On-line Construction of Compact Suffix Vectors and Maximal Repeats

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Journées Montoises

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Plan

Motivation

Detecting repeats in long biological sequences.

Adapted index structure.

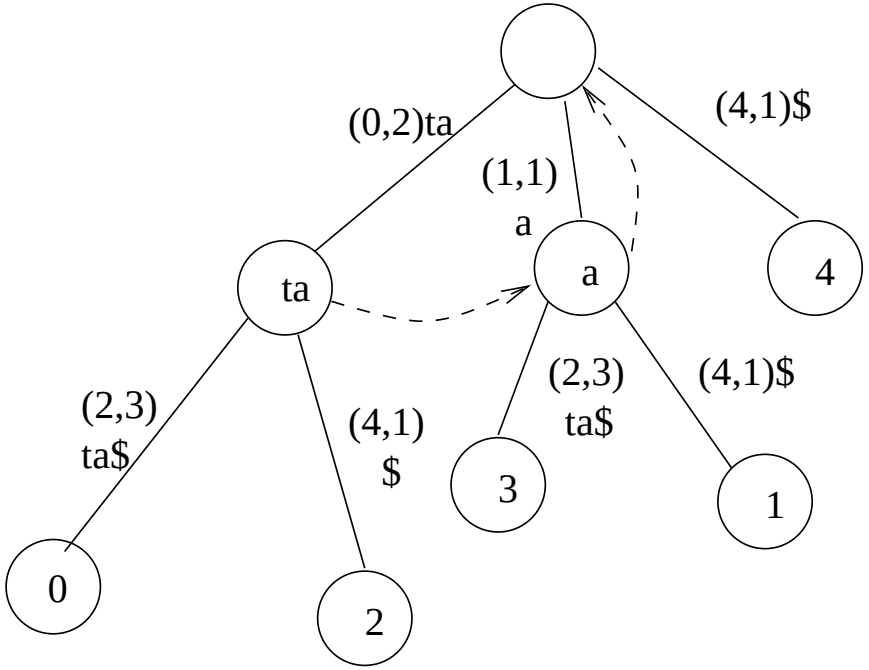
Notations

y is a sequence of length n on the alphabet A.
$ is a terminator symbol.

Suffix tree

- index structure;
- all substrings represented;
- edges labeled (begin position, length);
- leaves represent suffixes.

Suffix tree of `tata$`



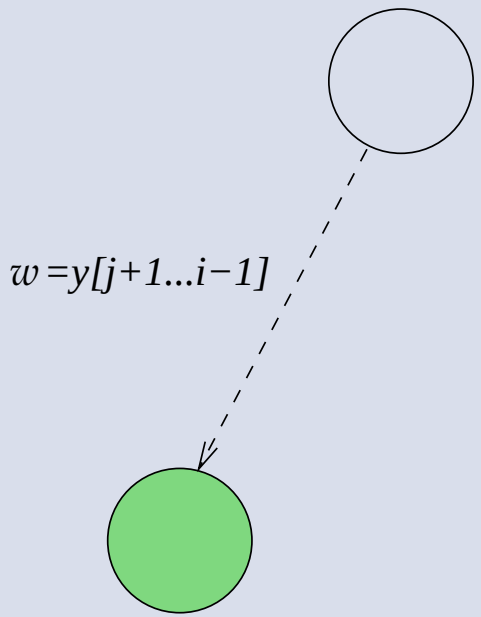
Ukkonen's algorithm

- On-line algorithm
- Construction split into n phases which are also split into extensions.
- During the phase i, construction of the implicit tree of $y[0..i]$ from the one of $y[0..i-1]$.
- During the extension j of the phase i, the suffix $y[j+1..i]$ is added to the tree.
- The last added substring is $w = y[j+1..i-1]$.

The 3 rules

Ukkonen's algorithm is based on 3 rules expressed by Gusfield[1]:

Rule 1



[1] *Algorithms on Strings, Trees and Sequences: Computer Science and Computational Biology*, Cambridge University Press, 1997

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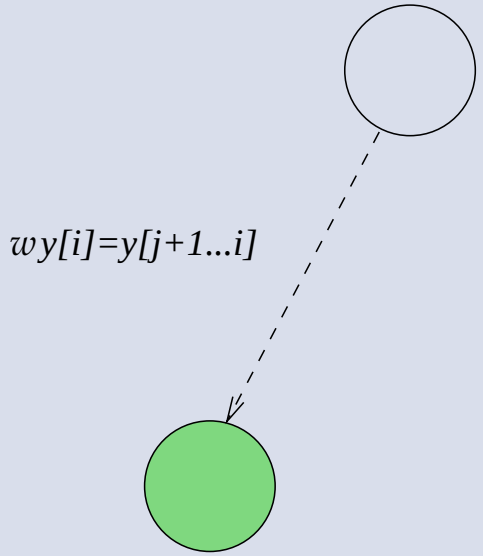

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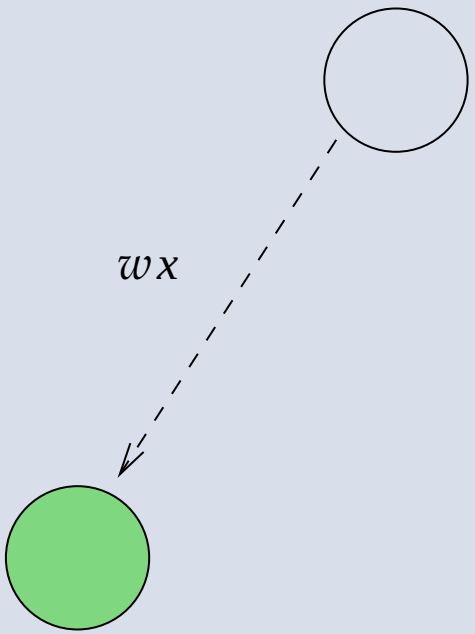
Rule 2



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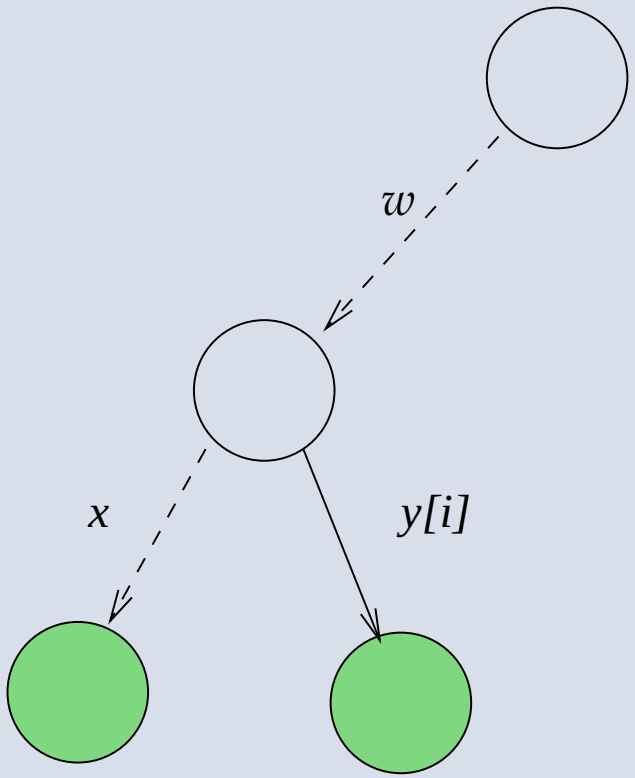

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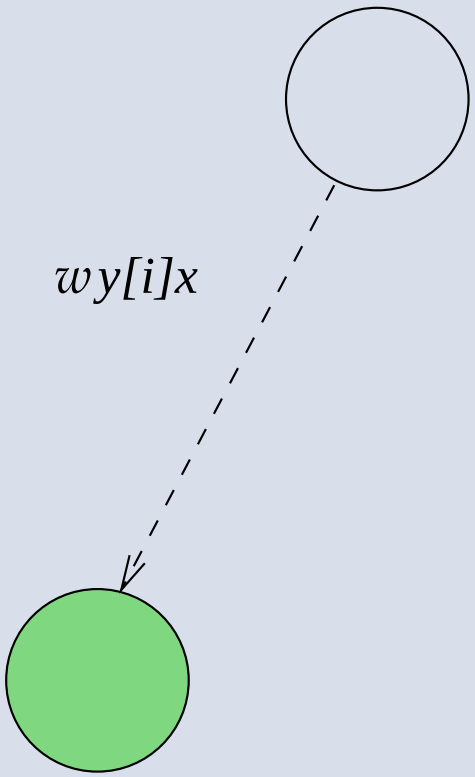
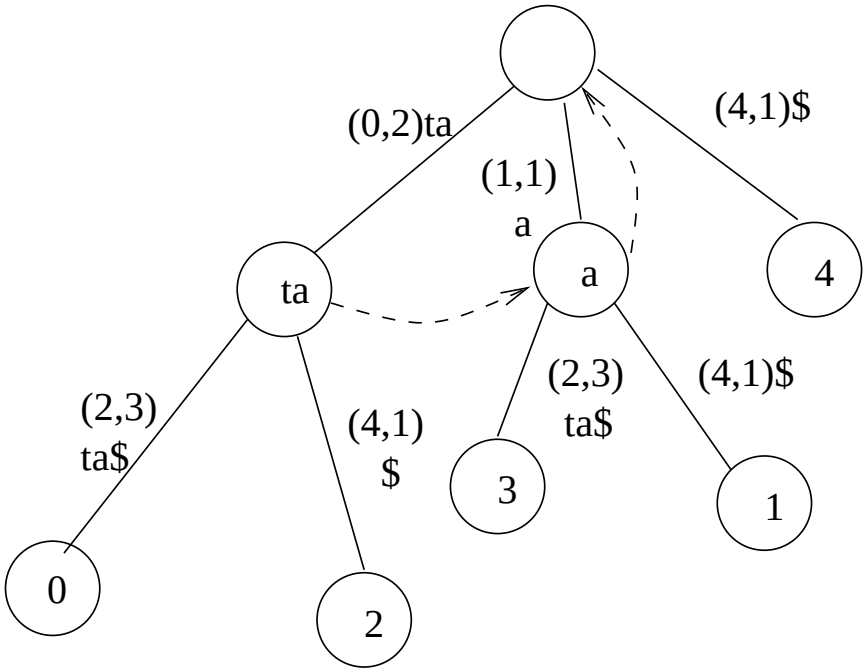


Some properties

- leaves are added in increasing order;
- rule 1 does not need any treatment;
- phase i begins at the extension $j_\ell + 1$, where j_ℓ is the number of the last created leaf;
- phase i ends at the first extension $j > j_\ell$ such that rule 3 is applied.

Introduction to suffix vectors



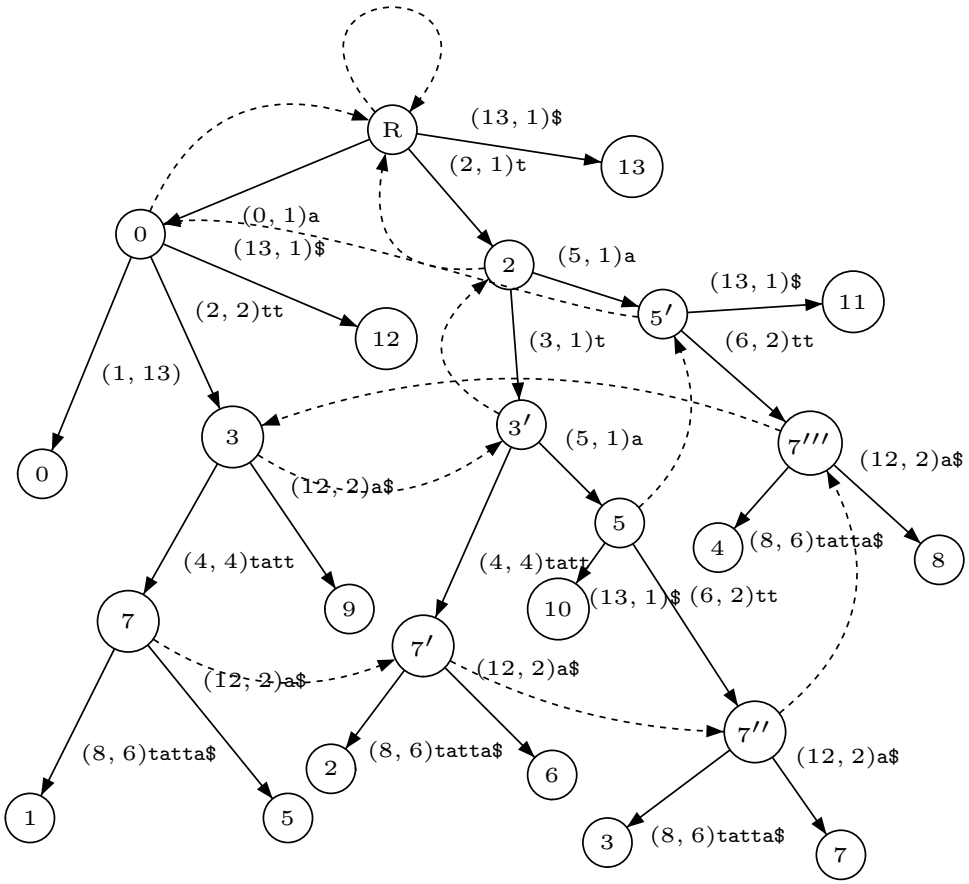
Root

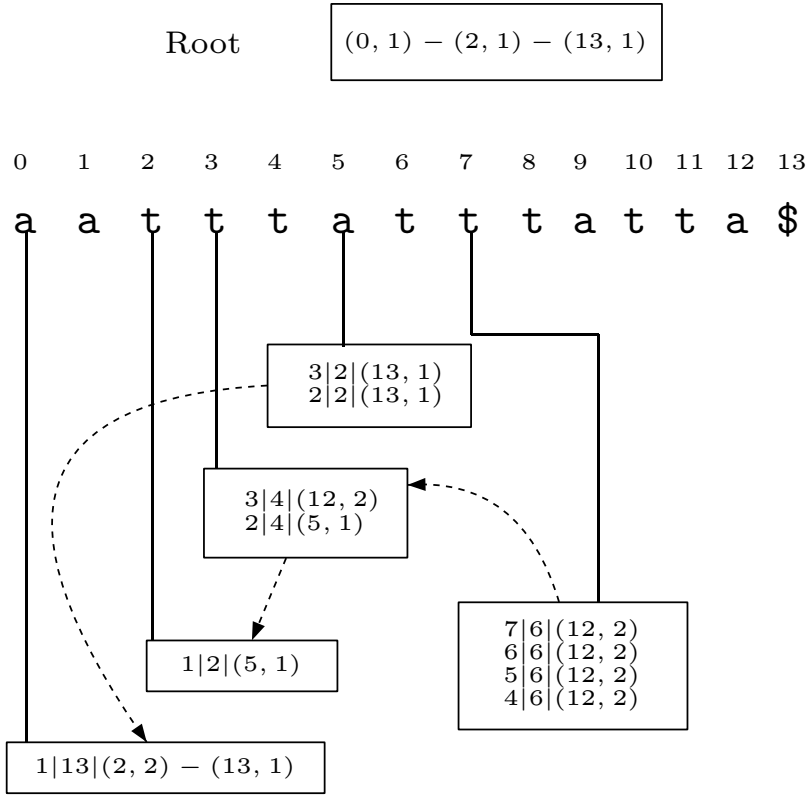
(0, 2) – (1,1) – (4, 1)

0	1	2	3	4
t	a	t	a	$

2	3	(4,1)
1	3	(4,1)

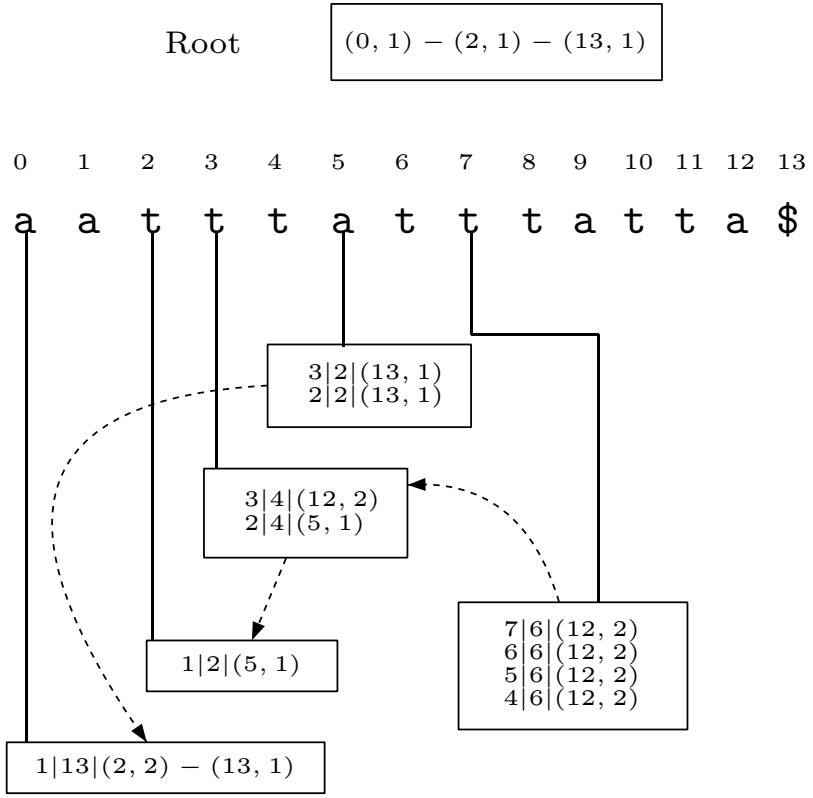
Introduction to suffix vectors





Introduction to suffix vectors

- Alternative data structure to suffix trees
- same information in reduced space
- introduced by K. Monostori in 2001



Introduction to suffix vectors

Definition

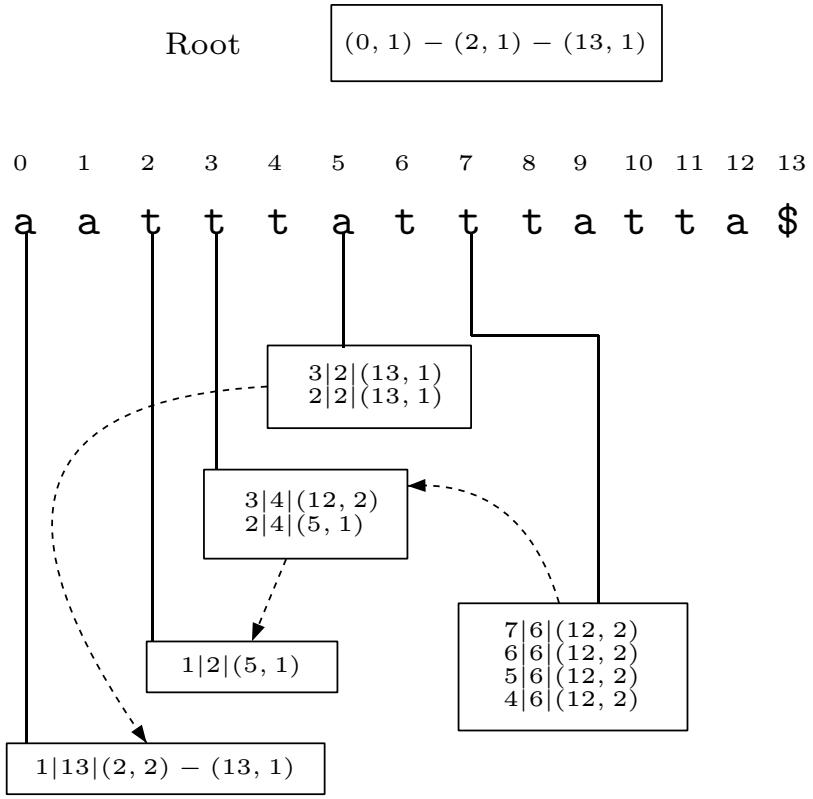
A succession of boxes whose lines contain:

- the depth of the node;
- the natural edge;
- the edge list.

The root is a special box.

Notations

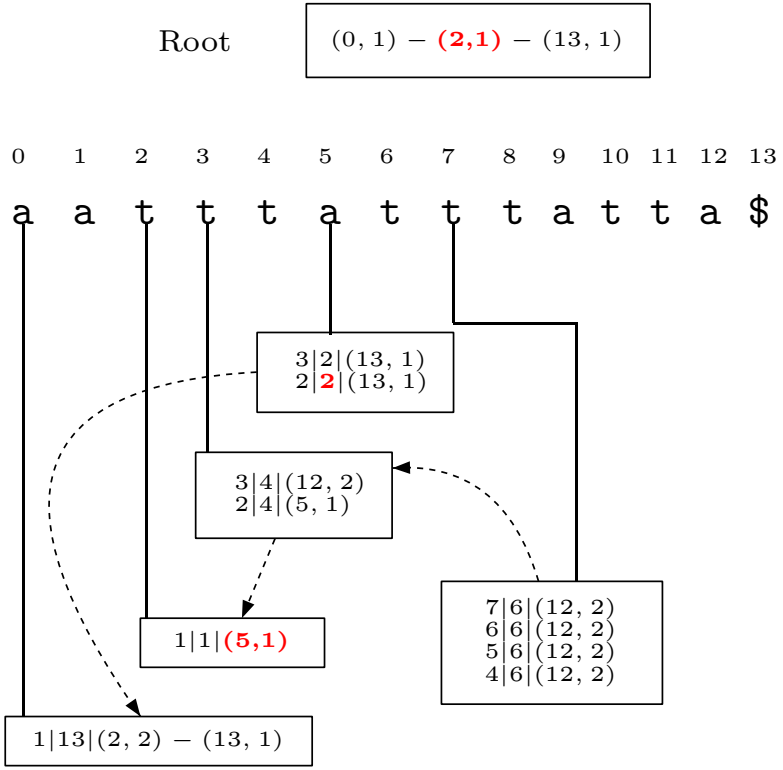
- B_j: box at position j in y,
- The natural edge of a line in B_j is the end position of the edge beginning by $y[j+1]$.



Introduction to suffix vectors

Example

tatt is a substring of y ?
The root contains the edge $(2, 1)$ beginning by **t** leading to B_2.
The edge $(5, 1)$ by **a** leads to B_5.
The natural edge begins by **tt**.



Compact a vector

Definition

A *group of nodes* is a set of nodes which are in the same box and have exactly the same edges.

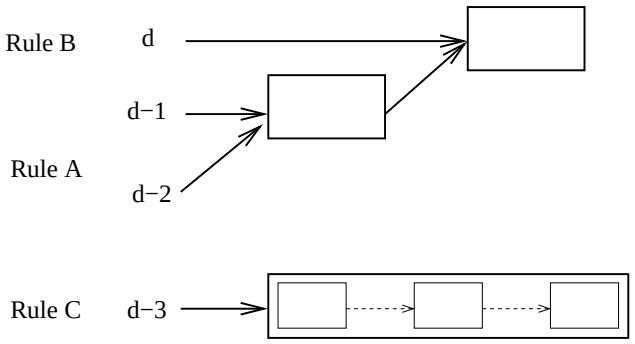
Compact suffix vectors

3 rules of compaction of a box:

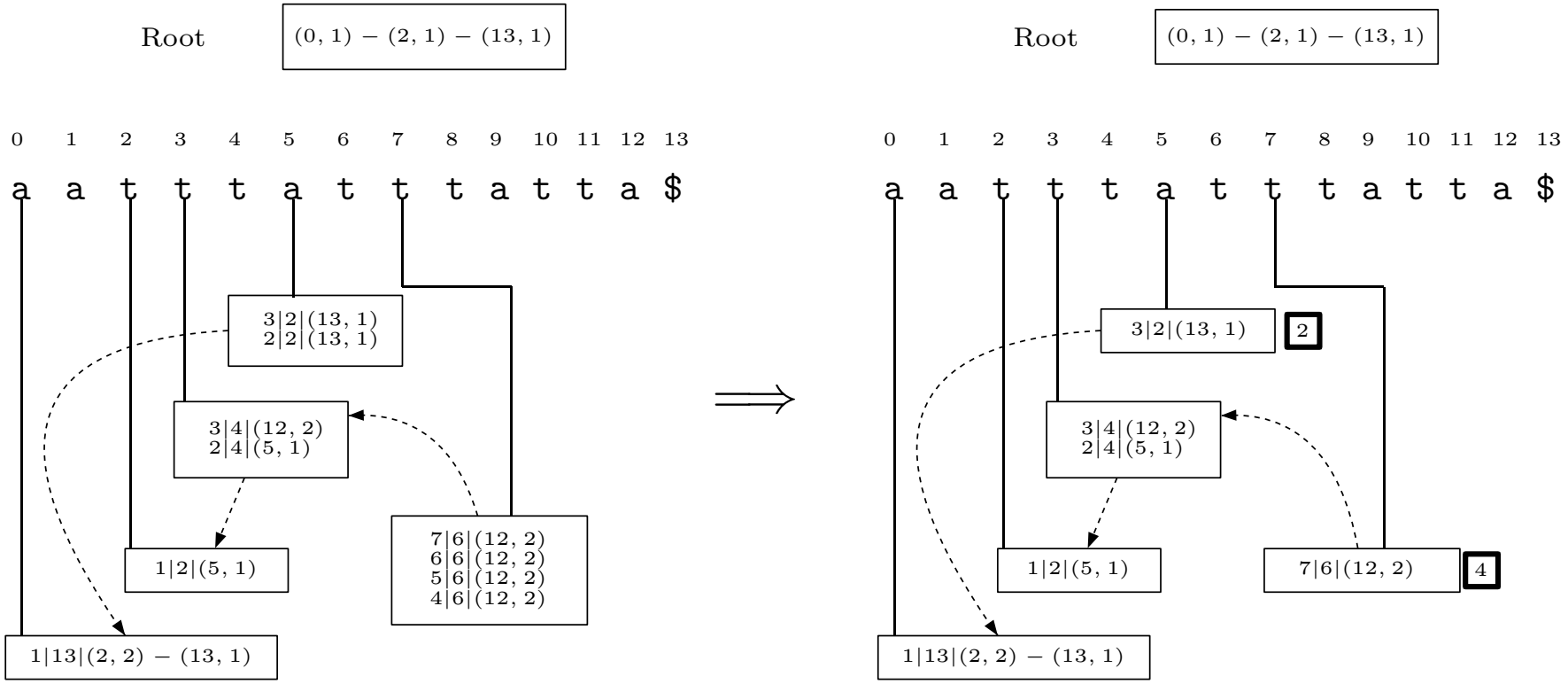
Rule A the node with depth $d-2$ has the same edges as the node with depth $d-1$,

Rule B the node with depth $d-1$ has the same edges as the node with depth d and some extra edges,

Rule C the node with depth $d-3$ has different edges to the node with depth $d-2$.

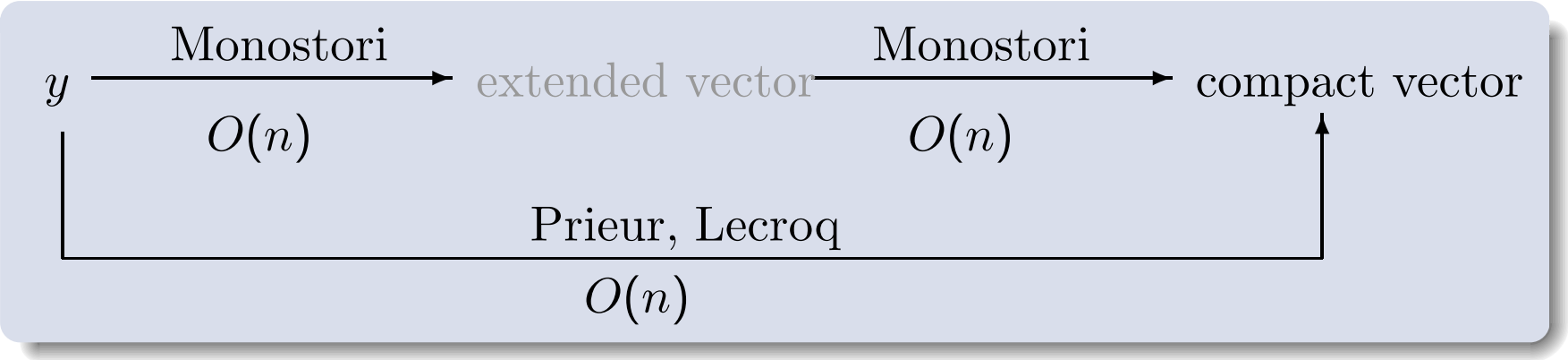


Compacting $\mathcal{V}$(aatttatttatta$)



$$y \xrightarrow[O(n)]{\text{Monostori}} \text{Extended vector} \xrightarrow[O(n)]{\text{Monostori}} \text{Compact vector}$$

On-line construction of a compact vector

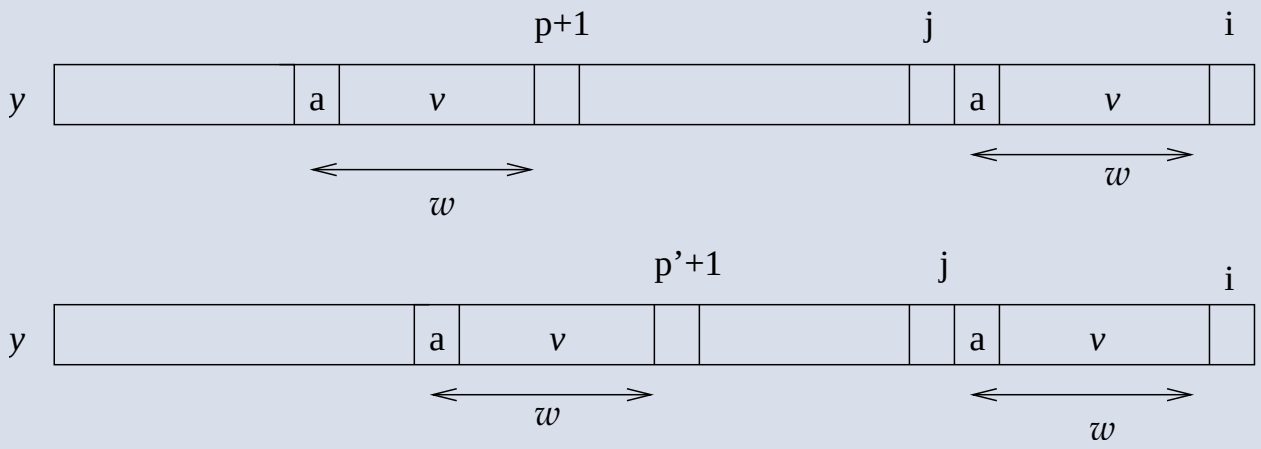


Faster and more space economical construction.

On-line construction of a compact vector

Proposition

When an edge is added to the node w of depth d in a box B_p, this edge will be added to all the nodes in B_p of depth smaller then d in the group of nodes of w.



On-line construction of a compact vector

Skip $k-1$ extensions where k is the number of the nodes in the group into the edge is added.

Definition

A maximal repeat in a string is a substring such that there exist at least 2 occurrences : a_1ub_1 and a_2ub_2 with $a_1 \neq a_2$, $b_1 \neq b_2$ and $a_1, a_2, b_1, b_2 \in A$.

Example

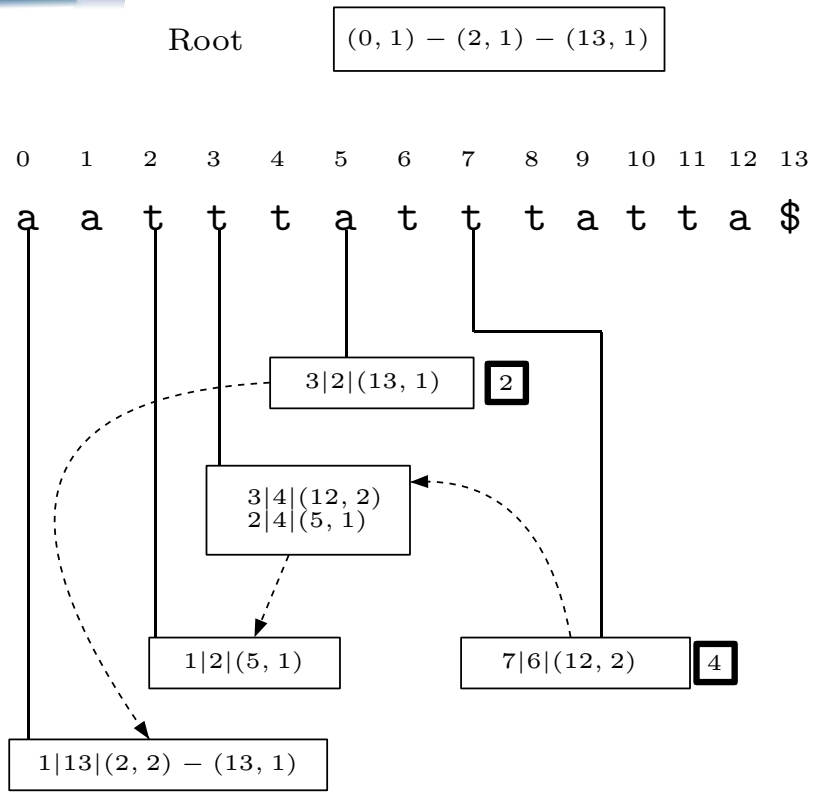
$y =$aatttatttatta\$

tta is a maximal repeat at positions 5 and 12.

Applying to suffix vectors

Proposition

The deepest node of each group of nodes represents a maximal repeat.



Example

Boxes 0, 2, 5 et 7 are reduced:
`a, t, tta, atttatt` are maximal repeats.
Box B_3 is extended, the 2 lines have different edges:
`att, tt` are maximal repeats.

Conclusion

More economical construction of the compact suffix vector.

Linear method to compute maximal repeats with a compact suffix vector.