Some inverse problems in electrocardiography

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- Models in electrocardiography and discretisation
- Discrete ill-posed general least-squares problem
- Time regularisation

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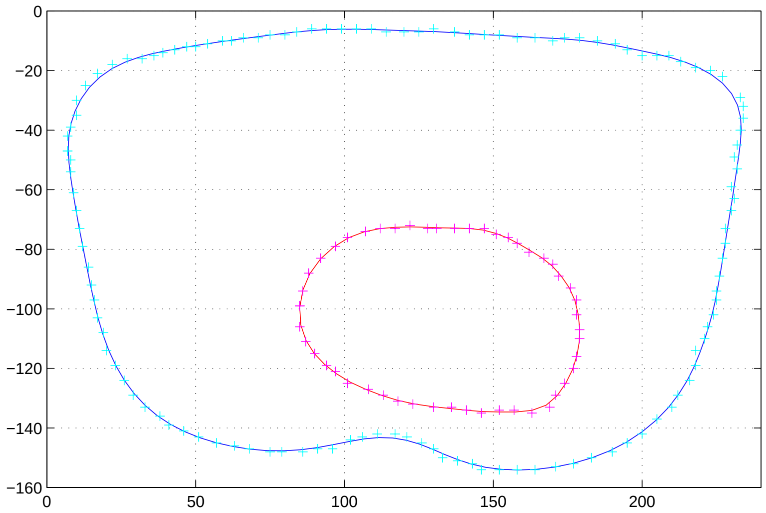
Electrocardiography



source of electrical current at the surface of the heart : Γ_E
propagation through the chest, bioelectric volume conductor : Ω
measure of potential at the surface of the torso : Γ_T

Direct and inverse models in ECG

Φ is the electrostatic potentiel

σ is the electrical conductivity tensor

Direct model

$$\begin{array}{l} \nabla.(\sigma\nabla\Phi) = 0 \text{ in } \Omega \\ \frac{\partial\Phi}{\partial n} = 0 \text{ on } \Gamma_T \\ \Phi = \Phi_E \text{ on } \Gamma_E \end{array} \quad (1)$$

Well-posed problem

Inverse model

$$\begin{array}{l} \nabla.(\sigma\nabla\Phi) = 0 \text{ in } \Omega \\ \frac{\partial\Phi}{\partial n} = 0 \text{ on } \Gamma_T \\ \Phi = \Phi_T \text{ on } \Sigma \subset \Gamma_T \end{array} \quad (2)$$

Unique solution but not continuous : ill-posed problem

Discretisation by Boundary Element Methods

Well-suited for homogeneous and isotropic case (σ scalar and constant)
n nodes on Γ_E and $m \geq n$ nodes on Γ_T
x_1 : discretisation of Φ_E
x_2 : discretisation of $\frac{\partial \Phi_E}{\partial n}$
y : discretisation of Φ_T

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y \end{pmatrix} = 0 \tag{3}$$

Dense small linear system

Discretisation by Finite Element Methods

Well-suited for heterogeneous or anisotropic case (σ tensor)
n nodes on Γ_E, N nodes in Ω and $m \geq n$ nodes on Γ_T
x_1 : discretisation of Φ_E
x_2 : discretisation of Φ in Ω
y : discretisation of Φ_T

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y \end{pmatrix} = 0 \tag{4}$$

Sparse large linear system (N large)

Direct problem

In BEM, we set $N = n$

Square linear system of order $N + m$

$$\begin{pmatrix} A_{12} & A_{13} \\ A_{22} & A_{23} \end{pmatrix} \begin{pmatrix} x_2 \\ y \end{pmatrix} = \begin{pmatrix} -A_{11} \\ -A_{21} \end{pmatrix} x_1 \tag{5}$$

which can be solved by

$$\begin{array}{l} (A_{23} - A_{22}A_{12}^{-1}A_{13})y = (A_{22}A_{12}^{-1}A_{11} - A_{21})x_1 \\ x_2 = -A_{12}^{-1}(A_{11}x_1 + A_{13}y) \end{array} \tag{6}$$

Transfer matrix : $y = Tx_1$, $T \in \mathbb{R}^{m \times n}$

Inverse problem

$N+n$ unknowns and $N+m$ equations : least-squares problem
y : measure of potential on the torso : errors
$y+e$ with e blank noise $(0,\mu^2 I)$

Approach used in most papers and software

$$\min_{x_1} \|y - Tx_1\|$$

Our proposal

$$\min_{x,e} \|e\| \text{ subject to } Ax + Be = By$$

$x = (x_1,x_2) \in \mathbb{R}^{n+N}$, $y \in \mathbb{R}^m$
$A \in \mathbb{R}^{(N+m)\times(n+N)}$, $B \in \mathbb{R}^{(N+m)\times m}$

General Gauss-Markov linear model

Regularisation

Algorithms for general linear models

Generalised Singular Value Decomposition GSVD
Generalised QR factorisation GQR (Paige 's algorithm)
Iterative methods of type LSQR?

Regularisation for discrete ill-posed problem

$$\min_{x,e}(\|e\|^2 + \lambda^2\|Cx\|^2) \text{ subject to } Ax + Be = By$$

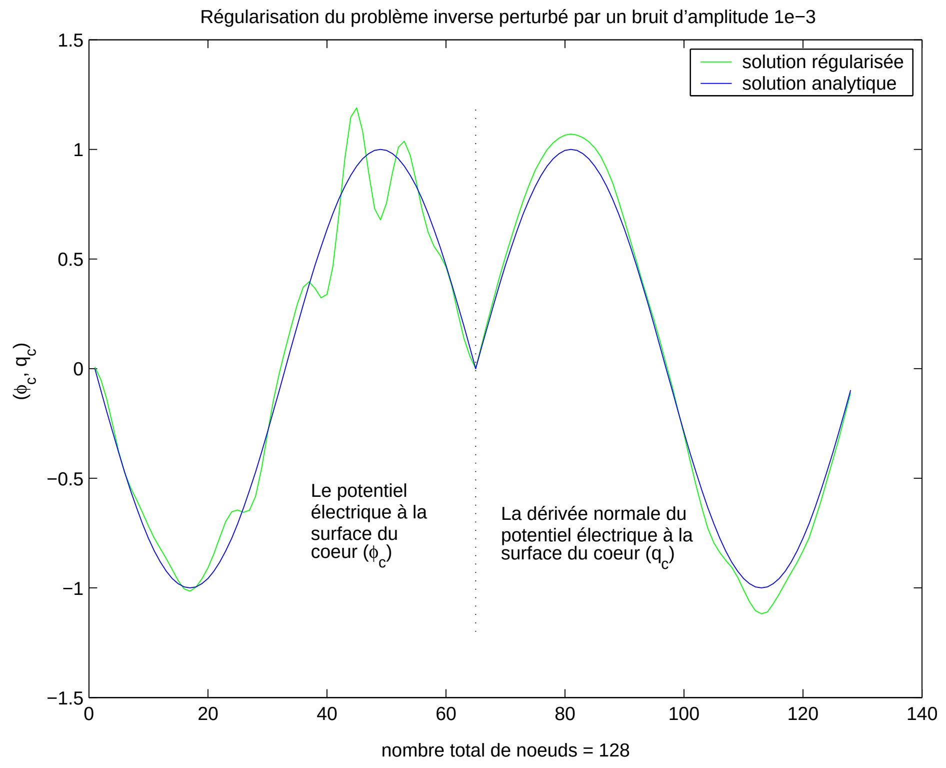
Ref : H. Zua and P.C. Hansen, 1990
Restricted SVD
Algorithm similar to GQR
Iterative methods of type LSQR?

Some very preliminary results

Discretisation by BEM - homogeneous and isotropic case - 2D

Linear model $\min_x \|z - Bx\|$ with $z = By$

Tychonov regularisation and parameter selection using an L-curve



Time dependent problem

Time dependent model

Measures during a time interval
Time discretisation $(t_1, \dots, t_p)$
Measure $Y = (y(t_1), \dots, y(t_p))$
Unknown $X = (x(t_1), \dots, x(t_p))$
Matrices A and B independent of time

$$AX = B(Y + E)$$

where $A \in \mathbb{R}^{(N+m)\times n+N}$, $B \in \mathbb{R}^{(N+m)\times m}$, $X \in \mathbb{R}^{(n+N)\times p}$, $Y \in \mathbb{R}^{m\times p}$

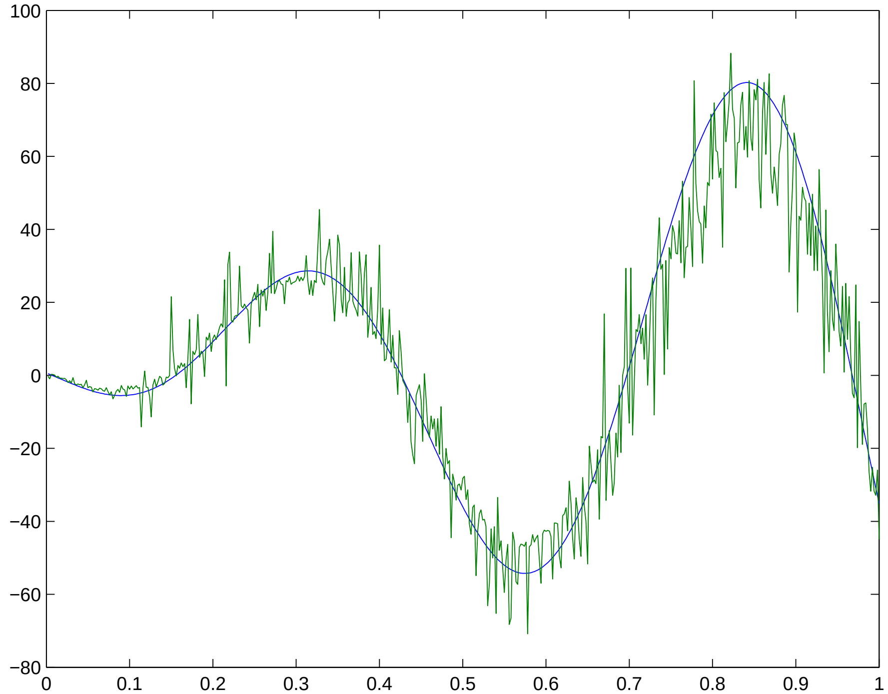
$$(I_p \otimes A)vec(X) = (I_p \otimes B)(vec(Y) + vec(E))$$

Case of total blank noise

p independent problems

$$Ax(t_k) = B(y(t_k) + e(t_k)),\ k = 1, \dots, p$$

Some very preliminary results



Same algorithm as before for each time step

Time dependent regularisation

Approach based on the SVD of Y with $m \leq t$

Ref : F. Greensite and G. Huiskamp, 1998

$Y = P(S\ 0)Q^T$ and $Q = (Q_1\ Q_2)$
$AXQ_1 = PS + EQ_1$ and $AXQ_2 = EQ_2$

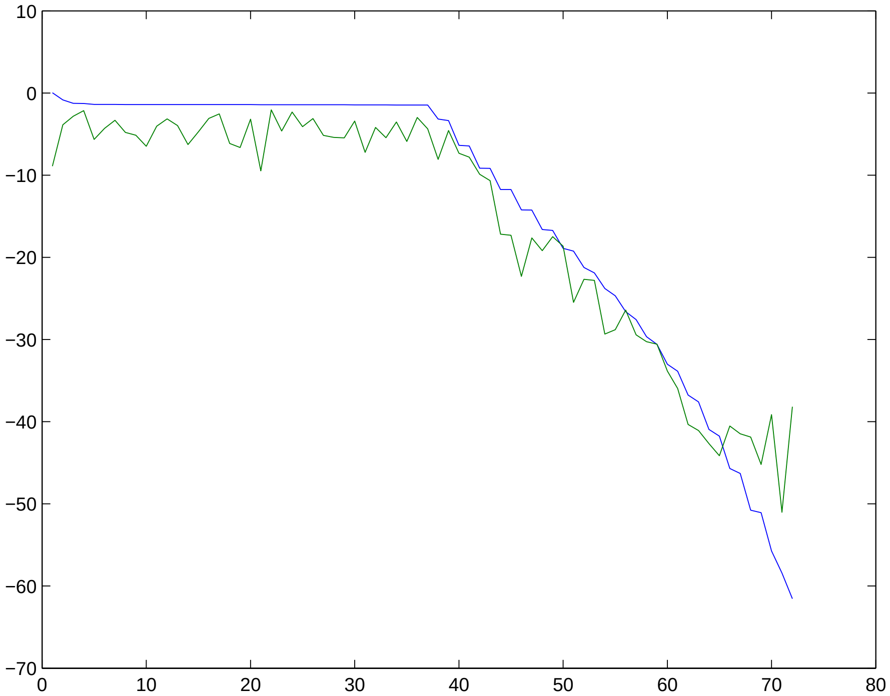
Preliminary method

Solve $\min_Z \|AZ - PS\|$ and take $X = ZQ_1^T$
Solve p independent problems

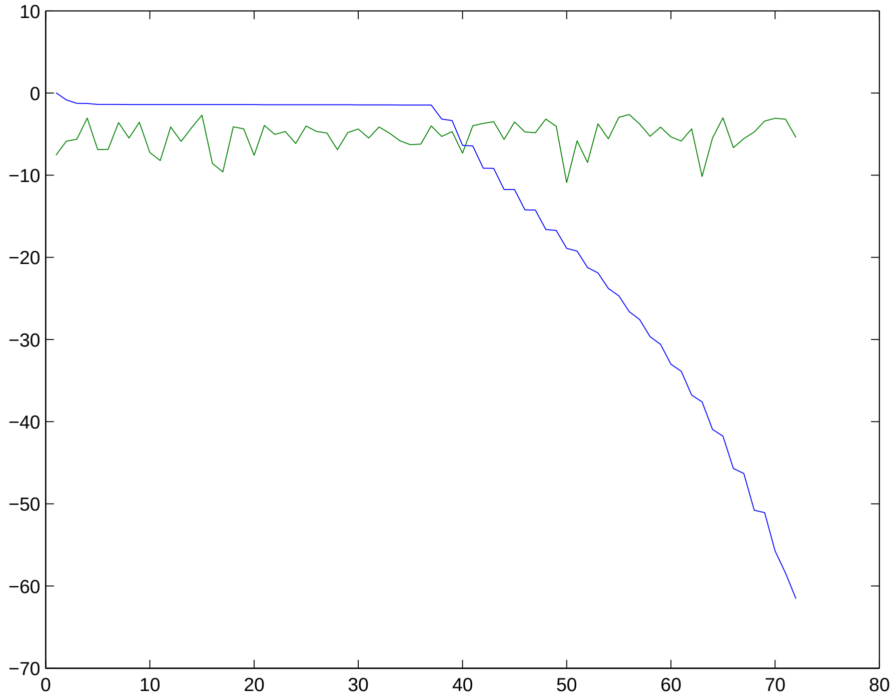
$$\min_{z(t_k)} \|Az(t_k) - (PS)(t_k)\|,\ k = 1, \ldots, p$$

Regularisation for each $(PS)(t_k)$
Solution discarded if the Discrete Picard condition is not satisfied

Some very preliminary results

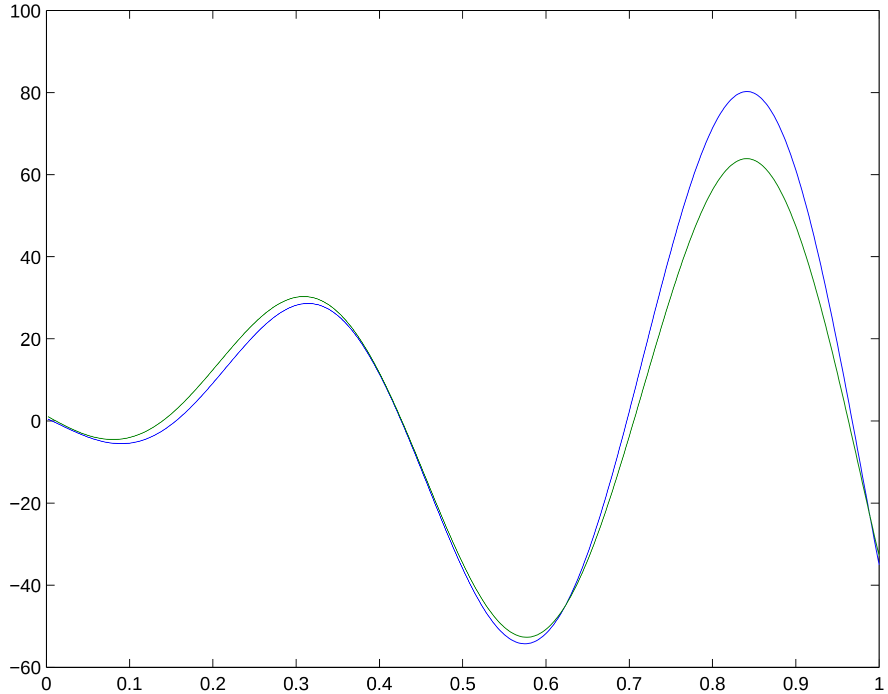


Discrete Picard condition satisfied



Discrete Picard condition not satisfied

Some very preliminary results



Solution using time regularisation

Some perspectives

- Regularisation of general linear model
- Analysis of time regularisation
- General time regularisation
- Comparison with the classical approach