Positional determinacy of games with infinitely many priorities

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(joint work with Igor Walukiewicz)

Quite a while ago Jonathan Swift reported on serious disagreements between the Big-Endians and the Little-Endians, two factions within the society of Lilliput, on a matter of fundamental importance: Big-Endians break their eggs at the larger end ("the primitive way"), while Little-Endians break it at the smaller end, and doing so support the view of the Emperor who had *"published an edict, commanding all his subjects, upon great penaltys, to break the smaller end of their eggs."* The issue was very serious indeed because the Big-Endians *"so highly resented this law, that our historys tell us there have been six rebellions raised on that account; wherein one Emperor lost his life, and another his crown."* [1, Part I, Chapter IV].

The fights between Big-Endians and Little-Endians continued and became even more vigorous when, with the advent of modern computing machines, new issues arose: should numbers be stored most or least significant byte first? Big-endians insist that the "big end" is stored first because this is the way God intended integers to be represented, most important part first. Those in the big-endian camp include the Java VM virtual computer, the Java binary file format, the IBM 360 and and most mainframes. Little-Endians assert that putting the low-order part first is more natural because when you do arithmetic, you start at the least significant part and work toward the most significant part. In the little-endian camp are the Intel 8080, 8086, 80286, Pentium and follow ons and the MOS 6502.

A yet more serious battle between the two factions arose when the people of Lilliput abandonded their traditional games like chess, soccer, Russian roulette, etc. and started to play *parity games.* The little-endian rules for playing parity games assert, quite reasonably, that the smallest priority occurring infinitely often in a play determines the winner. The Big-Endians on the other side insist, strangely enough, that it is the largest priority seen infinitely often that decides who wins the game.

The goal of this paper is to settle this issue once and for all and to prove that the Little-Endians are right. One method to do so would of course be to show the existence of an egg that can only be broken at the smaller end. However, it can be proved that the size of such an egg would necessarily have to be infinite; hence at least a constructive proof seems out of reach for present methodology. Instead we resolve this issue by way of parity games with infinitely many priorities. Whatever arguments there are between Little-Endians and Big-Endians, both factions agree that the right way to win parity games is via *positional strategies* or, if these are unavailable, *finite-memory strategies*, because the use of infinite memory *"is just not fair"*. We prove here that any parity game with priorities in ω, defined in the little-endian style, can indeed be won be means of a positional winning strategy. The Big-Endians, however, miserably fail on their variant of parity games, as soon as they admit infinitely many priorities.

References

[1] J. Swift, *Gulliver's Travels*, George Routledge & Sons, London, 1906. First published in 1726.