, l) = c(x) + c\left(\sum_{i=1}^{k} \delta_i\right) \mathbb{1}_{\left\{\sum_{i=1}^{k} c_i \geqslant \Theta\right\}} + \gamma C(\delta_k) - F_{\text{net}}(l).$$

In Definition 1, we can remark that θ is in $\mathbb{R}^{2N}$, δ_k is a real representing the difference between actual and contract consumptions, Θ is the threshold in terms of cumulus points in order to charge the extra-fee and to start the contract renegotiation, and x is the expected resource consumption as specified in the contract.

To express the user level of satisfaction, we introduce a utility function $U(x)$ when consuming x. In economics, the notion of utility function is used to rank user preferences. Here, like in network pricing papers, one can think of $U(x)$ as the amount of money the user is willing to pay to receive x [1]. At the kth period, the level of satisfaction of a user can then be defined by the utility of consuming $x + \delta_k$ minus the costs which are the charge from the contract $c(x)$, the extra charge $c(\sum_{i=1}^{k}\delta_i)$ from the over-consumption if the specific renegotiation threshold Θ is reached, and the cost from positive cumulus points $\gamma C(\delta_k)$.

This leads to the following definition:

Definition 2. The level of satisfaction of a user that requires x but consumes $x + \delta_k$ over the kth period is

$$G_u(x,\delta_k,\theta,k) = U(x+\delta_k) - c(x) - \gamma C(\delta_k) - c\left(\sum_{i=1}^{k}\delta_i\right)\mathbb{1}_{\left\{\sum_{i=1}^{k}c_i \geqslant \Theta\right\}}.$$

Note that the discrete random variable $C(\delta_k)$ depends not only on δ_i, but also on the thresholds θ_n.

2.3. CPS and incentives to cheat

From Point 3 of the assumptions over the cost functions in Section 2.1, the assumed role of the penalty function Ψ is to prevent the user from an a priori over- or under-estimation of his consumption. Indeed, a counter-attitude would penalize the provider in a phase of capacity planning. But the translation into financial terms of measurements and cumulus points requires further analysis. The following definition describes how a user has an incentive to truthfully reveal his anticipated consumption.

Definition 3. The property of truthful anticipated consumption revelation of the model is expressed by the following condition: $\forall\theta,k\ \exists x^* > 0$ such that

$$\arg\max_{x,\delta} G_u(x,\delta,\theta,k) = (x^*,0,\theta,k). \tag{3}$$

This means that a customer chooses his *expected* consumption optimizing his level of satisfaction and that, if the model behaves correctly, the *declared* consumption x is exactly this *expected* consumption (meaning that $\delta = 0$).

Now, we show in the next theorem that the CPS model does not verify this property of truthful revelation.

Theorem 4. *The CPS model defined in this section does not verify the property of truthful revelation of anticipated consumption expressed in Definition* 3.

Proof. Let x be a customer's declared expected resource consumption (as expressed in the contract). Suppose that this user consumes exactly the same quantity of resource $x + \delta_k$ during each period. Then

$$G_u(x,\delta_k,\theta,k) = U(x+\delta_k) - c(x) - \gamma C(\delta_k) - c(k\delta_k)\mathbb{1}_{\left\{\sum_{i=1}^{k}c_i \geqslant \Theta\right\}}.$$

To prevent an hysteresis due to the measurements, the first threshold θ_1 is assumed to be strictly positive. Suppose that δ_k is between 0 and θ_1. A necessary condition to obtain (3) is that $\forall k,\theta$, $\exists x > 0$, $\forall\delta_k \in [-x,+\infty[\backslash\{0\}$,

$$G_u(x,-\delta_k,\delta_k,\theta,k) < G_u(x,0,\theta,k),$$

meaning that the user benefits from declaring in his contract his expected consumption. As $0 \leqslant \delta_k < \theta_1$, $C_i(\delta_k) = 0\,\forall i$, we have $\sum_{i=1}^{k}C_i = 0$ and

$$G_u(x^*-\delta_k,\delta_k,\theta,k) - G_u(x^*,0,\theta,k) = c(x^*) - c(x^*-\delta_k) > 0,$$

which shows that (3) is not verified, so that the property of *truthful anticipated consumption revelation* is not satisfied by the model. □

Remark 5. Note that, interestingly, the result of this theorem is independent of the utility function.

Specifically, we have shown that the property is not verified since a customer has an incentive to under-estimate his consumption to $x - \delta$ with

$\delta \in [0, \theta_1[$ so that he will not be penalized by a cumulus point. In the following proposition, we present a sufficient condition on the first threshold θ_1 constraining the user to restrict their under-estimation to less than θ_1.

Proposition 6. *If the positive thresholds verify that*

$$\theta_1(x) < c^{-1}(\gamma + c(x)) - x \tag{4}$$

and for $j \geqslant 2$,

$$\theta_j(x) < c^{-1}((k-1)\gamma + c(x)) - x, \tag{5}$$

then $\forall \theta, k, x > 0,\ \delta_k \in [\,\theta_1, +\infty[$, *we have*

$$G_u(x, \delta_k, \theta, k) < G_u(x + \delta_k, 0, \theta, k).$$

This proposition implies that users do not have an incentive to over-use the resource by more than θ_1. More exactly, it ensures that the cost of being assigned $k-1$ points is higher than the difference of costs between a declared consumption $x + \theta_j$ and a declared consumption x.

Proof. Let $x > 0$ and $\delta_k \geqslant \theta_1$. The difference of user level of satisfaction, when the anticipated belief of consumption is $x + \delta_k$, between a false declaration x and a truthful revelation $x + \delta_k$ is

$$\begin{aligned} &G_u(x, \delta_k, \theta, k) - G_u(x + \delta_k, 0, \theta, k) \\ &\quad = c(x + \delta_k) - c(x) - \gamma C(\delta_k) \\ &\qquad - c\left(\sum_{i=1}^{k} \delta_i\right) \mathbb{1}_{\left\{\sum_{i=1}^{k} c_i \geqslant K\right\}}. \end{aligned}$$

A sufficient condition to make sure that this quantity is negative is that

$$D(\delta_k) = c(x + \delta_k) - c(x) - \gamma C(\delta_k) < 0 \quad \forall \delta_k \geqslant \theta_1 \tag{6}$$

(since this expression corresponds to the case when the extra-fee is not applied).

Assume first that $\delta_k = \theta_1$. From (4), we obtain that

$$D(\theta_1) = c(x + \theta_1) - c(x) - \gamma < 0.$$

Next, since function D is strictly increasing with δ_k when $\delta_k \in [\theta_{j-1}, \theta_j[$ $(k \geqslant 2)$, it is sufficient to make sure that condition (6) is verified when δ_k tends to θ_j. As

$$\begin{aligned} \lim_{\varepsilon \to 0} D(\theta_j - \varepsilon) &= D(\theta_j) + \gamma \\ &= c(x + \theta_j) - c(x) - \gamma(k-1), \end{aligned}$$

a sufficient condition is provided by inequality (5). □

In the next section, we modify the model in order to generate incentives for users to reveal their anticipated consumption. In particular, they will not even have an interest to under-estimate their consumption by an amount smaller than the first threshold θ_1.

3. The total penalty CPS

In order to prevent from a false declaration of the anticipated consumption, we introduce a small variation of the model described above. It consists in charging the penalty $c(\delta)$ at the end of each period instead of just when a number Θ of cumulus points have been assigned. This is a particular case of the precedent model where the extra-fee threshold Θ is fixed to 0 but where, now, we charge as soon as there is an over-consumption, i.e. as soon as $\delta > 0$. Then, the definitions of the network revenue and of the user level of satisfaction need to be modified. We present them in the next subsection.

3.1. Definitions

Definition 7. For any period of measurements, the network revenue for a user who requires x and consumes $x + \delta$ is

$$G_{\text{net}}(x, \delta, \theta, l) = c(x) + c(\delta)\mathbb{1}_{\{\delta > 0\}} + \gamma C(\delta) - F_{\text{net}}(l),$$

and the level of satisfaction of a user is

$$G_u(x, \delta, \theta) = U(x + \delta) - c(x) - c(\delta)\mathbb{1}_{\{\delta > 0\}} - \gamma C(\delta).$$

Remark that since the extra-fee is charged at the end of each measurement period k, the definitions, with respect to those of Section 2, become independent of k. The extra-fee allows to avoid incentives to cheat. We prove this result in the next subsection.

3.2. About incentives to cheat in the total penalty CPS

In this part, we analyze the property of *truthful revelation of anticipated consumption* of Definition 3 for this new model. In the next theorem we formulate a sufficient condition to obtain this property.

Theorem 8. *The total penalty cumulus pricing scheme satisfies the truthful revelation of anticipated consumption property if the negative thresholds verify*

$$\forall i \geqslant 1 \text{ and } x > 0, \quad c(\theta_{-i} + x) < c(x) - i\gamma.$$

Before proving this theorem, we first show the following result.

Lemma 1. *A sufficient condition for truthful anticipated consumption revelation for the total penalty CPS is that* $\forall x > 0$, $\forall \delta \in [-x, +\infty[\setminus \{0\}$,

$$c(x+\delta) - c(x) - c(\delta)\mathbb{1}_{\{\delta \geqslant 0\}} - \gamma C(\delta) < 0.$$

Proof of Lemma 1. A sufficient condition for truthful anticipated consumption revelation is that for all x and δ, the difference in level of satisfaction between a user who requires x and consumes $x + \delta$, and a user who requires $x + \delta$, and respects its contract, is negative, i.e.,

$$\forall \delta, x, \theta \quad G_u(x, \delta, \theta) - G_u(x + \delta, 0, \theta) < 0.$$

If we replace the expression of G_u from Definition 7 in this inequality, we immediately get Lemma 1. □

Proof of Theorem 8. We separate the cases of over- (i.e., $\delta > 0$) and under-use (i.e., $\delta < 0$). For all k and θ, in the case where δ is positive, we have

$$\begin{aligned} &G_u(x, \delta, \theta, k) - G_u(x + \delta, 0, \theta, k) \\ &\quad = c(x+\delta) - c(x) - c(\delta) - \gamma C(\delta) \\ &\quad \leqslant \Psi(x, x+\delta) < 0. \end{aligned}$$

Thus, from the properties over function Ψ, this difference is strictly negative when we have an over-use.

Next, let $x > 0$ and $\delta \in [-x, 0[$ (i.e., negative). Let i be the integer such that $\theta_{-i-1} < \delta \leqslant \theta_{-i}$ and thus $C(\delta) = -i$. Then $\forall x > 0$, $\forall \delta \in [-x, +\infty[\setminus \{0\}$,

$$\begin{aligned} &c(x+\delta) - c(x) - c(\delta)\mathbb{1}_{\{\delta \geqslant 0\}} - \gamma C(\delta) \\ &\quad = c(x+\delta) - c(x) + i\gamma \\ &\quad \leqslant c(x + \theta_{-i}) - c(x) + i\gamma < 0 \end{aligned}$$

using the assumption over the thresholds. Lemma 1 then gives the result. □

Note that this sufficient condition deals with the negative thresholds only. Another important remark is that it is independent of the utility function. Nevertheless, it is not unique. For instance, another condition is when, whatever the contract specification x is, the level of satisfaction of a user respecting his contract is higher than when deviating from it, i.e.,

$$G_u(x, \delta, \theta) - G_u(x, 0, \theta) < 0 \quad \forall \theta, \delta, x.$$

This leads to the following equations over the thresholds θ_{-i}: $\forall i > 0$, $\forall x > 0$, $\forall \delta \in [-x, +\infty[\setminus \{0\}$,

$$U(\theta_{-i} + x) - U(x) - c(\delta)\mathbb{1}_{\{\delta > 0\}} - \gamma C(\delta) < 0,$$

but it is less attractive since it depends on the utility function. Another important remark is that the thresholds depend on the contract specification x. This will have some consequences on the optimization of the network revenue in the next section.

4. Optimization of the provider's revenue

4.1. Optimization problem and minimization of the cost function $\boldsymbol{F}_{net}$

We have provided in Theorem 8 sufficient conditions over the negative thresholds that the model can satisfy in order to verify the property of *truthful anticipated consumption revelation*. Our goal now is to find out the configuration of thresholds optimizing the total revenue of the network, under the assumptions of Theorem 8.

We consider that users come into the network with a mean rate λ which is a function of the average level of satisfaction introduced in Definition 2. So, we define the total network revenue by

$$\int_0^{+\infty} \mathbb{E}(G_{\text{net}}(x,\delta,\theta,l))\,\mathrm{d}\lambda(\mathbb{E}(G_u(x,\delta,\theta))).$$

Under the sufficient conditions of Theorem 8 for truthful revelation of anticipated consumption (where the thresholds depend on the amount of resource x required in the contract), either a customer enters the network with its anticipated consumption x, or does not enter if it is too expensive for him. Anyway, he has no interest in entering at another declared level x'. Thus, the maximization of the total network revenue is equivalent to finding out

$$\max_{\theta}(\lambda(\mathbb{E}(G_u(x,\delta,\theta)))\mathbb{E}(G_{\text{net}}(x,\delta,\theta,l))), \tag{7}$$

for all x, subject to the constraint

$$\forall i \geqslant 1 \text{ and } x > 0, \quad c(\theta_{-i}(x)+x) < c(x) - i\gamma.$$

Assumption 1. We will assume from now and throughout the paper that the difference δ between the actual and the anticipated consumption is a random variable. Following the truthful revelation of anticipated consumption property, it is reasonable to assume that the density of δ is symmetric, so that its mean value is zero (assuming that it exists).

In order to maximize the network revenue given by Eq. (7), the provider has first to minimize the cost function $F_{\text{net}}(l) = \beta l + 2\mu(\widehat{S}q_\alpha/\sqrt{l})$ under the constraint defined by Eq. (2). Since F_{net} reaches its minimum at

$$l^* = \left(\frac{\widehat{S}q_\alpha\mu}{\beta}\right)^{2/3},$$

we have the following relation between l and the thresholds: if l^* is larger than $4\widehat{S}^2q_\alpha^2/\min(\theta_{i+1}-\theta_i)^2$ (the constraint value in (2)), i.e., if $\min(\theta_{i+1}-\theta_i) \geqslant 2(\beta/\mu)^{1/3}(\widehat{S}q_\alpha)^{2/3}$, then the minimum is obtained for $l = l^*$ and is equal to $3\beta^{1/3}(\mu\widehat{S}q_\alpha)^{2/3}$ (by replacing l by l^* in the formulas). Otherwise, the minimum is obtained for $l = 4(4\widehat{S}^2q_\alpha^2/\min(\theta_{i+1}-\theta_i)^2)$ and is equal to $4\beta(\widehat{S}^2q_\alpha^2/\min(\theta_{i+1}-\theta_i)^2) + \mu\min(\theta_{i+1}-\theta_i)$.

We can then express the cost function in terms of the thresholds by

$$F_{\text{net}}(\theta) = L\mathbb{1}_{\{\min(\theta_{i+1}-\theta_i)\geqslant\kappa\}} + \left(4\beta\frac{\widehat{S}^2q_\alpha^2}{\min(\theta_{i+1}-\theta_i)^2} + \mu\min(\theta_{i+1}-\theta_i)\right) \times \mathbb{1}_{\{\min(\theta_{i+1}-\theta_i)<\kappa\}}$$

with

$$\kappa = 2\left(\frac{\beta}{\mu}\right)^{1/3}(\widehat{S}q_\alpha)^{2/3} \tag{8}$$

and

$$L = 3\beta^{1/3}(\mu\widehat{S}q_\alpha)^{2/3}.$$

As l is actually a function of θ, we now write $G_{\text{net}}(x,\delta,\theta)$ instead of $G_{\text{net}}(x,\delta,\theta,l)$.

4.2. Optimization in a general setting

The difficulty with the optimization stems from the non-differentiability of the function $F_{\text{net}}(\theta)$. Thus, we need to resort to numerical approaches. In this paper, we use a simulated annealing algorithm. This class of continuous global optimization algorithms seems the most adapted for our objective function because, first, this method requires no specifications of differentiability and, second, it is specifically adapted to functions with multiple local optima. We use the algorithm called ASA for adaptive simulated annealing [9]. It is one of the most widely tested SA algorithm in the literature, and its code is publicly available at the web site http://www.ingber.com. We use more specifically ASAMIN which is a gateway function to ASA developed by Sakata [15]. One particularity of this simulated annealing algorithm is the use of the temperature to define the densities of the next candidate point [7].

In Section 5, we illustrate numerically this method on one example. Nevertheless, under some restrictions over the thresholds, we are able to obtain some analytic results.

4.3. Analytic results in particular cases

In this section, we investigate analytically the optimization problem in three particular cases of thresholds. First, we look at the configuration proposed by Reichl et al. in [12,17] where the thresholds are considered symmetric. This configuration seems realistic when users have no incentive to consume more or less than specified in the contract. Second, we consider the linear thresholds configuration to reward (resp. penalize) users proportionally to their under-use (resp. over-use). Finally, we take into account the special configuration where thresholds are fixed so that the probability of obtaining one or more positive points is uniform, with the same property for negative points.

4.3.1. Symmetric thresholds

We consider the configuration proposed by Reichl et al. in [12,17] where the thresholds verify, for all k, $\theta_k = -\theta_{-k}$. Then, using Assumption 1, the mean number of assigned cumulus points $\in\{0, \pm 1, \ldots, \pm N\}$ is zero and, when optimizing $\lambda(\mathbb{E}(G_u(x,\delta,\theta)))\mathbb{E}(G_{\text{net}}(x,\delta,\theta,l))$, the thresholds intervene only in the cost function F_{net} of the network revenue (since $\mathbb{E}(G_u(x,\delta,\theta)) = U(x+\delta) - c(x) - c(\delta)\mathbb{1}_{\{\delta>0\}}$ does not depend on θ). Consequently, maximizing the total network revenue is equivalent to maximizing the network revenue for one user, which is itself equivalent to minimizing the cost function $F_{\text{net}}(\theta)$ in terms of the minimal difference between two neighboring thresholds.

Thus, in order to optimize the total network revenue, we have to set the thresholds so that $\min(\theta_{i+1} - \theta_i) \geqslant \kappa$, meaning that $F_{\text{net}}(\theta)$ reaches its minimum without the limitation of constraint (2), where κ is given by Eq. (8). Note that the negative thresholds have to satisfy the sufficient condition defined in Theorem 8. Note also that, in this case, the total network revenue is independent of the number of thresholds (as it actually depends only on the minimal difference between successive thresholds), so we can take like Reichl et al. in [12], 3–5 thresholds on each direction for simplicity. This is illustrated in Section 5.

4.3.2. Linear thresholds

Let us now consider the case where the threshold values are linear, i.e.,

$$\forall i > 0, \quad \theta_i = i\theta_+ \quad \text{and} \quad \theta_{-i} = -i\theta_-.$$

The optimization is then carried out with respect to the variables $\theta_+ > 0$ and $\theta_- > 0$ representing the distance between positive thresholds and between negative thresholds respectively. Let F_δ be the cumulative distribution function δ of and let f_δ denote its density. The mean number of cumulus points obtained by a user, expressed in terms of those variables is

$$\begin{aligned}\mathbb{E}(C(\delta)) &= \sum_{k=1}^{N} k(F_\delta((k+1)\theta_+) - F_\delta(k\theta_+)) \\ &\quad + \sum_{k=1}^{N}(-k)(F_\delta(-k\theta_-) - F_\delta(-(k+1)\theta_-)) \\ &= N - \sum_{k=1}^{N} F_\delta(k\theta_+) - \sum_{k=1}^{N} F_\delta(-k\theta_-). \qquad (9)\end{aligned}$$

In order to optimize the network revenue, we consider separately the three following cases, $\theta_+ < \theta_-$, $\theta_+ > \theta_-$ and $\theta_+ = \theta_-$. A numerical comparison of the maxima obtained over these domains will provide the global maximum.

(a) $\theta_+ < \theta_-$. The network cost F_{net} then only depends on θ_+ since $\min(\theta_{i+1} - \theta_i) = \theta_+$. Note that, to simplify the expression, we will write $\lambda(\cdot)$ and $\mathbb{E}(G_{\text{net}})$ without their arguments in all following equations. A first equation of the system defined by the first order conditions of the optimization problem gives

$$\begin{aligned}&\frac{\partial}{\partial\theta_-}(\lambda(\cdot)\mathbb{E}(G_{\text{net}})) \\ &\quad = \mathbb{E}(G_{\text{net}})\lambda'\frac{\partial}{\partial\theta_-}\mathbb{E}(G_u) + \lambda\frac{\partial}{\partial\theta_-}\mathbb{E}(G_{\text{net}}) \\ &\quad = \mathbb{E}(G_{\text{net}})\lambda'\gamma\sum_{k=1}^{N}(-k)f_\delta(-k\theta_-) \\ &\qquad - \lambda\gamma\sum_{k=1}^{N}(-k)f_\delta(-k\theta_-) \\ &\quad = \left(\gamma\sum_{k=1}^{N}(-k)f_\delta(-k\theta_-)\right)(\mathbb{E}(G_{\text{net}})\lambda' - \lambda) = 0, \qquad (10)\end{aligned}$$

and the second one gives

$$\begin{aligned}\frac{\partial}{\partial\theta_+}(\lambda\mathbb{E}(G_{\text{net}})) &= \mathbb{E}(G_{\text{net}})\lambda'\frac{\partial}{\partial\theta_+}\mathbb{E}(G_u) + \lambda\frac{\partial}{\partial\theta_+}\mathbb{E}(G_{\text{net}}) \\ &= \mathbb{E}(G_{\text{net}})\lambda'\gamma\sum_{k=1}^{N} kf_\delta(k\theta_+) \\ &\quad - \lambda\gamma\sum_{k=1}^{N} kf_\delta(k\theta_+) - \lambda\frac{\partial}{\partial\theta_+}F_{\text{net}}(\theta_+) \\ &= \gamma\sum_{k=1}^{N} kf_\delta(k\theta_+)(\mathbb{E}(G_{\text{net}})\lambda' - \lambda) \\ &\quad - \lambda\frac{\partial}{\partial\theta_+}F_{\text{net}}(\theta_+) = 0. \end{aligned} \tag{11}$$

Eq. (10) gives that $\mathbb{E}(G_{\text{net}})\lambda' - \lambda = 0$ since $\gamma\sum_{k=1}^{N}(-k)f_\delta(-k\theta_-) \neq 0$, which, yields from (11) that $\frac{\partial}{\partial\theta_+}F_{\text{net}}(\theta_+) = 0$. Since

$$\frac{\partial}{\partial\theta_+}F_{\text{net}}(\theta_+) = \left(-8\beta\frac{\widehat{S}^2 q_\alpha^2}{\theta_+^3} + \mu\right)\mathbb{1}_{\{\theta_+<\kappa\}},$$

we get the solution $\theta_+^* = \kappa$ (with κ defined in Eq. (8)). Then, inserting $\theta_+^* = \kappa$ in Eq. (10), we can determine θ_-^*.

(b) Suppose now that $\theta_- < \theta_+$. We use the same idea than in the previous case to derive the solution, just by inverting the roles of θ_+ and θ_-, since F_{net} now depends on θ_-. It leads to $\theta_-^* = \kappa$, while θ_+^* is determined by inserting θ_-^* into one of the first order equations.

(c) Let $\theta_- = \theta_+$. In this case, F_{net} is not differentiable because of the minimum function. Nevertheless, the thresholds are symmetric; we are then in the particular case studied Section 4.3.1 and the solution gives $\theta_+^* = \theta_-^* = \kappa$.

Finding out which of the three local optimum solutions obtained in the above cases yields the global optimum can be realized by a numerical comparison.

4.3.3. Uniform thresholds

We consider here that we have N_+ positive thresholds and N_- negative thresholds, distributed according to the quartiles of the distribution of the random variable δ, so that the probability to obtain n points is equal (and a similar property is assumed over negative thresholds) for all $n \leqslant N_+$. Formally,

$$\theta_k = \begin{cases} F_\delta^{-1}\left(\frac{1}{2} + \frac{k}{2(N_+ + 1)}\right) & \text{if } k > 0, \\ F_\delta^{-1}\left(\frac{1}{2} + \frac{k}{2(N_- + 1)}\right) & \text{if } k < 0. \end{cases}$$

The expression of the mean number of assigned points is

$$\begin{aligned}\mathbb{E}(C(\delta)) &= \sum_{k=1}^{N_+} k(F_\delta(\theta_{k+1}) - F_\delta(\theta_k)) \\ &\quad + \sum_{k=1}^{N-}(-k)(F_\delta(-\theta_k) - F_\delta(\theta_{-k-1})) \\ &= \sum_{k=1}^{N_+}\frac{k}{2(N_+ + 1)} - \sum_{k=1}^{N-}\frac{k}{2(N_- + 1)} \\ &= \tfrac{1}{4}N_+ - \tfrac{1}{4}N_-. \end{aligned}$$

The optimization problem is then defined in terms of the two parameters N_+ and N_-.

If we consider the case where δ follows a uniform distribution over the interval $[-Z, Z]$, the thresholds are given by

$$\theta_k = \begin{cases} \frac{Z}{N_+ + 1} & \text{if } k > 0, \\ -\frac{Z}{N_- + 1} & \text{if } k < 0. \end{cases}$$

To get the values maximizing the revenue, we look at the first order conditions following the lines of the linear thresholds case. Also, we consider that N_+ and N_- are real in order to get derivatives.

(a) Assume that $N_+ > N_-$, so that minimal difference between consecutive thresholds is

$$\theta_1 = \frac{Z}{N_+ + 1}. \tag{12}$$

As in the linear case, first order conditions are

$$\begin{aligned}\frac{\partial}{\partial N_-}(\lambda\mathbb{E}(G_{\text{net}})) &= \mathbb{E}(G_{\text{net}})\lambda'\frac{\partial}{\partial N_-}\mathbb{E}(G_u) \\ &\quad + \lambda\frac{\partial}{\partial N_-}\mathbb{E}(G_{\text{net}}) \\ &= \frac{\partial}{\partial N_-}\mathbb{E}(G_u)(\mathbb{E}(G_{\text{net}})\lambda' - \lambda) = 0 \end{aligned}$$

since it can be observed that $\frac{\partial}{\partial N_-}\mathbb{E}(G_{\text{net}}) = -\frac{\partial}{\partial N_-}\mathbb{E}(G_u)$.

Similarly, since $\frac{\partial}{\partial N_+}\mathbb{E}(G_{\text{net}}) = -\frac{\partial}{\partial N_+}\mathbb{E}(G_u) - \frac{\partial}{\partial N_+}F_{\text{net}}$,

$$\frac{\partial}{\partial N_+}(\lambda\mathbb{E}(G_{\text{net}})) = \frac{\partial}{\partial N_+}\mathbb{E}(G_u)(\mathbb{E}(G_{\text{net}})\lambda' - \lambda) - \lambda\frac{\partial}{\partial N_+}F_{\text{net}}(\theta_1(N_+)) = 0.$$

As $\frac{\partial}{\partial N_-}\mathbb{E}(G_u) = \gamma/4 \neq 0$, this system of equations leads to

$$\frac{\partial}{\partial N_+}F_{\text{net}}(\theta_1(N_+)) = 0,$$

with the expression of θ_1 described in (12). It again leads to

$$\theta_1 = \kappa, \text{ so that } N_+^* = -\frac{1}{F_\delta(\kappa) - 1} - 1.$$

To obtain N_-^*, we replace N_+^* by its value in the first order condition

$$\frac{\partial}{\partial N_-}\lambda(\mathbb{E}(G_u(x,\delta,\theta)))\mathbb{E}(G_{\text{net}}) = \frac{\gamma}{4}\mathbb{E}(G_{\text{net}})\lambda' - \frac{\gamma}{4}\lambda = 0.$$

For instance, if we suppose that the arrival rate is expressed by $\lambda(x) = \sqrt{x}$ and that δ is uniformly distributed over $[-Z, Z]$, we have $N_+^* = (Z/\kappa) - 1$ and N_-^* is the solution of

$$\frac{\gamma}{4}\left(A - \frac{\gamma}{4}N_-\right)\frac{1}{2\sqrt{\mathbb{E}(U(x+\delta)) - A - L + (\gamma/4)N_-}} - \frac{\gamma}{4}\sqrt{\mathbb{E}(U(x+\delta)) - A - L + \frac{\gamma}{4}N_-} = 0$$

with $A = c(x) + \mathbb{E}(c(\delta)\mathbb{1}_{\{\delta>0\}}) + (\gamma/4)N_+^* - L$, i.e.,

$$N_-^* = \frac{16}{8\gamma + \gamma^2}\left(\frac{9\gamma}{4}A - 2\mathbb{E}(U(x+\delta)) + 2K\right).$$

(b) If $N_- > N_+$, we use exactly the same procedure, but invert the roles of N_- and N_+. The difference is that the minimum difference between consecutive thresholds is θ_{-1}. From the first order conditions

$$\frac{\partial}{\partial N_+}(\lambda\mathbb{E}(G_{\text{net}})) = \frac{\partial}{\partial N_+}\mathbb{E}(G_u)(\mathbb{E}(G_{\text{net}})\lambda' - \lambda) = 0,$$

and

$$\frac{\partial}{\partial N_-}(\lambda\mathbb{E}(G_{\text{net}})) = \frac{\partial}{\partial N_-}\mathbb{E}(G_u)(\mathbb{E}(G_{\text{net}})\lambda' - \lambda) - \lambda\frac{\partial}{\partial N_-}F_{\text{net}}(\theta_1(N_-)) = 0,$$

we obtained $N_-^* = 1/(F_\delta(\kappa) - 1) - 1$ while N_+^* is obtained by replacing N_-^* by its value in one of the first order conditions.

(c) If $N = N_+ = N_-$, the thresholds are symmetric. This is (again) a sub-case of the symmetric thresholds studied in Section 4.3.1. From $\frac{\partial}{\partial N}F_{\text{net}} \times (\theta_1(N)) = 0$, we get $N_+ = N_- = 1/(F_\delta(\kappa) - 1) - 1$.

Again, in order to find the maximum between the three local optimum solutions obtained in the different cases, we compare them numerically in the next section.

5. Numerical illustrations

In this section, we illustrate the results that can be obtained. Firstly, we are interested in the property of truthful anticipated consumption revelation: we compare the user level of satisfaction of the two CPS models. Secondly, we observe the results obtained with the simulated annealing algorithm and discuss the evolution of the total network revenue in terms of the number of thresholds. Finally, we look at the results obtained in the particular cases defined in Section 4.3.

In all the following examples, the demand function is defined by

$$\lambda(y) = \sqrt{y}$$

and, assuming the traffic elastic, the utility function is given (like in [8]) by

$$U(x) = 10\log(1 + x).$$

For the optimization of the network revenue in the symmetric and linear particular cases, we arbitrarily use the following parameters:

- user's required consumption $x = 100$,
- cost per cumulus point $\gamma = 100$,
- unit measurement cost $\beta = 0.1$,
- unit error cost $\mu = 0.1$.

For the uniform case, we use the same parameters, apart from the unit cost for measurement set to $\beta = 0.05$ and the unit error cost set to $\mu = 5$.

5.1. Truthful anticipated consumption revelation

We present here a numerical application illustrating the property of truthful anticipated consumption revelation. We plot the difference of level of satisfaction between a user who requires x but consumes $x + \delta$ and another who requires and consumes $x + \delta$. It is important to note that we present results with the first model where Θ is very large, to have the worst case for the network revenue and to use the sufficient condition of Proposition 6. The results are displayed in Fig. 2 where we have considered the thresholds $\theta = [-40, -20, 0, 10, 18, 40, 60, 90]$ so that sufficient conditions on the positive thresholds (Proposition 6) and on the negative thresholds (Theorem 8) for each model are satisfied. The user specified demand is $x = 100$ and the cost by cumulus point is $\gamma = 1$. We remark that, for the initial CPS model, the difference of level of satisfaction cannot be negative when the over-use δ is under the first threshold θ_1. This illustrates the fact that this model does not verify the property of *truthful anticipated consumption revelation*. But we see that for the total penalty CPS model, this difference is always negative. Furthermore, the difference is much larger in the case of over-use for the total penalty CPS, which is a desirable situation preventing even more from cheating.

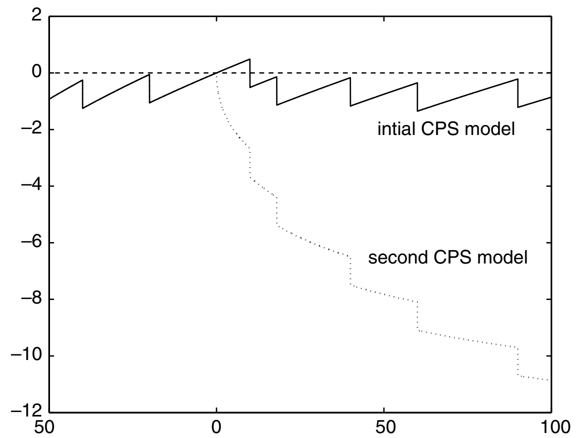

Fig. 2. Evolution of $G_u(x, \delta, \theta) - G_u(x + \delta, 0, \theta)$ in terms of δ.

5.2. Simulated annealing for the general case

We assume here that δ follows a Gaussian distribution with mean 0 and variance 1. In Table 1, we give the results obtained using simulated annealing.

We have fixed the number of thresholds from 1 to 10 and the best threshold values have been determined each time (but note that the number of positive or negative thresholds is not fixed). If we compare each case, we can observe that the maximum revenue is obtained when there are three thresholds, one being negative and two being positive. The small number of thresholds might be due to the measurement error cost: the larger the number of thresholds, the less the measurement precision has to be. Also, having at least one negative thresholds prevents users from cheating.

Table 1
Thresholds maximizing the total network revenue, obtained with simulated annealing

Number of thresholds	Thresholds positions	Total network revenue
1	−2.57	54.49
2	−2.57 0.84	111.17
3	−2.57 0.92 1.85	113.17
4	−1.89 0.85 1.71 2.57	112.22
5	−2.57 −1.74 0.82 1.65 2.47	111.63
6	−2.57 −1.89 −1.22 0.67 1.34 2.01	107.31
7	−2.55 −1.9 −1.08 0.64 1.28 1.92 2.57	106.13
8	−2.43 −1.89 −0.58 0.51 1.02 1.54 2.05 2.56	98.39
9	−2.49 −1.47 −1.2 −0.77 0.33 0.87 1.36 1.61 2.51	37.54
10	−2.56 −2.11 −1.66 −1.2 −0.75 0.45 0.89 1.34 1.8 2.52	91.98

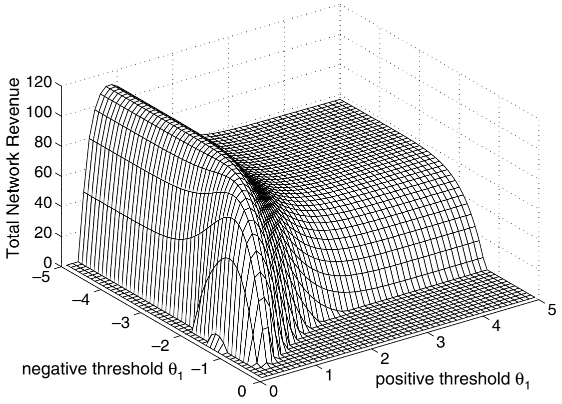

Fig. 3. Network total revenue when there are two thresholds: one positive and one negative.

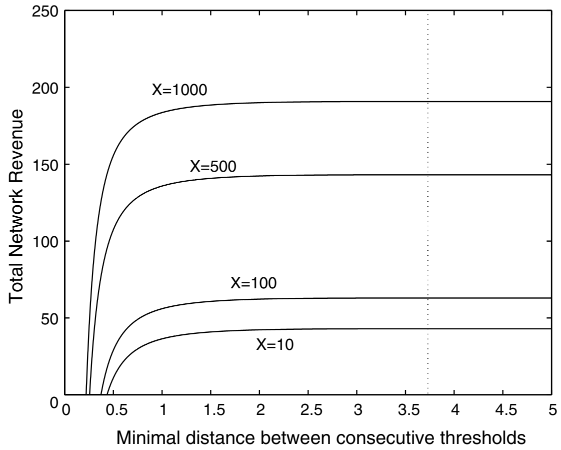

Fig. 4. Network total revenue with symmetric thresholds in terms of the minimum distance between consecutive thresholds.

In order to verify the relevance of our results, we have also plotted in Fig. 3 the revenue in the case when there are only two thresholds, one positive and one negative. We can verify that the value obtained in Table 1 (second line) is the maximum.

5.3. Special cases

In order to illustrate the theoretical results, we look at the special cases introduced before. First we suppose that the thresholds are symmetric, so that the mean number of cumulus points assigned in one monitoring period is null. Second, the thresholds are assumed to be linear, and finally they are supposed to be uniform. In those three cases, the numerical results are verified to be in accordance with theoretical ones.

5.3.1. Symmetric thresholds

In this special case, the total network revenue is expressed in terms of the minimum distance between consecutive thresholds. In Fig. 4, we plot the total network revenue in terms of the required quantity of resources x. The vertical line on the right represents the value κ (from Eq. (8)) and we see that behind this frontier, the total network revenue is constant. We remark as well that, as it could be expected, the revenue is increasing with x.

5.3.2. Linear thresholds

In this configuration, we obtain that $\kappa = 3.73$. We observe in Fig. 5, that the total network revenue is constant when θ_+ and θ_- are above κ, since the cost F_{net} is then independent of the thresholds. The local maximum revenue when $\theta_+ = \theta_-$ is reached for $\theta_+ = \theta_- = \kappa$ as we minimize the cost function F_{net}. One can observe that the global maximum is reached when $\theta_+ < \theta_-$ and that the optimal value obtained when $\theta_- < \theta_+$ gives actually a *negative* minimum, so that users are not allowed to enter and the revenue is therefore zero.

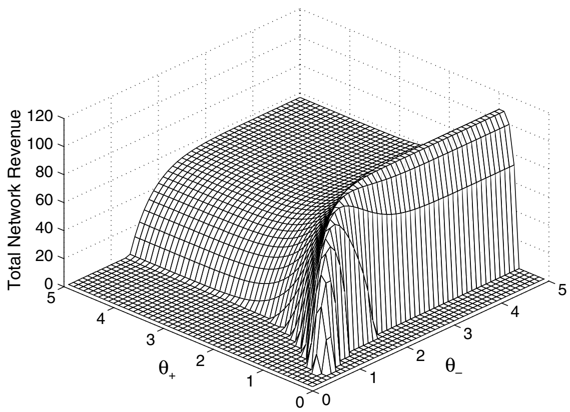

Fig. 5. Network total revenue with linear thresholds in terms of the distance between neighboring positive thresholds θ_+ and negative θ_-.

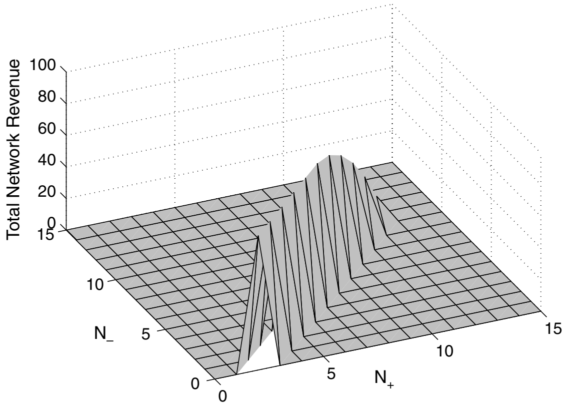

Fig. 6. Network total revenue with uniform thresholds in terms of the number of positive threshold N_+ and negative ones N_-.

5.3.3. Uniform threshold

For this numerical illustration, we use the same parameters as defined in the beginning of this section, except that $\beta = 0.05$ and $\mu = 5$. We display in Fig. 6 the total network revenue when the numbers of positive and negative thresholds vary. The distribution of δ is taken uniform over the interval $[-50, 50]$. Like in the previous case, we observe that one of the theoretical optimum solution (obtained when $N_+ > N_-$) is valid because it gives a positive value whereas and the other (obtained when $N_+ < N_-$) is not, because it gives a negative value. Moreover, note that the revenue is null when $N_+ = N_-$ as the mean number of cumulus points assigned is zero and because the user's level of satisfaction does not depend on the thresholds then.

It can be verified that the optimum is the same than given by equations in Section 4.3.3:

$N^*_+ = 5.57 \quad \text{and} \quad N^*_- = 2.56.$

6. Conclusions

This paper was motivated by the parameters optimization problem in the cumulus pricing scheme. First, after proving that the initial CPS provided an incentive to cheat, we have slightly modified the model to circumvent this problem. Second, we have analytically optimized the provider's revenue for this modified model under the following particular restrictions over the thresholds:

- *symmetric case*: the thresholds verify $\theta_k = -\theta_{-k}$ for all $k > 0$;
- *linear case*: the distances between the neighboring thresholds are equal;
- *uniform case*: the threshold are determined in order to equalize the *probabilities* of being assigned i cumulus points $\forall i$.

In the general case, we have used a simulated annealing algorithm to optimize the total network revenue in terms of the thresholds.

As future directions of research, one can be interested in time-scales: how does the model behave if the cumulus points are assigned every second, every hour or every month? We also plan to extend the model by including service differentiation, or traffic parameters like delay and jitter like in [14]. Finally, analyzing the CPS under competition between ISPs is an important aspect.

References

[1] C. Courcoubetis, R. Weber, Pricing Communications Networks, Wiley, New York, 2003.
[2] L.A. DaSilva, Pricing of QoS-enabled networks: a survey, IEEE Communications Surveys & Tutorials 3 (2) (2000).
[3] P. Dolan, Internet pricing. Is the end of the World Wide Wait in view? Communications & Strategies 37 (2000) 15–46.
[4] M. Falkner, M. Devetsikiotis, I. Lambadaris, An overview of pricing concepts for broadband IP networks, IEEE Communications Surveys & Tutorials 3 (2) (2000).
[5] J. Heinanen, F. Baker, W. Weiss, J. Wroclawski, Assured forwarding phb group, IETF: RFC 2597, 1999.
[6] T. Henderson, J. Crowcroft, S. Bhatti, Congestion pricing. Paying your way in communication networks, IEEE Internet Computing 5 (5) (2001) 85–89.
[7] L. Ingber, Simulated annealing: practice versus theory, Mathematical Computing Modelling 18 (11) (1993) 29–57.
[8] F.P. Kelly, A.K. Mauloo, D.K.H. Tan, Rate control in communication networks: shadow prices, proportional fairness and stability, Journal of the Operational Research Society 49 (1998) 237–252.
[9] M. Locatelli, Simulated annealing algorithms for continuous global optimization, Journal of Optimization Theory and Applications 104 (2000) 121–133.
[10] P. Reichl, P. Flury, J. Gerke, B. Stiller, How to overcome the feasibility problem for tariffing internet services: the cumulus pricing scheme, in: Proceedings of IEEE ICC 2001, vol. 7, 2001, pp. 2079–2083.

[11] J.W. Roberts, Quality of service guarantees and charging in multiservice networks, IEICE Transactions on Communications Electronics Information and Systems E 81 (5) (1998) 824–831.
[12] P. Reichl, B. Stiller, Edge pricing in space and time: theoretical and practical aspects of the cumulus pricing scheme, in: Proceedings of the 17th International Teletraffic Congress, 2001.
[13] P. Reichl, B. Stiller, Nil nove sub sole: Why internet charging schemes look like as they do, in: Proceedings of the 4th Berlin Internet Economic Workshop, 2001.
[14] P. Reichl, B. Stiller, T. Ziegler, Charging multi-dimensional QoS with the cumulus pricing scheme, in: Proceedings of ITCom'01, 2001.
[15] S. Sakata. Asamin, 2002. Available from <http://www.econ.lsa.umich.edu/ssakata/software>.
[16] B. Stiller, J. Gerke, P. Reichl, P. Flury, A generic and modular internet charging system for the cumulus pricing scheme, Journal of Network Management 3 (9) (2001) 293–325.
[17] B. Stiller, J. Gerke, P. Reichl, P. Flury, Management of differentiated services usage by the cumulus pricing scheme and a generic internet charging system, in: Proceedings of the Symposium on Integrated Network Management, 2001.
[18] B. Stiller, P. Reichl, S. Leinen, Pricing and cost recovery for internet services: practical review, classification, and application of relevant models, Netnomics 2 (1) (2000).
[19] B. Tuffin, Charging the Internet without bandwidth reservation: an overview and bibliography of mathematical approaches, Journal of Information Science and Engineering 19 (5) (2003).

Yezekael Hayel has a M.Sc. in Statistics and Stochatic Modeling and a M.Sc. in Computer Science both from University of Rennes 1, France in 2001 and 2002, respectively. He is currently a PhD student at Rennes 1, working at the INRIA/IRISA Labs. His research interests include pricing, network model, performance evaluation, queueing network, optimization.

Bruno Tuffin (IRISA/INRIA) received his PhD degree in applied mathematics from Rennes 1 University in 1997. Since, he has been with INRIA-Rennes, France. His research interests include developing Monte Carlo and quasi-Monte Carlo simulation techniques for the performance evaluation of computer and telecommunication systems, and more recently developing Internet active measurement techniques and new pricing schemes. On this last topic, he is the coordinator of the INRIA's cooperative research action PRIXNET (see http://www.irisa.fr/armor/Armor-Ext/RA/prixnet/ARC.htm).